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UNIVERSITY OF OXFORD

Summer School on Multidimensional Poverty Analysis

3–15 August 2015

Georgetown University, Washington, DC, USA

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Robustness analysis and statistical inference

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Session I, 11 August 2015

Focus of This Lecture

- How accurate are the estimates?
- If they are used for policy, what is the chance that they are mistaken?
  - How sensitive policy prescriptions are to choices of parameters used for designing the measure (**Robustness Analyses**)
  - How accurate policy prescriptions are subject to the sample from which the they are computed (**Statistical Inferences**)

Policy Prescriptions Often of Interest

- A central government wants to allocate budget to the poor according to the MPI in each **region** of the country
  - Need to test if the regional comparisons are robust and statistically significant

- A minister wants to show the steepest decrease in poverty in their region/dimension
  - Need to test if the inter-temporal comparisons are robust and statistically significant

Importance of Robustness Analyses

Comparisons may <u>alter</u> when parameters vary

– An example with the Global MPI

For *k* = 1/3

– MPI for Zambia is 0.328 > MPI of Nigeria is 0.310

For *k* = 1/2

– MPI of Nigeria is 0.232 > MPI for Zambia is 0.214

k : The poverty cutoff. A person with a deprivation score equal to or greater than what is identified as poor

How are Statistical Tests Important?

Differences in estimates may be of the same magnitude, but statistical inferences may not be the same

– An example comparing Indian states

State	**Adjusted Headcount Ratio (*M_0*)**	**Difference**	**Statistically Significant?**
Goa	0.057	0.31	**Yes**
Punjab	0.088		
Maharashtra	0.194	0.32	**No**
Tripura	0.226		

Source: Alkire and Seth (2013)

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Robustness Analyses

Parameters of M_0

The M_0 measure and its partial indices are based on the following parameter values:

- Poverty cutoff (*k*)
- Weighting vector (*w*)
- Deprivation cutoffs (*z*)

An extreme form of robustness is *dominance*

Robustness Anlysis

1. Dominance Analysis for Changes in the Poverty Cutoff
   - With respect to poverty cutoff (analogous to unidimensional dominance)
   - Multidimensional dominance

2. Rank Robustness Analysis
   - With respect to weights
   - With respect to deprivation cutoffs

Dominance for H and M_0

Question: When can we say that a distribution has higher H or M_0 for any poverty cutoff (k), for a given weight vector and a given deprivation cutoff vector?

Hint: The concept can be borrowed from unidimensional stochastic dominance

Alkire and Foster (2011)

Dominance for H and M_0 in AF

Consider the following deprivation matrix

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity
$g^0 =$	0	0	0	0
	1	0	0	1
	1	1	1	1
	0	1	0	0
$z =$	500	12	1	1

Dominance for H and M_0 in AF

- For equal weight, the deprivation count vector is c

$g^0 =$

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	**c**
	0	0	0	0	**0**
	1	0	0	1	**0.5**
	1	1	1	1	**1**
	0	1	0	0	**0.25**
$z =$	**500**	**12**	**1**	**1**	

Complementary CDF (CCDF)

CDF of a distribution x is denoted by F_x

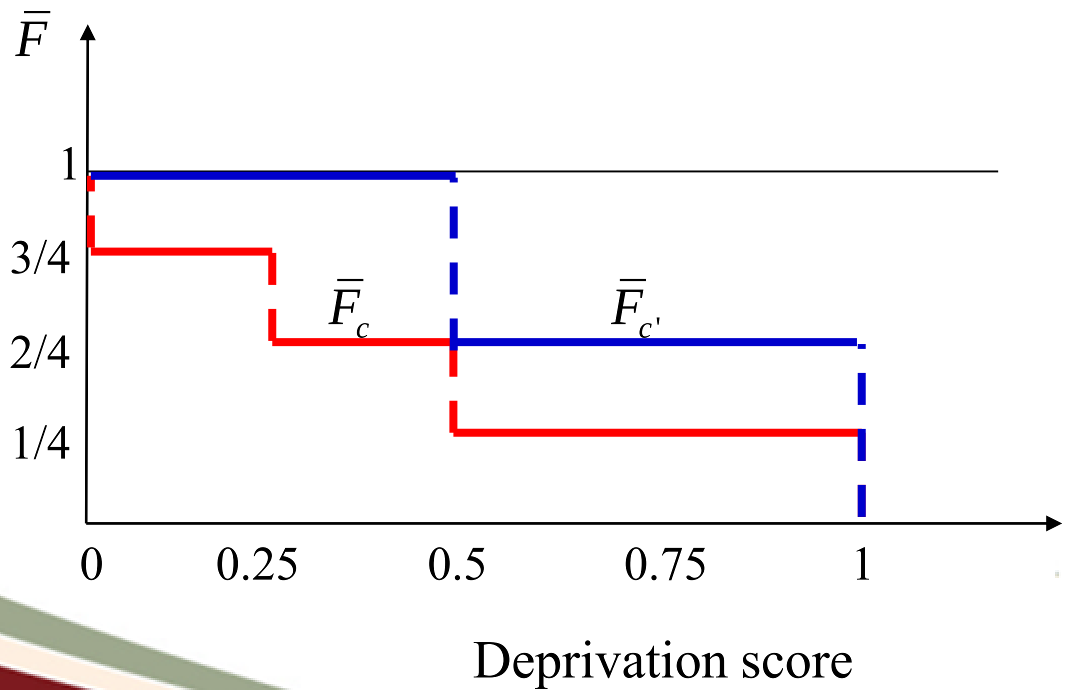
Complementary CDF (CCDF) of a distribution x is $\bar{F}_x$

