HDCA Summer School on Capability and Multidimensional Poverty

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Unidimensional Poverty Measurement

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Oxford Poverty & Human Development Initiative (OPHI)
Main Sources of this Lecture

• Foster and Sen (1997), Annexe of “On Economic Inequality”.
• Foster (2006) “Poverty Indices”
• There are others: please see the readings list.
Preliminaries
Preliminaries

• Single dimension
  – Income (referred as ‘achievement’)

• Achievements of a society or a country can be represented by a vector or a distribution

• Unit of analysis may be individual or household
Achievement Vector

Suppose there are four persons in a society with incomes $9, $4, $15 and $8

- Then \( x = (9, 4, 15, 8) \) is a vector representing the incomes of the society

Ordered Achievement Vector

An ordered vector ranks or orders individuals by their achievements

- Ordered vector \( x^{\text{ord}} \) of \( x \) is \( x^{\text{ord}} = (4, 8, 9, 15) \)
Preliminaries

Cumulative Distribution Function (CDF)

The ordered vector of $x = (9, 4, 15, 8)$ can be represented by a cdf. The cdf of distribution $x$ is denoted by $F_x$

$$F_x = \begin{cases} 
0 & \text{for } b<4 \\
1/4 & \text{for } 4 \leq b<8 \\
2/4 & \text{for } 8 \leq b<9 \\
3/4 & \text{for } 9 \leq b<15 \\
1 & \text{For } b \geq 15
\end{cases}$$
Cumulative Distribution Function (CDF)

For a society with large population size, a typical cdf looks like

What does a CDF tell us?

It tells us the share of the population having income less than a particular income level

e.g., $F_x(4)$ is the share of the population having income less than $4$
A policy maker is generally interested in the following three aspects of a distribution or a vector:

- **Size (Welfare)**, e.g. per-capita income
- **Spread (Inequality)**, e.g. Gini coefficient
- **Base (Poverty)**
  - Welfare of the population below a certain level of income

In this summer school, we focus on the third aspect.
Unidimensional Poverty Measurement
Poverty Measurement

Unidimensional poverty measurement involves two steps (Sen 1976): Identification and Aggregation

Identification: Who is poor?

This step dichotomises the population into poor and non-poor.

The main tool: Poverty Line (z)

Person $i$ is poor if $x_i < z$ and is non-poor if $x_i \geq z$

$x_i$ is the $i^{th}$ element of vector $x$
Types of Poverty Lines

**Absolute Poverty Line** \((z_a)\): Does not depend on the size of the entire distribution. Rather usually based on the cost of a set of goods and services considered necessary for having a satisfactory life.

*Example*: a food poverty line: 2100 calories a day equivalent of consumption expenditure.

**Relative Poverty Line** \((z_r)\): Depends on the size of the entire distribution.

*Example*: half of the median income.

**Hybrid Poverty Line** \((z_h)\): Combinations of absolute and relative poverty lines.

*Examples*: \(z = (z_r)^\rho(z_a)^{(1-\rho)}\) for \(0 < \rho < 1\) (Foster, 1998);

\[z = \max(z_a, \alpha + kM),\] with \(M\) being the median income, (Ravallion and Chen, 2009).
Significance of Poverty Line (z)

Poverty line enables policy makers to **identify** a group of people, who are subject to different social assistance programs or ‘**targeting**’

Poverty line as ‘**a benchmark**’: the objective of a policy maker should be to bring the poor at z

– For poverty analysis, thus, additional achievement of the non-poor **above** the poverty line is ignored
Significance of Poverty Line (z)

Censored Distribution of Achievements

Having z as benchmark, allows us to create a censored distribution of x, referred as $x^*$, where

$$x_i^* = \begin{cases} x_i & \text{if } x_i < z \\ z & \text{if } x_i \geq z \end{cases}$$

**Example**: If $z = 10$ and $x = (9, 4, 15, 8)$, then $x^* = (9, 4, 10, 8)$
Second Step: Aggregation

**Aggregation:** *How poor is the society?*

This step constructs an index of poverty summarizing the information in the censored achievement vector $x^*$. For each distribution $x$ and poverty line $z$, $P(x;z)$ or $P(x^*)$ indicates the level of poverty in the distribution.

