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Multidimensional Poverty Measurement and Analysis: Chapter 9 – Distribution and Dynamics

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Abstract

For meaningful policy analysis, it is important not only to look at overall poverty, and compare countries or regions at a single point in time, but also to understand the distribution among the poor, the disparity across subgroups, and the dynamics of poverty. This extends the methodological toolkit presented in Chapter 5. First we present a new measure of inequality among the poor and its axiomatic justification. The same measure, a form of variance, can be extended to analyse horizontal inequalities, which are disparities across different population subgroups. The next two sections provide methodological extensions that are required for inter-temporal analysis of poverty using repeated cross-sectional data, and for the analysis of dynamic subgroups. The last section elaborates a measure chronic multidimensional poverty, which uses a counting approach

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both across dimensions and across periods, and provide its consistent partial indices, including a new statistic on the duration of poverty.

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9   Distribution and Dynamics

This chapter provides techniques required to measure and analyse inequality among the poor (section 9.1), to describe changes over time using repeated cross-sectional data (section 9.2), to understand changes across dynamic subgroups (section 9.3) and to measure chronic multidimensional poverty (section 9.4). Each of these sections extends the M_0 methodological toolkit beyond the partial indices presented in Chapter 5, to address common empirical problems such as poverty comparisons, and illustrate these with examples. We build upon and do not repeat material presented in earlier chapters, and as in other chapters, confine attention to issues that are distinctive in multidimensional poverty measures.

9.1   Inequality among the Poor

Given the long-standing interest in inequality among the poor, we first enquire whether M_0 can be extended to reflect inequality among the poor. To make a long story short, it can easily do so. But the problem is that the resulting measure loses the property of dimensional breakdown that provides critical information for policy. So, taking a step back, we consider key properties a measure should have in order to reflect inequality among the poor and be analysed in tandem with M_0. Our chosen measure uses the distribution of censored deprivation scores to compute a form of variance across the multidimensionally poor. We also illustrate interesting related applications of this measure: for example to assess horizontal disparities across groups.

Chapter 5 showed that the Adjusted Headcount Ratio (M_0) can be expressed as a product of the incidence of poverty (H) and the intensity of poverty (A) among the poor. Thus, M_0 captures two very important components of poverty—incidence and intensity. But it remains silent on a third important component: inequality across the poor. Now, the ultimate objective is to eradicate poverty—not merely reduce inequality among the poor. However, the consideration of inequality is important because the same average intensity can hide widely varying levels of inequality among the poor. For this reason,

following the seminal article by Sen (1976), numerous efforts were made to incorporate inequality into unidimensional and latterly multidimensional poverty measures.[1]

This section explores how inequality among the poor can be examined when poverty analyses are conducted using the M_0 measure (Alkire and Foster 2013, Seth and Alkire 2014a, b).[2]

9.1.1 Integrating Inequality into Poverty Measures

Section 5.7.2 already presented one way of bringing inequality into multidimensional poverty measures. This was achieved by using M_2 or some other gap measure applied to cardinal data, where the exponent on the normalized gap is strictly greater than one. Such an approach is linked to Kolm (1977) and generalizes the notion of a progressive transfer (or more broadly a Lorenz comparison) to the multidimensional setting by applying the same bistochastic matrix to every variable to smooth out the distribution of each variable (the powered normalized gap) while preserving its mean.[3] Poverty measures that are sensitive to inequality fall (or at least do not rise) in this case.

A second form of multidimensional inequality is linked to the work of Atkinson and Bourguignon (1982) and relies on patterns of achievements *across* dimensions. Imagine a case where one poor person initially has more of everything than another poor person and the two persons switch achievements for a single dimension in which both are deprived. This can be interpreted as a progressive transfer that preserves the marginal distribution of each variable and lowers inequality by relaxing the positive association across variables under the assumption that the dimensions are substitutes. The resulting transfer principle specifies conditions under which this alternative form of progressive transfer among the poor should lower poverty, or at least not raise it. The transfer properties are motivated by the idea that poverty should be sensitive to the level of inequality among the poor, with greater inequality being associated with a higher (or at least not lower) level of poverty.[4] Alkire and Foster (2011a) observe that the AF class of measures can be easily adjusted to respect the strict version of the second kind of

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[1] For inequality-adjusted poverty measures in the unidimensional context, see Thon (1979), Clark, Hemming, and Ulph (1981), Chakravarty (1983b), Foster, Greer, and Thorbecke (1984), and Shorrocks (1995). For inequality-adjusted multidimensional poverty measures, i.e., those that satisfy transfer and/or strict rearrangement properties, see section section 3.6.

[2] This section summarizes these two papers.

[3] In order to say that one multidimensional distribution is more equal than another, each must be smoothed using the *same* bistochastic matrix.

[4] See Sen (1976) and Foster and Sen (1997).

transfer (the strong deprivation rearrangement property as discussed in section 5.2.5) involving a change in association between dimensions by replacing the deprivation count or score $M_\alpha(x_i.; z)$ with a related individual poverty function $[M_\alpha(x_i.; z)]^\beta$ for some $\beta > 0$, and averaging across persons.[5]

Many multidimensional poverty measures that employ cardinal data, including M_α satisfy one or both of these transfer principles.[6] Alkire and Foster (2013) formulate a strict version of distribution sensitivity — 'dimensional transfer' (defined in 5.2.5)—which is applicable to poverty measures such as M_0 that use ordinal data. This property follows the Atkinson–Bourguignon type of distribution sensitivity, in which greater inequality among the poor strictly raises poverty. Alkire and Foster (2013) also prove a general result establishing that 'the highly desirable and practical properties of subgroup decomposability, dimensional breakdown, and symmetry prevent a poverty measure from satisfying the dimensional transfer property'. In other words, M_0 does not reflect inequality among the poor, and, furthermore, no measure that satisfies dimensional breakdown and symmetry will be found that *does* satisfy dimensional transfer.