CCDF is also known as survival function or reliability function in other branch of literature

$\bar{F}_x(b)$ denotes the proportion of population with values equal or larger than b

Example

Let the two deprivation score (count) vectors be $c = (0, 0.25, 0.5, 1)$ and $c' = (0.5, 0.5, 1, 1)$

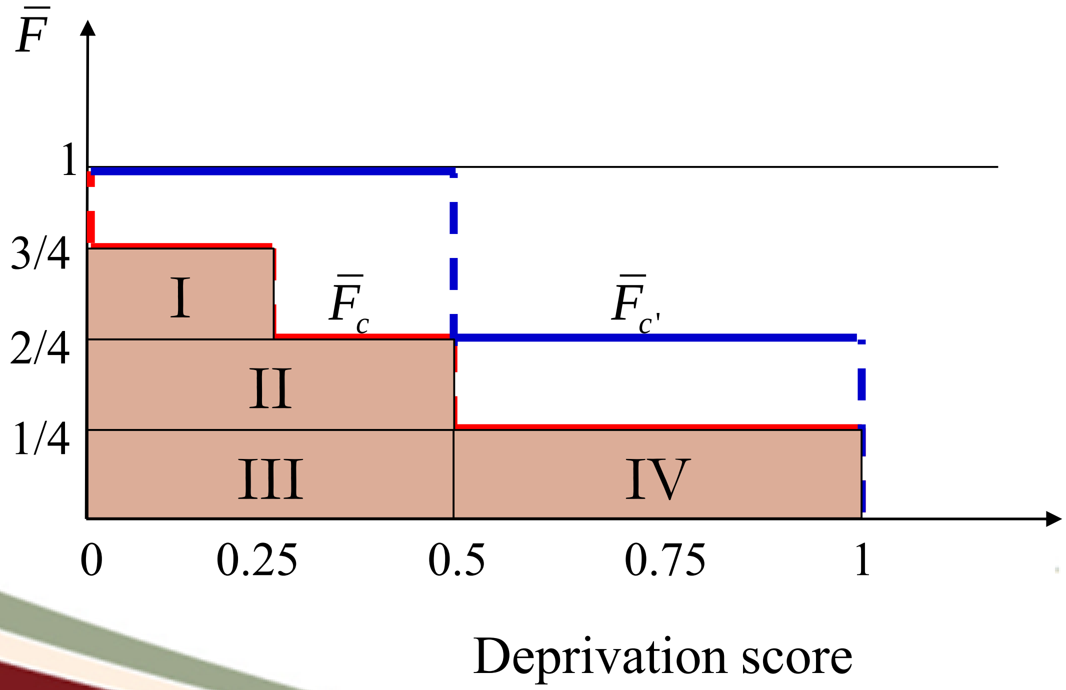


Is there any poverty cutoff (k) for which there is more H in c than in c' ?

H dominance implies M_0 dominance

How is M0 Computed for *c*?