We will adopt an absolute $z$ approach and focus the discussion in terms of the indices.
Axioms
Axioms
(Classification of Foster, 2006)

- Invariance Axioms
- Dominance Axioms
- Continuity
- Subgroup Axioms (Consistency and Decomposability)
Invariance Axioms

Symmetry (Anonymity): If vector $y$ is obtained from vector $x$ by a permutation of incomes and the poverty line remains unchanged, then $P(y;z) = P(x;z)$

$y$ is obtained from $x$ by a permutation of incomes if $y = Px$, where $P$ is a permutation matrix.

Example: $z = 10$, $x = (9, 4, 15, 8)$ and $y = (9, 15, 4, 8)$

Why is this axiom important?
Invariance Axioms

**Permutation Matrix**

A square matrix with entries 0 or 1, with rows and columns summing up to one

**Example:**

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{pmatrix}
\]
Invariance Axioms

Replication Invariance (Population Principle): If vector \( y \) is obtained from vector \( x \) by a replication and the poverty line remains unchanged, then \( P(y;z) = P(x;z) \)

\( y \) is obtained from \( x \) by a replication if the incomes in \( y \) are simply the incomes in \( x \) repeated a finite number of times.

Example: \( z = 10, \ x = (9,4,15,8), \ y = (9,9,4,4,15,15,8,8) \)

Why is this axiom important?
Invariance Axioms

**Focus:** If $y$ is obtained from $x$ by an increment to a non-poor person’s income and the poverty line remains unchanged, then $P(y;z) = P(x;z)$

**Example:** $z = 10$, $x = (9,4,15,8)$, $y = (9,4,16,8)$

Income of the non-poor person increases from $15$ to $16$, which should not alter a poverty index

Why is this axiom important?
Invariance Axioms

**Scale Invariance (Homogeneity of Zero-Degree):**
If all incomes in vector $x$ and the poverty line $z$ are changed by the same proportion $\alpha > 0$, then $P(\alpha x; \alpha z) = P(x; z)$.

*Example:* $z = 10$ and $x = (9,4,15,8)$; if $\alpha = 2$, then $\alpha z = 20$ and $\alpha x = (18,8,30,16)$

Why is this axiom important?
Invariance Axioms

Normalization: As long as everybody is non-poor in vector $x$ for any poverty line $z$, then $P(x;z) = 0$. In other words, if $\min\{x\} \geq z$, then $P(x;z) = 0$

Example 1: $z = 4$ and $x = (9,4,15,8)$
Example 2: $z = 2$ and $x = (9,4,15,8)$
Dominance Axioms

**Monotonicity:** If \( y \) is obtained from \( x \) by a decrement of incomes among the poor and the poverty line remains unchanged, then \( P(y,z) > P(x,z) \)

*Example 1:* \( z = 10, x = (9,4,15,8); y = (9,4,15,7) \)

*Example 2:* \( z = 10, x = (4,8,9,15); y = (3,8,6,15) \)

Why is this axiom important?
Dominance Axioms

**Transfer**: If $y$ is obtained from $x$ by a progressive transfer among the poor, then $P(y;z) < P(x;z)$.

If income is transferred from a person to another who is not richer than the former, keeping mean income same, the transfer is called a *progressive transfer*

**Example**: $z = 10$, $x = (9,4,15,8)$; $y = (9,5,15,7)$
Dominance Axioms

Transfer: Is there a limit on the amount of transfer in this axiom?

There is a limit on the amount of transfer. Post-transfer income cannot fall below the lower pre-transfer income.

What is the implication of this axiom for non-transferable dimensions?
Continuity

A technical assumption. It prevents poverty measures from changing abruptly for changes in distribution of achievements

*Continuity*: For any sequence $x$, if $x'$ converges to $x$, then $P(x';z)$ converges to $P(x;z)$
Continuity

*Example*: $x' = (9,4,15,8)$ and $z = 10$

Suppose the income of the poorest person with income $4$ is increased gradually.

No change in $H$ is observed until the income reaches $10$ [$x = (9,10,15,8)$]

Then, $H$ falls suddenly from $3/4$ to $2/4$ without taking any intermediate value.
Subgroup Axioms

**Subgroup Consistency:** If \( P(y';z) > P(x';z) \) and \( P(y'';z) = P(x'';z) \), and \( n(x') = n(y') \), \( n(y'') = n(x'') \), then \( P(y;z) > P(x;z) \)

**Example:** Let \( z=10 \), \( x=(9,4,15,8) \), \( x'=(9,4) \), \( x''=(15,8) \)

Say \( y' \) is obtained from \( x' \) such that \( y'=(6,4) \), while \( y'' \) is obtained from \( x'' \) such that \( y'' = x'' \)

Then \( P(y';z) > P(x';z) \) by *monotonicity*, but \( P(y'';z) = P(x'';z) \), and so one would expect \( P(y;z) > P(x;z) \)
Subgroup Consistency

For which practical reason may this axiom be important?