Given that it is necessary to choose between measures that satisfy dimensional transfer and those that can be broken down by dimension, and given that both properties are arguably important, how should empirical studies proceed? The first option is to employ the class of measures that respect dimensional breakdown and to supplement these with associated inequality measures. The second is to employ poverty measures that are inequality-sensitive but cannot be broken down by dimension, and to supplement them with separate dimensional analyses.

9.1.2 Analyzing Inequality Separately: A Descriptive Tool

While both should be explored, this book favours the first route in applied work for several reasons. Dimensional breakdown enriches the informational content of poverty measures for policy, enabling them to be used to tailor policies to the composition of poverty, to monitor changes by dimension, and to make comparisons across time and

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[5] Bourguignon and Chakravarty (2003) and Datt (2013) also propose a similar class of indices but using a union identification criterion.

[6] We have already shown that our multidimensional measure M_α satisfies weak transfer, the first type of transfer property, for $\alpha \geq 1$, and the second type of transfer property, weak rearrangement, for $\alpha \geq 0$. Chakravarty, Mukherjee, and Ranade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Chakravarty and D'Ambrosio (2006), Maasoumi and Lugo (2008), Aaberge and Peluso (2012), Bossert, Chakravarty, and D'Ambrosio (2013), and Silber and Yalonetzky (2014).

space. Poverty reduction in measures respecting dimensional breakdown can be accounted for in terms of changes in deprivations among the poor and analysed by region and dimension. This creates positive feedback loops that reward effective policies. Also, the inequality-adjusted poverty measures may lack the intuitive appeal of the M_0 measure. Some of the inequality-adjusted measures (Chakravarty and D'Ambrosio, 2006, Rippin, 2012) are broken down into different components separately capturing incidence, intensity, and inequality, but without clarifying the relative weights attached to these components.

Whether or not an inequality measure is not computed, M_0 measures can be supplemented by direct descriptions of inequality among the poor. A first descriptive but powerfully informative tool is to report subsets of poor people which have mutually exclusive and collectively exhaustive graded bands of deprivation scores. This is possible by effectively ordering all q poor persons according to the value of their deprivation score $c_i(k)$ and dividing them into groups. If the poverty cutoff is 30%, the analysis might then report the percentage of poor people whose deprivation scores fall in the band of 30–39.9% of deprivations, 40–49.9%, and so on to 100%. The percentage of people who experience different intensity gradients of poverty across regions and time can be compared to see how inequality among the poor is evolving.[7] Figure 9.1 presents an example of two countries—Madagascar and Rwanda—which have similar multidimensional headcount ratios (H) and MPIs. However, the distributions of intensities across the poor are quite different. Also, data permitting, these intensity groups can be decomposed by population subgroups such as region or ethnicity. The comparisons can be enriched by applying a dimensional breakdown to examine the dimensional composition of poverty experienced by those having different ranges of deprivation scores.

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[7] For empirical examples, see Alkire, Roche, and Seth (2013), who compare countries across four gradients of poverty.

Figure 9.1 Distribution of Intensities among the Poor in Madagascar and Rwanda

Madagascar (2009)
MPI = 0.357, *H* = 67%, *A* = 53%

Rwanda (2010)
MPI = 0.350, *H* = 69% *A* = 50.8%
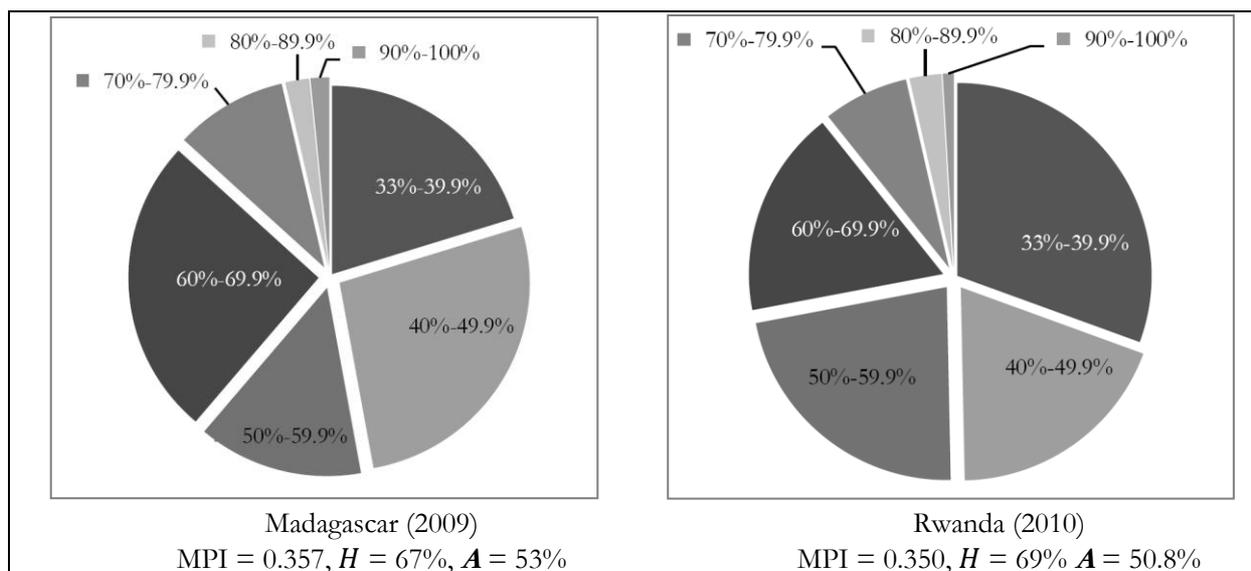
Source: Country briefings, MPI 2013, http://www.ophi.org.uk/multidimensional-poverty-index/mpi-data-bank/mpi-country-briefings.