$c = (0, 0.25, 0.5, 1)$



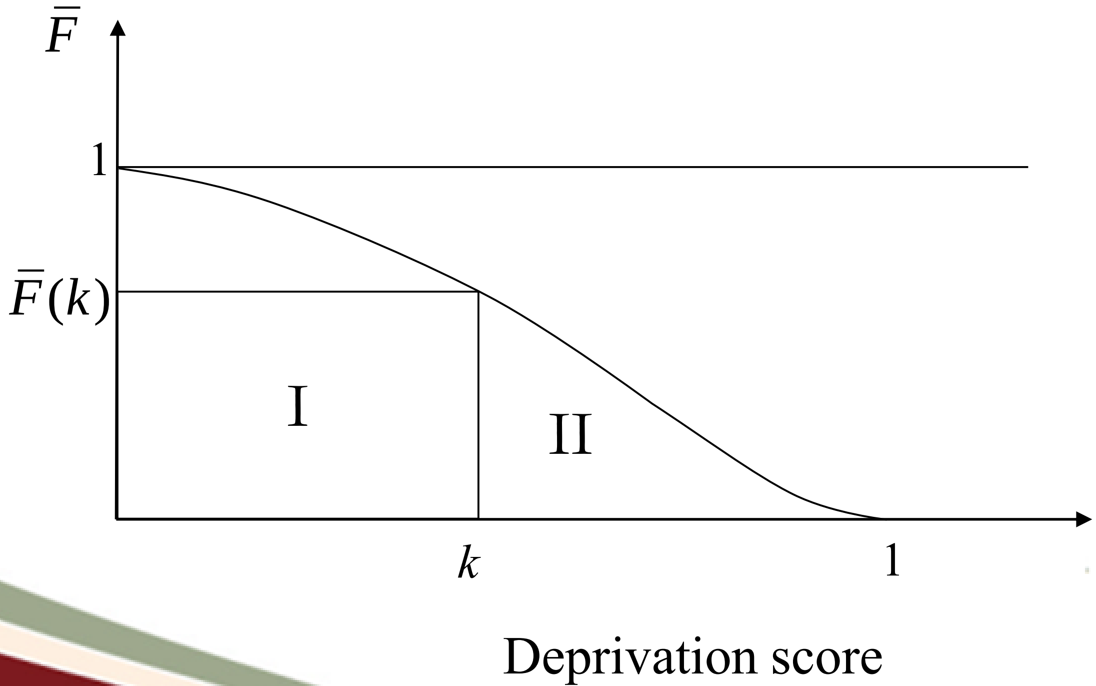
For union approach
M0 = I+II+III+IV
= (0+0.25+0.5+1)/4

For $k \in (0.25, 0.5]$
M0 = II+III+IV
= (0+0+0.5+1)/4

For $k = 1$
M0 = III + IV
= (0+0+0+1)/4

How is M0 Computed for *c*?