– Evaluation of poverty reduction programs!
– It can be seen as an extension of monotonicity

Monotonicity requires poverty to fall when one person’s poverty level is reduced. SC requires aggregate poverty to fall when one group’s poverty level is reduced.

However, a reduction in one group’s poverty may be accompanied by both increase and fall in individual incomes.
Subgroup Axioms

**Additive Decomposability**: A poverty measure is additive decomposable if:

\[ P(x) = \frac{n(x')}{n} P(x') + \frac{n(x'')}{n} P(x'') \]

(Extendable to any number of groups)

Then, one can calculate the contribution of each group to overall poverty:

\[ C(x') = \frac{n(x')P(x')}{nP(x)} \]

*Additive decomposability implies subgroup consistency, but the converse does not hold*
P is a continuous, subgroup consistent poverty index if and only if P is a continuous, increasing transformation of a continuous, decomposable poverty index.

(Foster and Shorrocks, 1991)
Other Advanced Axioms

Transfer Sensitivity: If a transfer \( t > 0 \) of income takes place from a poor person with income \( x_i \) to a poor person with income \( x_i + d \), then the magnitude of the increase in poverty must be smaller for larger \( x_i \). (Kakwani, 1980)

In other words, a poverty measure that satisfies transfer-sensitivity places greater emphasis on progressive (regressive) transfers at the lower end of the distribution of poor than at the upper end of the distribution of poor.
Poverty Measures
Classification of Measures

Basic Measures

- Headcount Ratio
- Income Gap Ratio
- Poverty Gap Ratio

Advanced Measures

- Squared Poverty Gap (Foster-Greer-Thorbecke)
- Sen-Shorrocks-Thon Measure
- Watts Measure
- Clark-Hemming-Ulph-Chakravarty Class of Measures
Basic Poverty Measures

The Headcount Ratio (H)

The most commonly and widely used measure of poverty

It reports the proportion of the population that is poor

Thus, ranges between 0 and 1

If \( q \) is the number of poor in vector \( x \) with population size \( n \), then \( H = q/n \)

\( Example: \) Let \( z = 10 \) and \( x = (9,4,15,8) \), then \( H = 3/4 \)
Basic Poverty Measures

The Headcount Ratio (H)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, scale invariance, focus, normalization, and subgroup consistency

Does not satisfy – monotonicity, transfer, transfer sensitivity, continuity

Policy Implication?

Encourages a policy maker, with limited budget, to assist the marginally poor only instead of the severely poor
Basic Poverty Measures

Income Gap Ratio (I)

It reports the average normalized income shortfall of the poor from the poverty line; *ranges between 0 and 1*

The average normalized income shortfall of the $i^{th}$ poor is $(z - x_i)/z$. The average income shortfall of the poor is:

$$I = \frac{1}{q} \sum_{q} (z - x_i)/z = \frac{(z - \mu_p)}{z}$$

*Example:* $x=(9,4,15,8); z=10; \mu_p=(4+8+9)/3=7;$

Thus, $I = (10 - 7)/10 = 0.3$
Basic Poverty Measures

Income Gap Ratio (I)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, scale invariance, focus, normalization, monotonicity and subgroup consistency

Does not satisfy –transfer, transfer sensitivity, continuity

Policy Implication?

Counter intuitive: if a poor person’s income increases and the person becomes non-poor, poverty increases!
Basic Poverty Measures

Poverty Gap Ratio (PG)

This measure repairs some of the problems of the headcount ratio and income gap ratio.

It reports the average normalized income shortfall from the poverty line using the *censored* distribution $x^*$.