9.1.3 Using a Separate Inequality Measure

Another tool is to supplement M_0 with a measure of inequality among the poor. Using the distribution of (censored) deprivation scores across the poor or some transformation of these, it is actually elementary to create an inequality measure, much in the same way that traditional inequality measures such as Atkinson, Theil, or Gini are constructed. Such measures will offer a window onto one type of multidimensional inequality—one that is oriented to the breadth of deprivations people experience. This approach is quite different from other constructions of multidimensional inequality, but it is useful, particularly when data are ordinal. Building on Chakravarty (2001), Seth and Alkire (2014a,b) propose such an inequality measure that is founded on certain properties. Note that these are properties of *inequality* measures, and are defined differently from those presented in Chapter 2 (despite similar names), but introduced intuitively below. Let us briefly discuss these properties before introducing the measure.

9.1.3.1 Properties

The first property, **translation invariance***,* requires inequality not to change if the deprivation score of every poor person increases by the same amount. Implicitly, we assume that the measure reflects *absolute* inequality. Seth and Alkire (2014) argue that measures reflecting absolute inequality are more appropriate when each deprivation is judged to be of intrinsic importance. In addition, the use of the absolute inequality

measure ensures that inequality remains the same whether poverty is measured by counting the number of deprivations or by counting the number of attainments. The use of the relative inequality measure is more common in the case of income inequality, where it is often assumed that as long as people's *relative* incomes remain unchanged, inequality should not change. However, it is difficult to argue that inequality between two poor persons who are deprived in one and two dimensions respectively is the same as the inequality between two poor persons who are deprived in five and ten dimensions, respectively, if these deprivations referred to, for example, serious human rights violations. Any relative inequality measure, such as the Generalized Entropy measures (which include the Squared Coefficient of Variation associated with the FGT2 index) or Gini Coefficient, would evaluate these two situations as having identical inequality across the poor. Moreover, a relative inequality measure may provide counterintuitive conclusion while assessing inequality within a counting approach framework. In fact, no non-constant inequality measure exists that is simultaneously invariant to absolute as well as relative changes in a distribution.

The second property requires that the inequality measure should be **additively decomposable** so that overall inequality in any society can be broken down into within-group and between-group components. This can be quite useful for policy (Stewart 2010). We have shown in Chapter 5 that the additive structure of the indices in the AF class allows the overall poverty figure to be decomposed across various population subgroups. A country or a region with same level of overall poverty may have very different poverty levels across different subgroups, or a country may have the same level of poverty across two time periods, but the distribution of poverty across different subgroups may change over time. Furthermore, within each population subgroup, there may be different distributions of deprivation scores across poor persons living within that subgroup, thus reflecting various levels of within-group inequality.

The third property, **within-group mean independence**, requires that overall within-group inequality should be expressed as a weighted average of the subgroup inequalities, where the weight attached to a subgroup is equal to the population share of that subgroup. This assumption makes the interpretation and analysis of the inequality measure more intuitive.

Four additional properties are commonly satisfied when constructing any inequality measure. The **anonymity** property requires that a permutation of deprivation scores

should not alter inequality. According to the **replication invariance** property, a mere replication of population leaves the inequality measure unaltered. The **normalization** property requires that the inequality measure should be equal to zero when the deprivation scores are equal for all. The **transfer** property requires that a progressive dimensional rearrangement among the poor should decrease inequality.

9.1.3.2 A Decomposable Measure

The proposed inequality measure, which is the only one to satisfy those properties, takes the general form

$$I\left(y; t, \tilde{\beta}\right) = \frac{\tilde{\beta}}{t} \sum_{i=1}^{t} [y_i - \mu(y)]^2, \tag{9.1}$$

where $y = (y_1, \dots, y_t)$ is a vector with t elements. Relevant applications using our familiar notation will be provided in equations (9.6) and (9.7) below, but we first present the general form and notation. As we will show, in relevant applications an element y_i may be the deprivation score of a person c_i or $c_i(k)$ or the average poverty level of a region. The size of the vector y for an entire population would be $t = n$ and for the poor it would be $t = q$.[8] The functional form in equation (9.1) is a positive multiple $(\tilde{\beta})$ of the variance. The measure reflects the average squared difference between person i's deprivation score and the mean of the deprivation scores in y. The value of parameter $\tilde{\beta}$ can be chosen in such a way that it normalizes the inequality measure to lie between 0 and 1.

The overall inequality in y may be decomposed into two components: total within-group inequality and between-group inequality. Following the notation in Chapter 2, suppose there are $m \geq 2$ population subgroups. The deprivation score vector of subgroup ℓ is denoted by y^ℓ with t^ℓ elements. The decomposition expression is given as follows:

$$I(y) = \sum_{\ell=1}^{t} \frac{t^\ell}{t} I\left(y^\ell; t^\ell, \tilde{\beta}\right) + I\left(\mu(y^1), \dots, \mu(y^m); \frac{t^1}{t}, \dots, \frac{t^m}{t}, \tilde{\beta}\right), \tag{9.2}$$

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[8] There are many variations. For example, if data are relatively accurate one might consider inequality using the uncensored deprivation score vector c, and alternatively if one only wishes to capture inequality within some subgroup ℓ, then $y = c^\ell$ and $\spadesuit = n^\ell$.