Now suppose c is a more continuous distribution of deprivation scores

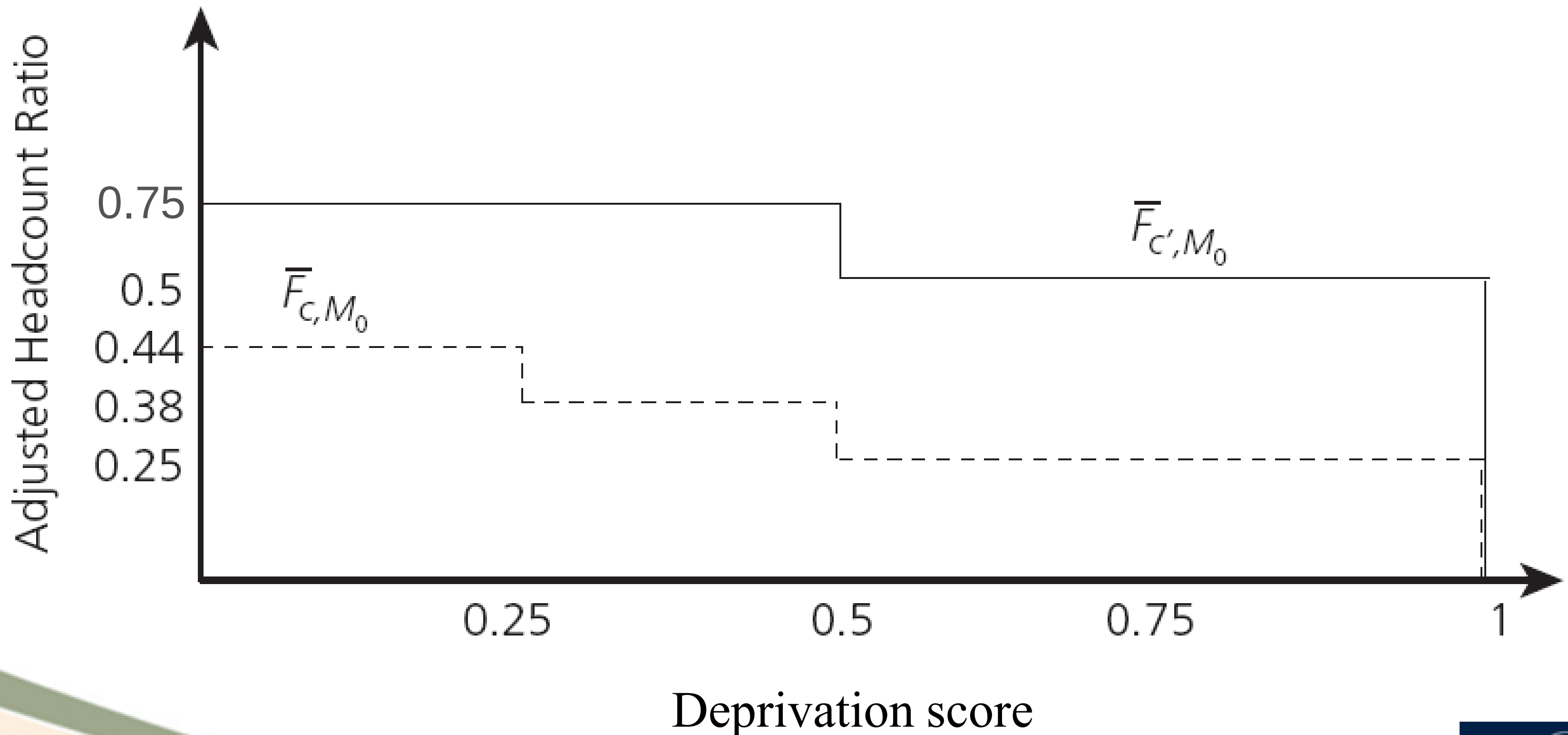


For poverty cutoff k,

M0 = I + II

M0 Curves

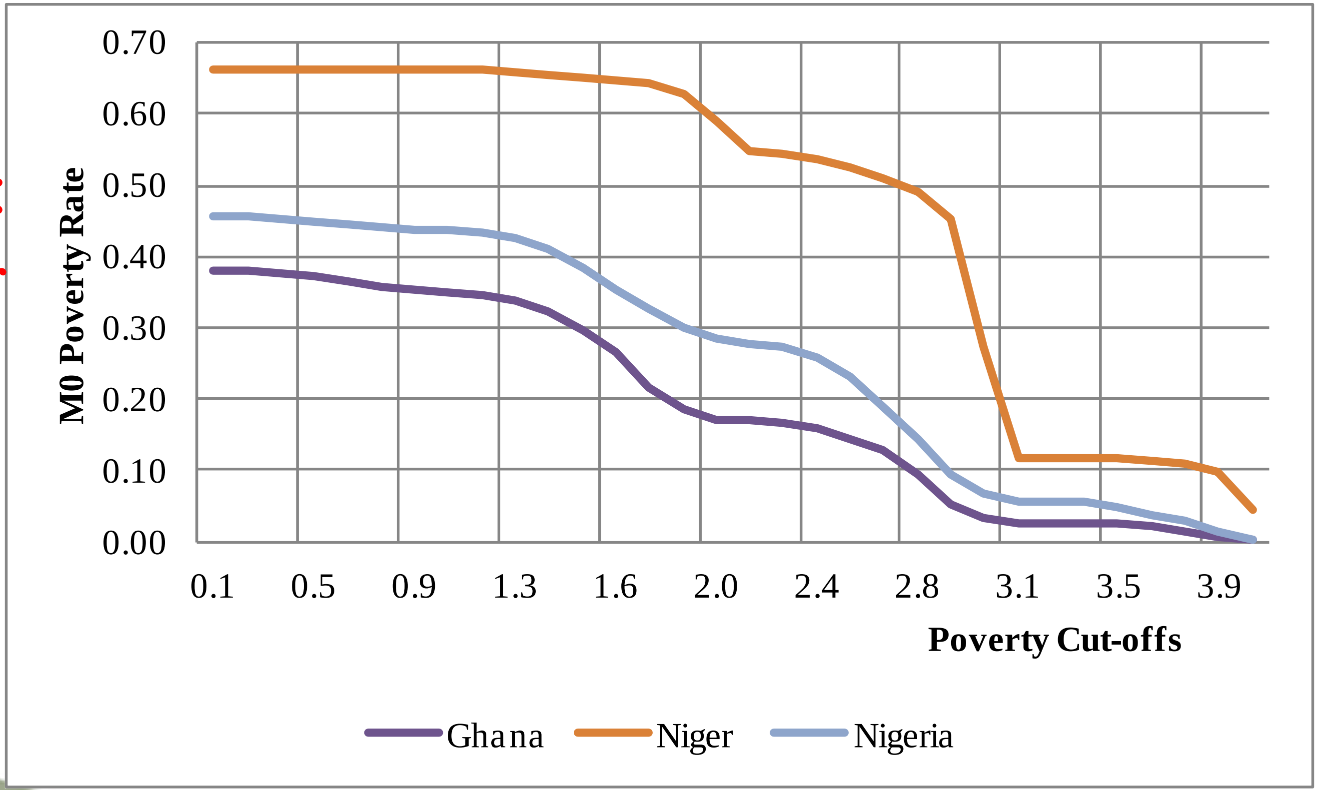
M0 curves for c = (0, 0.25, 0.5,1) and c' = (0.5, 0.5, 1,1)