The average normalized shortfall of the $i^{th}$ person is $g_i^* = (z - x_i^*)/z$. The average income normalized shortfall:

$$PG = \frac{1}{n} \sum g_i^* = \frac{z - \mu^*}{z} = H \times I$$
Basic Poverty Measures

Poverty Gap Ratio (PG)

Example: \( x=(9,4,15,8) \); \( z=10 \). Then \( x^*=(9,4,10,8) \) and \( g^*=(0.1,0.6,0,0.2) \). So, \( PG = 0.9/4 = 0.225 \)

Alternatively, \( \mu^* = \) average of elements in \( x^* \). So, \( \mu^* = 7.75 \). Thus, \( PG = (10 – 7.75)/10 = 0.225 \)

PG ranges between 0 and 1.

1: when everybody is poor with no income at all
0: when there is no poor
Basic Poverty Measures

Poverty Gap Ratio (PG)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, scale invariance, focus, normalization, monotonicity, continuity and subgroup consistency

Does not satisfy – transfer, transfer sensitivity

Policy Implication?

Does not encourage a policy maker to distinguish between a marginally poor and severely poor while assisting
Advance Poverty Measures

Squared Poverty Gap (SG)

It reports the average of squared normalized income shortfalls from the poverty line using the censored distribution $x^*$. Also, known as Foster-Greer-Thorbecke measure.

The average normalized shortfall of the $i^{th}$ person is $g_i^* = (z - x_i^*)/z$. The average of squared normalized income shortfalls:

$$SG = \frac{1}{n}\sum_{i=1}^{n} (g_i^*)^2$$
Advance Poverty Measures

Squared Poverty Gap (SG)

*Example:* $x=(9,4,15,8)$; $z=10$. Then $x^*=(9,4,\mathbf{10},8)$ and $g^*=(0.1,0.6,0,0.2)$ and squares of poverty gap are $sg^*=(0.1^2,0.6^2,0^2,0.2^2)=(0.01,0.36,0,0.04)$. Thus, $SG = 0.41/4 = 0.102$.

SG *ranges between 0 and 1*.

1: when everybody is poor with *no income* at all
0: when there is no poor
Advance Poverty Measures

Squared Poverty Gap (SG)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, transfer, and subgroup consistency

Does not satisfy –transfer sensitivity

This measure can be presented as: \( SG = H[I^2 + (1 - I)^2 \times C_p^2] \), where \( C_p \) is the coefficient of variation of income across the poor. *Policy implication: care for the severe poor first*
The FGT class of measures is defined as

$$\text{FGT}_\alpha = (1/n) \sum_n (g_i^*)^\alpha$$

where $\alpha$ is a parameter and $g_i^*$ is the normalized income gap of the $i^{th}$ person in $x^*$.

For $\alpha = 0$, FGT is the Headcount Ratio.
For $\alpha = 1$, FGT is the Poverty Gap Ratio.
For $\alpha = 2$, FGT is the Squared Poverty Gap.
Advance Poverty Measures

Sen-Shorrocks-Thon Measure (SST)

Like SG, SST is also sensitive to inequality across the poor and is written as

\[ \text{SST} = \text{HI} + (1 - \text{HI}) \times \text{Gini}(x^*) \]

where \( \text{Gini}(x^*) \) is the Gini Coefficient of the censored distribution

It lies between zero and one.
Advance Poverty Measures

Sen-Shorrocks-Thon Measure (SST)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, and transfer

Does not satisfy – transfer sensitivity, subgroup consistency

Not subgroup consistent means it is possible that poverty of a region *falls* and that of other remains *unchanged* but the overall poverty of the society as a whole *increases*
Advance Poverty Measures

Watts Measure (W)

Watts (1968) proposed the following measure of poverty

\[ W = \frac{1}{n} \sum_{i} (\ln z - \ln x_i^*) = \frac{1}{n} \sum_{i} \ln \left( \frac{z}{x_i^*} \right) \]

where \( \ln \) stands for *natural logarithm*

The lower bound of the measure is zero, it does not have any upper bound
Advance Poverty Measures

**Watts Measure (W)**

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, transfer, transfer sensitivity, and subgroup consistency

Also, Additively Decomposable

A perfect measure except for few *limitations*. 
Advance Poverty Measures

Watts Measure (W)

Limitations?