Total within-group   Total between-group

where $\frac{t^\ell}{t}$ is the population share of subgroup ℓ in the overall population and $\mu(y^\ell)$ is the mean of all elements in y^ℓ for all $\ell = 1, \dots, m$.

The **between-group inequality** component $I(\mu(y^1), \dots, \mu(y^m); \frac{t^1}{t}, \dots, \frac{t^m}{t}, \tilde{\beta})$ in (9.2) can be computed as

$$I\left(\mu(y^1), \dots, \mu(y^m); \frac{t^1}{t}, \dots, \frac{t^m}{t}, \tilde{\beta}\right) = \tilde{\beta} \sum_{\ell=1}^{m} \frac{t^\ell}{t}\left(\mu(y^\ell) - \mu(y)\right)^2, \tag{9.3}$$

where $\mu(y)$ is the mean of all elements in y.

The **within-group inequality** component of subgroup ℓ can be computed using (9.1) as

$$I(y^\ell; t^\ell, \tilde{\beta}) = \frac{\tilde{\beta}}{t^\ell} \sum_{i=1}^{t^\ell} [y_i^\ell - \mu(y^\ell)]^2; \tag{9.4}$$

and thus the **total within-group inequality** component in (9.2) can be computed as

$$\sum_{\ell=1}^{t} \frac{t^\ell}{t} I(y^\ell; t^\ell, \tilde{\beta}) = \frac{\tilde{\beta}}{t} \sum_{\ell=1}^{t} \sum_{i=1}^{t^\ell} [y_i^\ell - \mu(y^\ell)]^2. \tag{9.5}$$

9.1.3.3 Two Important Applications

There are different relevant applications of this inequality framework to multidimensional poverty analyses based on M_0. The first central case is to assess inequality among the poor. To do so we suppose that the deprivation scores are ordered in a descending order and the first q persons are identified as poor. The elements are taken from the censored deprivation score vector, $y = (c_1(k), \dots, c_q(k))$. We choose vector y such that it contains only the deprivation scores of the poor ($t = q$). The average of all elements in y then is the intensity of poverty which for q persons is $\mu(y) = A$. We can then denote the inequality measure that reflects inequality in multiple deprivations only among the poor by I^q, which can be expressed as

$$I^q = \frac{\tilde{\beta}}{q} \sum_{i=1}^{q} [c_i(k) - A]^2 . \,^9 \tag{9.6}$$

The I^q measure effectively summarizes the information underlying Figure 9.1. It goes well beyond that figure because each individual deprivation score is used, which effectively creates a much finer gradation of intensity than that figure portrays. Furthermore, it can be decomposed by subgroup, to permit comparisons of within-subgroup inequalities among the poor. It can also be used over time to show how inequality among the poor changed.

Our second central case considers inequalities in poverty levels across population subgroups. It is motivated by studies of horizontal inequalities that find group-based inequalities to predict tension and in some cases conflict (Stewart 2010). Essentially, the measure reflects population-weighted disparities in poverty levels across population subgroups.

Suppose the censored deprivation score vector of subgroup ℓ is denoted by $c^\ell(k)$ with n^ℓ elements. If instead of only considering the deprivation scores of the poor, we now sum across the whole population so $(t = n)$, then we realize that $\mu(c^\ell(k))$ or the average of *all* elements in $c^\ell(k)$ is actually the M_0 of subgroup ℓ, which for simplicity we denote by M_0^ℓ. The between-group component of $I^n(c(k))$ shows the disparity in the national Adjusted Headcount Ratio (M_0) across subgroups and is written using (9.3) as

$$I^n \left(M_0^1, \dots, M_0^m; \frac{n^1}{n}, \dots, \frac{n^m}{n}, \tilde{\beta} \right) = \tilde{\beta} \sum_{\ell=1}^{m} \frac{n^\ell}{n} \left(M_0^\ell - M_0 \right)^2 \tag{9.7}$$

Thus, equation (9.7) captures the disparity in M_0s across m population subgroups, which can be used to detect patterns in horizontal disparities over time. Naturally, the number and population share of the subgroups must be considered in such comparisons.

While studying disparity in MPIs across sub-national regions, Alkire, Roche, and Seth (2011) found that the national MPIs masked a large amount of sub-national disparity within countries, and Alkire and Seth (2013) and Alkire, Roche, and Vaz (2014) found

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[9] If one is interested in decomposing (9.6) into within and between group components, then the total within-group inequality term can be computed as $\frac{\tilde{\beta}}{q} \sum_{\ell=1}^{m} \sum_{i=1}^{q^\ell} \left[c_i^\ell(k) - A^\ell \right]^2$ and the total between-group inequality term can be computed as $\tilde{\beta} \sum_{\ell=1}^{m} \frac{q^\ell}{q} \left(A^\ell - A \right)^2$, where A^ℓ the intensity of each subgroup ℓ such that $A^\ell = \mu(y^\ell)$ and q^ℓ/q is the share of all poor in subgroup ℓ.

considerable disparity in poverty trends across subnational groups. In some countries, the overall situation of the poor improved, but not all subgroups shared the equal fruit of success in poverty reduction and indeed poverty levels may have stagnated or risen in some groups. Therefore, it is also important to look at inequality or disparity in poverty across population subgroups. This separate inequality measure, elaborated in Seth and Alkire (2014), provides such framework.

9.1.3.4 An Illustration

Table 9.1 presents two pair-wise comparisons. For the inequality measure, we choose $\tilde{\beta} = 4$ because the deprivation scores are bounded between 0 and 1; hence the maximum possible variance is 0.25. $\tilde{\beta} = 4$ ensures that the inequality measure lies between 0 and 1. The first pair of countries, India and Yemen, have exactly the same levels of MPI. The multidimensional headcount ratios and the intensities of poverty are also similar. However, the inequality among the poor (computed using equation (9.1)) is much higher in Yemen than in India. We also measure disparity across sub-national regions. Yemen has twenty-one sub-national regions whereas, India has twenty-nine sub-national regions. We find that, like the national MPIs, the disparities across subnational MPIs—computed using equation 9.7)—are similar. This means that the inequality in Yemen is not primarily due to regional disparities in poverty levels, but may be affected by non-geographic divides such as cultural or rural–urban.