Similar Concepts: M_0 Curves

Dominance holds in terms of M_0 for all *k*

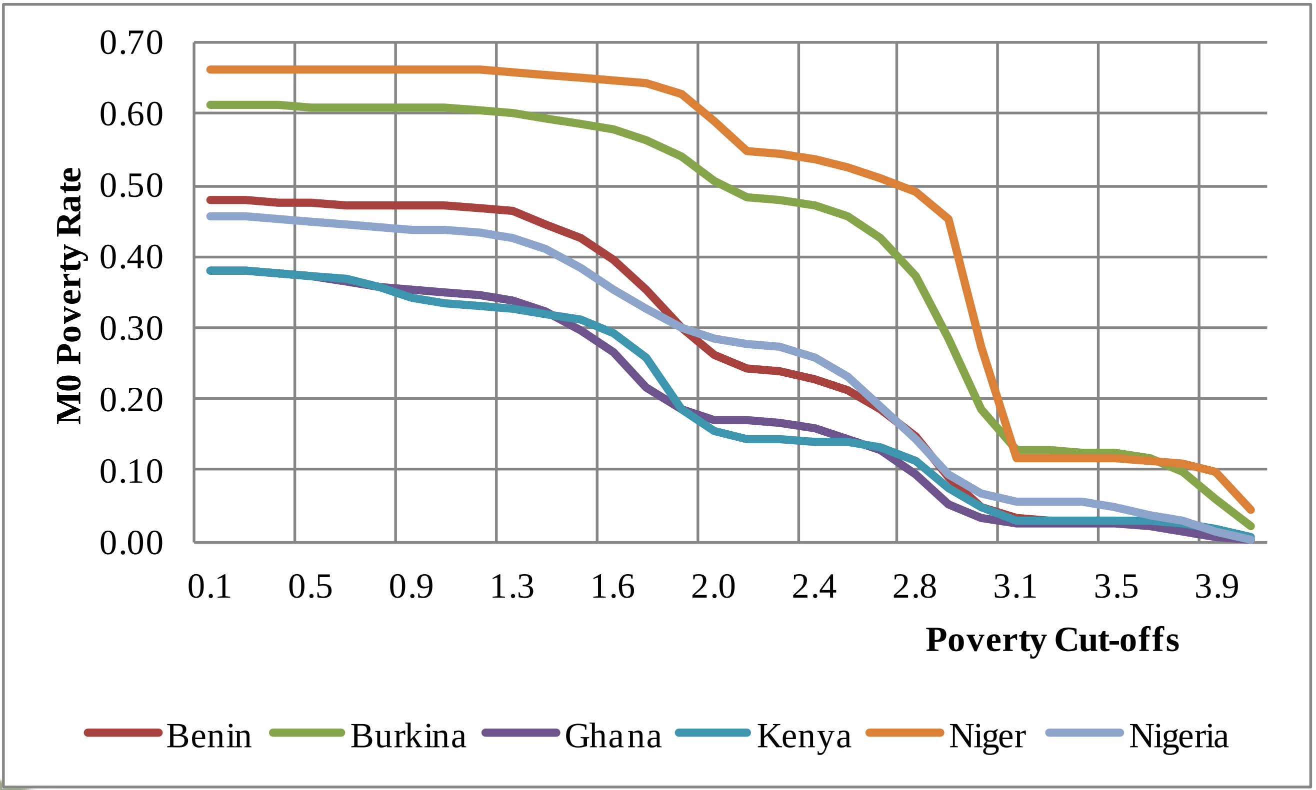
Sample vs population: revisit later



Source: Batana (2013)

M_0 Curves May, However, Cross

Note below that not all countries stochastically dominate each other (Batana 2013)



Rank Robustness Analyses

Dominance across all comparison values is an extreme form of robustness

Stochastic Dominance (SD) conditions are useful for pair by pair analysis and provides the strongest possible comparisons

SD conditions, however, may be too stringent and may not hold for the majority of countries

Rank Robustness Analyses

Until now, we have compared the robustness of comparisons across countries or regions to varying poverty cutoffs.