1. It is only defined for positive income, zero incomes are not allowed
2. It does not have nice intuitive interpretation or breakdown like other poverty measures
3. It does not have any upper bound like other measures
The CHUC class of poverty measures are defined as:

\[
C_\beta(x;z) = \begin{cases} 
1 - \left[ \frac{1}{n} \sum_{i=1}^{n} \left( \frac{x_i^*}{z} \right)^\beta \right]^{1/\beta} & \beta \leq 1, \beta \neq 0 \\
1 - \left[ \frac{1}{n} \prod_{i=1}^{n} \left( \frac{x_i^*}{z} \right) \right]^{1/n} & \beta = 0
\end{cases}
\]

For \( \beta = 0 \), monotonic transformation of \( W \)

For \( \beta = 1 \), Poverty Gap Ratio
Advance Poverty Measures

Clark-Hemming-Ulph-Chakravarty Class (CHUC)

What axioms does this measure satisfy?

For $\beta < 1$, satisfies - symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, transfer, transfer sensitivity, and subgroup consistency.

Cons: The intuitive interpretation of the measure for $\beta < 1$ is not so easy
Unidimensional Dominance
Main Sources of this Lecture

• Foster and Shorrocks (1988)
• Foster and Sen (1997), Annexe of “On Economic Inequality”.
• Atkinson (1987)
• There are others: please see the readings list.
When measuring poverty...

Identification → Poverty Line

Aggregation → Poverty Measure

Entail some arbitrariness
Which Poverty Line?

Does the choice of poverty line alters the ranking of distributions? Yes

*Example*: Consider two distributions:

\[ x = (4,8,9,15) \text{ and } y = (3,6,12,17) \]

Which distribution has more poverty by \( H \), if \( z = 7 \)?

\[ H(x;z) = \frac{1}{4} \text{ and } H(y;z) = \frac{2}{4} \]

Which distribution has more poverty by \( H \), if \( z = 10 \)?

\[ H(x;z) = \frac{3}{4} \text{ and } H(y;z) = \frac{2}{4} \]
Which Measure?

Does all measures rank any two distributions in the same manner? No

Example: Consider the same two distributions:

\[ x = (4, 8, 9, 15) \] and \[ y = (3, 6, 12, 17) \]

Which distribution has more poverty by H, if \( z = 10 \)?

\[ H(x;z) = \frac{3}{4} \] and \[ H(y;z) = \frac{2}{4} \]

Which distribution has more poverty by PG, if \( z = 10 \)?

\[ PG(x;z) = 0.225 \] and \[ PG(y;z) = 0.275 \]
Dominance Approach

To verify if one distribution dominates another distribution, do we need to calculate all measures for all possible poverty lines?
– This would be a tedious task!

Is there any useful tool for these purposes?
– Yes. A tool known as **Stochastic Dominance**
– This is closely linked to poverty ordering, where we rank different distributions
Dominance Approach

Two main types of poverty orderings:

1. Variable-line poverty orderings (focus on the identification step)

2. Variable-measure poverty orderings (address aggregation).
Variable-Line Poverty Orderings
(Foster and Shorrocks, 1988)

Main procedure:

1. Choose a measure

2. Identify the condition that two distributions must satisfy so as to be able to say that one has more poverty than the other.
Definition of Poverty Ordering

\[ x P y \text{ if and only if } \]
\[ P(x;z) \leq P(y;z) \text{ for all } z \text{ and } \]
\[ P(x;z) < P(y;z) \text{ for some } z \]

\[ x P y \text{ means that } x \text{ has } \textit{unambiguously less poverty than } y \text{ with respect to poverty index } P. \]
FGT Poverty Orderings

Foster and Shorrocks (1988) developed the conditions of poverty orderings for three members of the FGT family: H, PG and SG.
Poverty Ordering Based on H

Recall the Concept of CDF

For a society with large population size, a typical cdf looks like

What does a CDF tell us?

It tells us the share of the population having income less than a particular income level.

e.g., $F_x(4)$ is the share of the population having income less than $4$.
Recall the Concept of CDF

For a society with large population size, a typical cdf looks like

If the poverty line is $z=4$, then what is $H$?

$H$ is equal to $F_x(4)$ in this situation.

The diagram to the left may be presented as:
Poverty Ordering Based on H

Recall the Concept of CDF

For a society with large population size, a typical cdf looks like

If the poverty line is $z=4$, then what is $H$?

$H$ is equal to $F_x(4)$ in this situation.