A contrasting finding for regional disparity is obtained across Togo and Bangladesh. As before, the MPIs, headcount ratios, and intensities are quite similar across two countries—but with two differences. The inequality among the poor is very similar, but the regional disparities are stark. Even though both countries have similar number of sub-national regions, the level of sub-national disparity is much higher in Togo than that in Bangladesh.

Table 9.1: Countries with Similar Levels of MPI but Different Levels of Inequality among the Poor and Different Levels of Disparity across Regional MPIs

Country	Year	M0	A	H	Inequality Among The Poor	Disparity Between MPIs	Number of Regions
Yemen	2006	0.283	53.9%	52.5%	0.122	0.052	21
India	2005	0.283	52.7%	53.7%	0.104	0.050	29
Togo	2010	0.250	50.3%	49.8%	0.086	0.042	6
Bangladesh	2011	0.253	49.5%	51.2%	0.084	0.005	7

Source: Seth and Alkire (2014).

9.2   Descriptive Analysis of Changes over Time

A strong motivation for computing multidimensional poverty is to track and analyse changes over time. Most data available to study changes over time are repeated cross-sectional data, which compare the characteristics of representative samples drawn at different periods with sampling errors, but do not track specific individuals across time. This section describes how to compare M_0 and its associated partial indices over time with repeated cross-sectional data. It offers a standard methodology of computing such changes, and an array of small examples. This section does not treat the data issues underlying poverty comparisons, and readers are expected to know standard techniques that are required for such rigorous empirical comparisons. For example, the definition of indicators, cutoffs, weights, etc. must be strictly harmonized for meaningful comparisons across time, which always requires close verification of survey questions and response structures, and may require amending or dropping indicators. The sample designs of the surveys must be such that they can be meaningfully compared, and basic issues like the representativeness and structure of the data must be thoroughly understood and respected. We presume this background in what follows. This section focuses on changes across two time periods; naturally the comparisons can be easily extended across more than two time periods.

9.2.1   Changes in M$_0$, H and A across Two Time Periods

The basic component of poverty comparisons is the absolute pace of change across periods.[10] The absolute rate of change is the difference in levels between two periods. Changes (increases or decreases) in poverty across two time periods can also be reported as a relative rate. The relative rate of change is the difference in levels across two periods as a percentage of the initial period.

For example, if the M_0 has gone down from 0.5 to 0.4 between two consecutive years, then the absolute rate of change is $(0.5 - 0.4) = 0.1$. It tells us how much level of poverty (M_0) has changed: 10% of the total possible set of deprivations that poor people in that society could have experienced has been eradicated; 40% remains. The relative rate of change is $(0.5 - 0.4)/0.5 = 20\%$, which tells us that M_0 has gone down by 20% with respect to the initial level. While absolute changes are in some sense prior, because they are easy to understand and compare, both absolute and relative rates may be important

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[10] This section draws on Alkire, Roche, and Vaz (2014).

to report and analyse. The value-added of the relative changes is evident in relatively low-poverty regions. A region or country with a high initial level of poverty may be able to reduce poverty in absolute terms much more than one having a low initial level of poverty. It is however possible that although a region or country with low initial poverty levels did not show a large absolute reduction, the reduction was large *relative* to its initial level and thus it should not be discounted for its slower absolute reduction.[11] The analysis of both absolute and relative changes gives a clear sense of overall progress.

In expressing changes across two periods, we denote the initial period by t^1 and the final period by t^2. This section mostly presents the expressions for M_0, but they are equally applicable to its partial indices: incidence (H), intensity (A), censored headcount ratios ($h_j(k)$), and uncensored headcount ratios (h_j). The achievement matrices for period t^1 and t^2 are denoted by X_{t^1} and X_{t^2}, respectively. As presented in Chapter 5, M_0 and its partial indices depend on a set of parameters: deprivation cutoff vector z, weight vector w and poverty cutoff k. For simplicity of notation though, we present M_0 and its partial indices only as a function of the achievement matrix. For strict intertemporal comparability, it is important that the same set of parameters be used across two periods.

The **absolute rate of change** (Δ) is simply the difference in Adjusted Headcount Ratios between two periods and is computed as

$$\Delta M_0 = M_0(X_{t^2}) - M_0(X_{t^1}). \tag{9.8}$$

Similarly, for H and A:

$$\Delta H = H(X_{t^2}) - H(X_{t^1}). \tag{9.9}$$

$$\Delta A = A(X_{t^2}) - A(X_{t^1}). \tag{9.10}$$

The **relative rate of change** (δ) is the difference in Adjusted Headcount Ratios as a percentage of the initial poverty level and is computed for $M_0, H,$ and A (only M_0 shown) as

$$\delta M_0 = \frac{M_0(X_{t^2}) - M_0(X_{t^1})}{M_0(X_{t^1})} \times 100. \tag{9.11}$$

If one is interested in comparing changes over time for the same reference period, the expressions (5.7) and (9.11) are appropriate. However, in cross-country exercises, one

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[11] Tables of the absolute levels and absolute rates of change make this feature visible; reporting the relative rate of change underscores this more precisely.

may often be interested in comparing the rates of poverty reduction across countries that have different periods of references. For example the reference period of one country may be five years; whereas the reference period for another country is three years. It is evident in Table 9.2 that the reference period of Nepal is five years (2006–2011); whereas that of Peru is only three years (2005-–2008). In such cases, it is essential to annualize the change in order to preserve strict comparison.