How can we evaluate the <u>ranking</u> of a set of countries or regions, when

- the poverty cut-off varies
- the weights vary
- the deprivation cutoffs vary

Rank Robustness of Comparisons

A useful method for comparing robustness of ranking is to compute rank correlation coefficients

- Spearman's rank correlation coefficient
- Kendall's rank correlation coefficient
- Percentage of pair-wise comparisons that are robust

First, different rankings of countries or regions are generated for different specifications of parameters

- Different weighting vectors, different poverty/deprivation cutoffs

Next, the pair-wise ranks and rank correlation coefficients are computed

Kendall's Tau

- For each pair, we find whether the comparison is *concordant* or *discordant*
  - 10 countries means 45 pair-wise comparisons

- The comparison between a pair of countries is *concordant* if one dominates the other for both specifications (C)

- The comparison between a pair of countries is *discordant* if one dominates the other for one specification, but is dominated for the other specification (D)

Kendall's Tau

- The Kendall's Tau rank correlation coefficient (*t*) is equal to

$$\tau = \frac{C - D}{C + D}$$

- It lies between -1 and +1
- If there are ties, this measure should be adjusted for ties
  - The tie adjusted Tau is known as tau-b

Spearman's Rho

- The Spearman's Rho also measures rank correlation but is slightly different from Tau
  - First, countries are ranked for two specifications
  - Then, for each country the difference in the two ranks are computed (r_i for country i)

- The Spearman's Rho (r) is

$$\rho = 1 - \frac{6\sum_{i=1}^{n} r_i^2}{n(n^2 - 1)}$$

Some Illustrations using the MPI

Robustness to weights

Re-weight each dimension:

– 33%	**50%**	25%	25%
– 33%	25%	**50%**	25%
– 33%	25%	25%	**50%**

Robustness to Weights

			MPI Weights 1	MPI Weights 2	MPI Weights 3
			Equal weights: 33% each **(Selected Measure)**	50% Education 25% Health 25% LS	50% Health 25% Education 25% LS
MPI Weights 2	50% Education	Pearson	0.992		
	25% Health	Spearman	0.979		
	25% LS	Kendall (Taub)	0.893		
MPI Weights 3	50% Health	Pearson	0.995	0.984	
	25% Education	Spearman	0.987	0.954	
	25% LS	Kendall (Taub)	0.918	0.829	
MPI Weights 4	50% LS	Pearson	0.987	0.965	0.975
	25% Education	Spearman	0.985	0.973	0.968
	25% Health	Kendall (Taub)	0.904	0.863	0.854
Number of countries:		**109**			

Alkire and Santos (2010, 2014)

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Statistical Inferences

Common Concerns

1. Does the overall poverty measure of a country amount to *P*?
2. Is the overall poverty larger or smaller in one region than another region?
3. Has the overall poverty increased or decreased over time?

One often needs to infer these conclusions (related to population) from a sample (as collecting data from the population is too expensive)

Some Terminologies

Inferential statistics like standard error (SE) and confidence intervals (CI) deal with inferences about populations **based on** the behavior of **samples**.

Both SEs and CIs will help us determine how likely it is that results based on a sample (or samples) are the same results that would have been obtained for the entire population

How to Obtain the Standard Error

To compute the standard error, we can use:

1. **Analytical approach**: "Formulas" which either provide the exact or the asymptotic approximation of the standard error (Yalonetzky, 2010).

2. **Resampling approach**: Standard errors and confidence intervals may be computed through bootstrap (Alkire & Santos, 2014).