The diagram to the left may be presented as:
Poverty Ordering Based on H

For any poverty line \( z \), the CDF of \( x \) gives the headcount ratio

If the \( \text{cdf} \) of another distribution \( y \), \( F_y \), lies nowhere to the right of the \( \text{cdf} \) of \( x \), then \( y \) has no lower headcount ratio than \( x \) for each and every poverty line.
Poverty Ordering Based on H

For any poverty line $z$, the CDF of $x$ gives the headcount ratio

If the CDF of another distribution $y$, $F_y$, lies nowhere to the right of the CDF of $x$, then $y$ has no lower headcount ratio than $x$ for each and every poverty line.

First order Stochastic Dominance (FSD)

$x \text{ FSD } y$
Poverty Ordering Based on H

Example of FSD: Let \( x=(2,4,6,10) \) and \( y=(1,4,5,10) \)

No part of \( y \) lies to the right of \( x \)

Thus, \( x \) FSD \( y \) in this case, which means \( x \) has unambiguously less poverty than \( y \) according to H.
Definition of FSD

For distributions x and y, x FSD y if and only if $F_x(b) \leq F_y(b)$ for all $b$ and $F_x(b) < F_y(b)$ for some $b$, where $b$ is income.

How strong is the FSD result?

If FSD holds, then there is agreement for all continuous poverty measures satisfying symmetry, focus, scale and replication invariance and monotonicity for all $z$. 
What Happens When CDFs Cross?

In this case, FSD does not hold for all z and H no more provides an unambiguous ranking.

Can distributions be ranked in this situation based on other measures for all z?

*Second order stochastic dominance (SSD)*

This requires comparing the area under the CDFs.

The area under a CDF is closely linked to PG (Foster and Shorrocks, 1988).
Poverty Ordering Based on PG

Area B under CDF of x is equal to z times PG
Poverty Ordering Based on PG

Area B under CDF of x is equal to z times PG

\[ x \text{ SSD } y \]
Poverty Ordering Based on PG

How to calculate the area under the CDF?

Example: Let $x=(2,4,6,10)$ and $z = 6$

Area\(=(1/4)[(6-2)+(6-4)]=\)
\[=(B_1+B_2)=1.5\]

$D_x$ is called deficit curve of $x$
Definition of SSD

For distributions x and y, x SSD y if and only if $D_x(b) \leq D_y(b)$ for all $b$ and $D_x(b) < D_y(b)$ for some $b$, where $b$ is income.

How strong is the SSD result?

If SSD holds, then there is agreement for \textit{all} continuous poverty measures satisfying symmetry, focus, scale and replication invariance, monotonicity and \textit{transfer} for all $z$.
What Happens When $D_x$ and $D_y$ Cross?

In this case, SSD does not hold for all $z$ and PG no more provides unambiguous ranking.

Can distributions be ranked in this situation based on other measures for all $z$?

*Third order Stochastic Dominance (TSD)*

This requires comparing the area under deficit curves.

The area under a deficit curve is closely linked to $SG$ (Foster and Shorrocks, 1988).
Poverty Ordering Based on SG

Area B under the deficit curve is equal to $z^2 \times SG$
Poverty Ordering Based on SG

Area B under the deficit curve is equal to \( z^2 \times SG \times \text{TSD}_y \)
Definition of TSD

For distributions x and y, x TSD y if and only if $S_x(b) \leq S_y(b)$ for all $b$ and $S_x(b) < S_y(b)$ for some $b$, where $b$ is income.

How strong is the TSD result?

If TSD holds, then there is agreement for all continuous poverty measures satisfying symmetry, focus, scale and replication invariance, monotonicity, transfer and transfer sensitivity for all z.
Limited Range Poverty Orderings

While deciding the precise value of the poverty line may be difficult, agreement is likely to occur on an interval $Z$.

So now the poverty ordering would be defined as $xP(Z)y$ when

$$P(y;z) \geq P(x;z) \text{ for all } z \text{ in } Z$$

and $> \text{ for some } z \text{ in } Z$

By restricting the values of $z$, the obtained ranking $P(Z)$ will be “more complete” than the $P$ ranking (but less general)
Limited Range Poverty Orderings

Indeed, looking at extremely high poverty lines, does not make sense. So now we can set an upper bound $z^*$, so that the relevant range is $Z^*=(0,z^*)$, and $P^*$ being the poverty ordering.

One can, then, work with the censored distribution, ‘ignoring’ incomes above $z^*$, i.e., replacing them by $z^*$:

$$x_i(z^*) = \min(x_i, z^*) \text{ for } i=1,\ldots,n(x)$$