The **annualized absolute rate of change** $(\overline{\Delta})$ is the difference in Adjusted Headcount Ratios between two periods divided by the difference in the two time periods $(t^2 - t^1)$ and is computed for M_0 as

$$\overline{\Delta}M_0 = \frac{M_0(X_{t^2}) - M_0(X_{t^1})}{t^2 - t^1}. \tag{9.12}$$

The **annualized relative rate of change** $(\overline{\delta})$ is the compound rate of reduction in M_0 per year between the initial and the final periods, and is computed for M_0 as

$$\overline{\delta}M_0 = \left[\left(\frac{M_0(X_{t^2})}{M_0(X_{t^1})} \right)^{\frac{1}{t^2 - t^1}} - 1 \right] \times 100. \tag{9.13}$$

As formula (9.8) has been used to compute the changes in H and A using formulae (9.9) and (9.10), formulae (9.11) to (9.13) can be used to compute and report annualized changes in the other partial indices, namely $H, A,\ h_j(k)$ or h_j.

9.2.2 An Example: Analyzing Changes in Global MPI for Four Countries

Table 9.2 presents both the annualized absolute and annualized relative rates of change in Global MPI, as outlined in Chapter 5, and its two partial indices—H and A—for four countries: Nepal, Peru, Rwanda, and Senegal, drawing from Alkire, Roche and Vaz (2014). Taking the survey design into account, we also present the standard errors (in parentheses) and the levels of statistical significance of the rates of reduction, as described in the Appendix of Chapter 8. The figures in the first four columns present the values and standard errors for M_0, H, and A in both time periods. The results show that Peru had the lowest MPI with 0.085 in the initial year, while Rwanda had the highest with 0.460.

Under the heading 'annualized change', Table 9.2 provides the annualized absolute and annualized relative reduction for M_0, H and A, which are computed using equations

(9.12) and (9.13). It shows, for example, that Nepal with a much lower initial poverty level than Rwanda, has experienced a greater absolute annualized poverty reduction of −0.027. In relative terms Nepal outperformed Rwanda. Peru had a low initial poverty level, and reduced it in absolute terms by only −0.006 per year, which means that the share of all possible deprivations among poor people that were removed was only less than one-fourth that of Nepal or Rwanda. But relative to its initial level of poverty, its progress was second only to Nepal. It is thus important to report both absolute and relative changes and to understand their interpretation. The same results for *H* and *A* are provided in the Panel II and III of the table. We see that Nepal reduced the percentage of people who were poor by 4.1 percentage points per year—for example, if the first year 64.7% of people were poor, the next year it would be 60.6%. Peru cut the poverty incidence by 1.3 percentage points per year. Relative to their starting levels, they had similar relative rates of reduction of the headcount ratio. Note that when estimates are reported in percentages, the absolute changes are reported in 'percentage points' and *not* in 'percentages'. Thus, Nepal's reduction in *H* from 64.7% to 44.2% is equivalent to an annualized absolute reduction of 4.1 percentage points and an annualized relative reduction of 6.3%.

Table 9.2: Reduction in Multidimensional Poverty Index, Headcount Ratio and Intensity of Poverty in Nepal, Peru, Rwanda and Senegal

	Year 1	Year 2	Statistical Significance of the Change	Annualized Change	
				Absolute	Relative
Panel I: Multidimensional Poverty Index (MPI)					
Nepal 2006-2011	0.35 -(0.013)	0.217 -(0.012)	***	-0.027	-9.1%
Peru 2005-2008	0.085 -(0.007)	0.066 -(0.004)	*	-0.006	-8.0%
Rwanda 2005-2010	0.46 -(0.005)	0.33 -(0.006)	***	-0.026	-6.4%
Senegal 2005-2010/11	0.44 -(0.019)	0.423 -(0.010)		-0.003	-0.7%
Panel II: Multidimensional Headcount Ratio (*H*)					
Nepal 2006-2011	64.70% -(2.0)	44.20% -(2.0)	***	-4.1	-7.4%
Peru 2005-2008	19.50% -(1.5)	15.70% -(0.8)	*	-1.3	-6.9%
Rwanda 2005-2010	82.90% -(0.8)	66.10% -(1.0)	***	-3.4	-4.4%
Senegal 2005-2010/11	71.30% -(2.4)	70.80% -(1.5)		-0.1	-0.1%
Panel III: Intensity of Poverty (*A*)					
Nepal 2006-2011	54.00% -(0.6)	49.00% -(0.7)	***	-1	-1.9%
Peru 2005-2008	43.60% -(0.5)	42.20% -(0.4)	**	-0.5	-1.1%
Rwanda 2005-2010	55.50% -(0.3)	49.90% -(0.3)	***	-1.1	-2.1%
Senegal 2005-2010/11	61.70% -(1.0)	59.70% -(0.7)	*	-0.4	-0.6%

Note: *** statistically significant at $\omega=0.01$, ** statistically significant at $\omega=0.05$, * statistically significant at $\omega=0.10$. These figures have been computed so as to be strictly comparable with harmonized indicator definitions, and therefore do not match the 'MPI' values released in UNDP reports.
Source: Alkire Roche and Vaz (2014).

The third column provides the results for the hypothesis tests which assess if the reduction between both years is statistically significant.[12] The reductions in M_0 in Nepal and Rwanda are significant at $\omega=0.01$, but the same in Peru is only significant at $\omega=0.10$. Interestingly, the reduction in intensity of poverty in Peru is significant at $\omega=0.05$. The case of Senegal is different in that the small reduction in M_0 is not even significantly different at $\omega=0.10$, preventing the null hypothesis that poverty level in both years remained unchanged from being rejected.