Analytical Approach

The analytical approach is based on two assumptions

1. Samples are drawn from a population that is infinitely large
   - Superpopulation approach

2. We treat each sample as drawn from the population with replacement

Resampling Approach

Bootstrap method (Efron and Tibshirani (1993), chapters 12 and 16)

1. Random artificial sample are drawn from the dataset
2. An estimate is produced from the artificial sample and stored
3. Assuming the artificial samples are iid, the standard error is computed using the artificial sample estimates

Analytical vs. Resampling Approach

In Bootstrap method

1. The inference on summary statistics does not rely on the CLT as the analytical approach
2. Natural bounds of measures are automatically taken into account
3. Computation of SEs may become complex when the estimator has complicated form (Biewen 2002)
4. Achieves same accuracy as the delta-method (Biewen 2002; Davidson and Flachaire 2007)

Statistical Inference: Cross-Section

One sample test: Can we reject the claim 'Goa's M_0 is 0.05'?

State	M_0	95% Confidence Interval	
		Lower Bound	Upper Bound
Goa	0.057	0.045	0.069

Two sample test: Can we reject the claim 'Punjab's M_0 is the same as Goa's M_0'?

State	M_0	Difference	Statistically Significant?
Goa	0.057	0.31	
Punjab	0.088		

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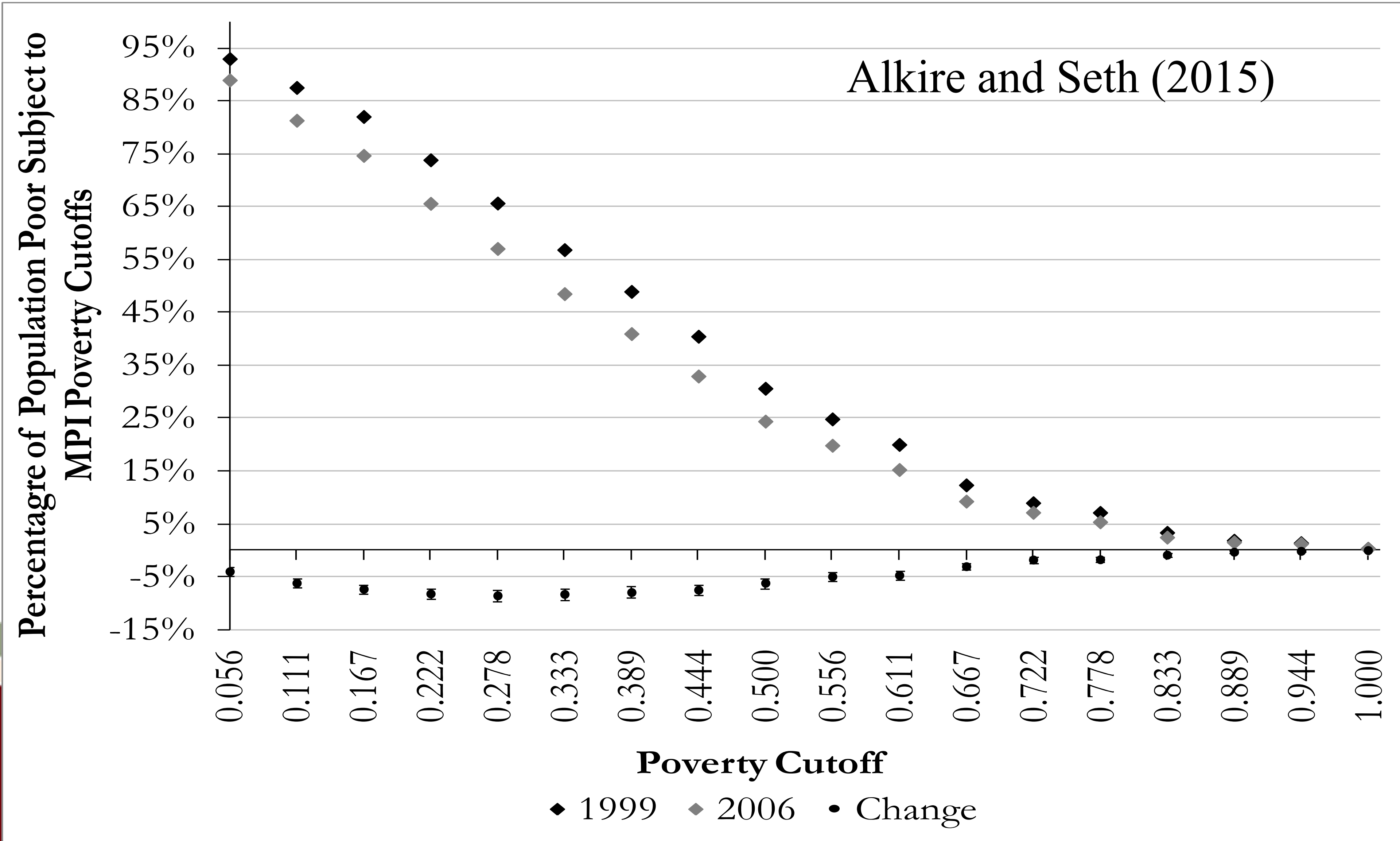
Statistical Inference: Inter-temporal