9.2.3 Population Growth and Change in the Number of Multidimensionally Poor

Besides comparing the rate of reduction in M_0, H and A as in Table 9.2, one should also examine whether the number of poor people is decreasing over time. It may be possible that the population growth is large enough to offset the rate of poverty reduction. Table 9.3 uses the same four countries as Table 9.2 but adds demographic information. Nepal had an annual population growth of 1.2% between 2006 and 2011, moving from 25.6 to 27.2 million people, and reduced the headcount ratio from 64.7% to 44.2%. This means that Nepal reduced the absolute number of poor by 4.6 million between 2006 and 2011.

Table 9.3 Changes in the Number of Poor Accounting for Population Growth

	Population			Total MPI Poor		
	Year 1	Year 2	Annual Growth in Population	Year1	Year2	Absolute Reduction
	(in Thousands)			(in Thousands)		
Nepal 2006-2011	25,634	27,156	1.20%	16,585	12,003	-4,582
Peru 2005-2008	27,723	28,626	0.60%	5,406	4,494	-912
Rwanda 2005-2010	9,429	10,837	2.80%	7,817	7,163	-654
Senegal 2005-2010/11	11,271	13,141	3.10%	8,036	9,304	1,267

Note: Population figures correspond to United Nations, Department of Economic and Social Affairs, Population Division (2013), World Population Prospects: The 2012 Revision, DVD Edition. Figures for Senegal 2010/11 correspond to the average between both years.
Source: Authors' presentation, based on Alkire, Roche andVaz (2014).

In order to reduce the absolute number of poor people, the rate of reduction in the headcount ratio needs to be faster than the population growth. The largest reduction in the number of multidimensionally poor has taken place in Nepal. A moderate reduction in the number of poor has taken place in Peru and Rwanda. In contrast, there has been an increase in the total number of multidimensional poor in Senegal, from 8 million to over 9 million between 2005 and 2011.

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[12] For small samples, one needs to conduct hypothesis tests using the Student-t distribution which are very similar to the hypothesis tests described in Chapter 8 that use the Standard Normal distribution.

9.2.4 Dimensional Changes (Uncensored and Censored Headcount Ratios)

The reductions in $M_0, H,$ or A can be broken down to reveal which dimensions have been responsible for the change in poverty. This can be seen by looking at changes in the uncensored headcount ratios (h_j) and censored headcount ratios $(h_j(k))$ described in section 5.5.3. We present the uncensored and censored headcount ratios of MPI indicators for Nepal in Table 9.4 for years 2006 and 2011 and analyse their changes over time. For definitions of indicators and their deprivation cutoffs, see section 5.6. Panel I gives levels and changes in uncensored headcount ratios, i.e. the percentage of people that are deprived in each indicator irrespective of deprivations in other indicators. Panel II provides levels and changes in the censored headcount ratios, i.e. the percentage of people that are multidimensionally poor and simultaneously deprived in each indicator. By definition, the uncensored headcount ratio of an indicator is equal to or higher than the censored headcount of that indicator. The standard errors are reported in the parenthesis.

Table 9.4: Uncensored and Censored Headcount Ratios of the Global MPI, Nepal 2006–2011

	2006	2011	Statistical Significance of the Change	Annualized Reduction	
				Absolute	**Relative**
Panel I: Uncensored Headcount Ratio					
Schooling	30.30% (3.3)	22.20% -(2.6)	***	-1.6	-6.0%
Attendance	16.10% (3.1)	8.40% -(1.9)	***	-1.5	-12.1%
Mortality	32.60% (3.5)	22.60% -(1.9)	***	-2	-7.1%
Nutrition	44.00% (3.5)	32.10% -(2.9)	***	-2.4	-6.1%
Electricity	50.70% (5.6)	24.40% -(4.5)	***	-5.3	-13.6%
Sanitation	75.60% (3.0)	60.30% -(4.0)	***	-3.1	-4.4%
Water	17.10% (3.3)	12.90% -(2.9)	*	-0.8	-5.5%
Flooring	76.70% (4.1)	70.00% -(4.0)	**	-1.3	-1.8%
Cooking fuel	86.80% (2.8)	79.30% -(3.2)	***	-1.5	-1.8%
Assets	59.20% (.0)	28.50% -(2.7)	***	-6.2	-13.6%
Panel II: Censored Headcount Ratio					
Schooling	29.20% (3.3)	20.30% -(2.7)	***	-1.8	-7.0%
Attendance	15.60% (3.2)	8.10% -(1.9)	***	-1.5	-12.3%
Mortality	30.00% (3.8)	18.60% -(2.0)	***	-2.3	-9.2%
Nutrition	40.30% (3.9)	25.30% -(2.9)	***	-3	-8.9%
Electricity	43.40% (5.0)	20.10% -(4.0)	***	-4.7	-14.3%
Sanitation	56.30% (4.2)	38.00% -(4.3)	***	-3.7	-7.6%
Water	14.40% (3.0)	8.80% -(2.4)	***	-1.1	-9.5%
Flooring	60.10% (4.5)	41.80% -(4.1)	***	-3.7	-7.0%
Cooking fuel	63.40% (3.9)	43.00% -(4.1)	***	-4.1	-7.5%
Assets	46.70% (.0)	21.70% -(2.7)	***	-5	-14.2%
Panel III: Dimensional Contribution to MPI					
	2006	**2011**			
Schooling	13.90%	15.60%			

Attendance	7.40%	6.20%
Mortality	14.30%	14.30%
Nutrition	19.20%	19.40%
Electricity	6.90%	5.20%
Sanitation	8.90%	9.70%
Water	2.30%	2.20%
Flooring	9.50%	10.70%
Cooking fuel	10.10%	11.00%
Assets	7.40%	5.60%

Note: *** statistically significant at ω=0.01, ** statistically significant at ω=0.05, * statistically significant at ω=0.10
Source: Alkire, Roche and Vaz (2014).