	Year 1		Year 2		Statistical significance of the change
Panel I: Multidimensional Poverty Index (MPI)					
Nepal 2006–2011	0.35	–(0.013)	0.217	–(0.012)	***
Peru 2005–2008	0.085	–(0.007)	0.066	–(0.004)	*
Rwanda 2005–2010	0.46	–(0.005)	0.33	–(0.006)	***
Senegal 2005–2010/11	0.44	–(0.019)	0.423	–(0.010)	

Source: Alkire, Roche, Vaz (2015), ***statistically significant at α=1%, **statistically significant at α=5%, *statistically significant at α=10%

Dominance with Statistical Inference



Robustness with Statistical Inference

Proportion of robust pairwise comparisons (Alkire and Santos (2014))

- Compute the MPIs
- Consider a comparison between a pair of countries robust if the difference is statistically significant
- Else the comparison is not robust
- Look at the proportion of robust comparisons

```
*** ROBUSTNESS, SENSITIVITY AND STANDARD ERRORS ********************************

* We open the clean dataset
clear
use "MyFirstMPI.dta"


*** For rank robustness comparisons, use the following commands:
* ktau
* spearman


*** For dominance, please compute the values of H and M0 for all k (1 to 100), and plot in Excel


*** Standars Errors

* Set the characteristics of the survey
svyset psu [pw = weight], strata(strata)


* Headcount Ratio (H)
//For details and discussions see equations (8.13) and (8.31), chapter 8 of the OPHI book

forvalue k = 10(10)100 {

    svy: mean multid_poor_`k'
    gen se_H_`k' = (_se[multid_poor_`k'])
    gen lb_H_`k' = _b[multid_poor_`k'] - 1.96 * se_H_`k'
    gen ub_H_`k' = _b[multid_poor_`k'] + 1.96 * se_H_`k'
    }

sum multid_poor_* lb_H_* ub_H_* [aw = weight]

* Adjusted Headcount Ratio (M0)
//For details and discussions see equations (8.11) and (8.30), chapter 8 of the OPHI book

forvalue k = 10(10)100 {
```

```
    gen ub_H_`k' = _b[multid_poor_`k'] + 1.96 * se_H_`k'
    }

sum multid_poor_* lb_H_* ub_H_* [aw = weight]

* Adjusted Headcount Ratio (M0)
//For details and discussions see equations (8.11) and (8.30), chapter 8 of the OPHI book

forvalue k = 10(10)100 {

    svy: mean cens_c_vector_`k'
    gen se_M0_`k' = (_se[cens_c_vector_`k'])
    gen lb_M0_`k' = _b[cens_c_vector_`k'] - 1.96* se_M0_`k'
    gen ub_M0_`k' = _b[cens_c_vector_`k'] + 1.96* se_M0_`k'
    }

sum cens_c_vector_* lb_M0_* ub_M0_* [aw = weight]

** Average Deprivation among the Poor (A)
//For details and discussions see equations (8.19), (8.35) and (8.36), chapter 8 of the OPHI book

forvalue k = 10(10)100 {

    svy: mean multid_poor_`k' cens_c_vector_`k'
    mat cov = e(V)
    loc cov = cov[2,1]
    loc var_H = cov[1,1]
    loc var_MPI = cov[2,2]

    gen se_A_`k' = ((`var_MPI'/_b[multid_poor_`k']^2) + (((_b[cens_c_vector_`k']/_b[multid_poor_`k']^2)^2)*(`var_H')) ///
    - 2*((_b[cens_c_vector_`k']/_b[multid_poor_`k']^3)*`cov'))^0.5

    gen lb_A_`k' = (_b[cens_c_vector_`k']/_b[multid_poor_`k']) - 1.96 * se_A_`k'
    gen ub_A_`k' = (_b[cens_c_vector_`k']/_b[multid_poor_`k']) + 1.96 * se_A_`k'
    }

//For discussions regarding statistical tests, please see Sections 8.2.2 and 8.2.3 of the OPHI Book
//For Robustness and dominance analysis with Statistical inference, see Section 8.3 of the OPHI Book.
//For those who are interested in bootstrap, see the Appendix of Chapter 8 of the OPHI Book.
```