As we can see in the table, Nepal made statistically significant reductions in all indicators in terms of both uncensored and censored headcount ratios. The larger reductions in censored headcount are observed in electricity, assets, cooking fuel, flooring, and sanitation; all censored headcount ratios have decreased by more than 3 percentage points. Nutrition, mortality, schooling and attendance follow with annual reductions of 3, 2.3, 1.8, and 1.5 percentage points, respectively.

The changes in censored headcount ratios depict changes in deprivations among the poor. Recall that the overall M_0 is the weighted sum of censored headcount ratios of the indicators as presented in equation (5.13) and the contribution of each indicator to the M_0 can be computed by equation (5.14). Because of this relationship, the **absolute rate of reduction** in M_0 in equation (9.8) and the **annualized absolute rate of reduction** in M_0 in equation (9.12) can be expressed as weighted averages of **absolute rate of reductions** in censored headcount ratios and **annualized absolute rate of reductions** in censored headcount ratios, respectively. When different indicators are assigned different weights, the effects of their changes on the change in M_0 reflect these weights.[13] For example, in the MPI, the nutrition indicator is assigned a three times more weight than electricity. This implies that a one percentage point reduction in nutrition *ceteris paribus* would lead to an absolute reduction in M_0 that is three times larger than a one percentage point reduction in the electricity indicator.

Recall that it is straightforward to compute the contribution of each indicator to M_0 using its weighted censored headcount ratio as given in equation (5.14). Note that interpreting the real on-the-ground contribution of each indicator to the *change* in M_0 is not so mechanical. Why? A reduction in the censored headcount ratio of an indicator is not independent of the changes in other indicators. It is possible that the reduction in the

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[13] Normative issues in assigning weights were discussed in details in Chapter 6.

censored headcount ratio of a certain indicator j occurred because a poor person became non-deprived in indicator j. But it is also possible that the reduction occurred because a person who had been deprived in j became non-poor due to reductions in other indicators, even though they remain deprived in j. In the second period, their deprivation in j is now censored because they are non-poor (their deprivation score does not exceed k). The comparison between the uncensored and censored headcount distinguishes these situations. For example, we can see from Panel I of Table 9.4 that the reductions in the uncensored headcount ratios of flooring and cooking fuel are lower than the annualized reductions of the censored headcount ratios of the these two indicators. Thus some non-poor people are deprived in these indicators. In intertemporal analysis it is useful to compare the corresponding censored and uncensored headcount ratios to analyse the relation between the dimensional changes among the poor and the society-wide changes in deprivations. Of course in repeated cross-sectional data, this comparison will also be affected by migration and demographic shifts as well as changes in the deprivation profiles of the non-poor.

Panel III of Table 9.5 presents the contribution of the indicators to the M_0 for Nepal in 2006 and in 2011. The contributions of assets, electricity, and attendance have gone down; whereas the contributions of flooring, cooking fuel, sanitation, and schooling have gone up. The contributions of water, nutrition and mortality have not shown large changes. Dimensional analyses is vital and motivating because any real reduction in a dimensional deprivation will *certainly* reduce M_0. Real reductions are normally those which are visible both in raw and censored headcounts.[14]

9.2.5 Subgroup Decomposition of Change in Poverty

One important property that the adjusted-FGT measures satisfy is population subgroup decomposability, so that the overall M_0 can be expressed as: $M_0 = \sum_{\ell=1}^{m} v^\ell M_0(X^\ell)$, where $M_0(X^\ell)$ denotes the Adjusted Headcount Ratio and $v^\ell = n^\ell/n$ the population share of subgroup ℓ as in equation (5.14). It is extremely useful to analyse poverty changes by population subgroups, to see if the poorest subgroups reduced poverty faster than less poor subgroups, and to see the dimensional composition of reduction across subgroups (Alkire and Seth 2013b; Alkire and Roche 2013; Alkire, Roche and Vaz 2014).

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[14] Comparisons of reductions in both raw and censored headcounts may be supplemented by information on migration, demographic shifts, or exogenous shocks, for example.

Population shares for each time period must be analysed alongside subgroup trends. For example, let us decompose the Indian population into four caste categories: Scheduled Castes (SC), Scheduled Tribes (ST), Other Backward Classes (OBC), and the General category. As Table 9.5 shows, M_0 as well as H have gone down statistically significantly at the national level and across all four subgroups, which is good news. However, the reduction was slowest among STs who were the poorest as a group in 1999, and their intensity showed no significant decrease. Thus, the poorest subgroup registered slowest progress in terms of reducing poverty.

Table 9.5 Decomposition of M_0, H and A across Castes in India

	1999				2006				Change		
	Pop. Share	M_0	H	A	Pop. Share	M_0	H	A	M_0	H	A
SC	18.3%	0.378	68.8%	55.0%	19.1%	0.307	58.3%	52.6%	-0.071 ***	-10.5% ***	-2.3% ***
ST	8.9%	0.458	80.3%	57.0%	8.5%	0.417	74.0%	56.3%	-0.041 ***	-6.3% ***	-0.7%
OBC	32.6%	0.301	57.9%	52.1%	40.2%	0.258	50.8%	50.8%	-0.043 ***	-7.1% ***	-1.3% ***
General	40.1%	0.229	45.2%	50.6%	32.2%	0.164	33.0%	49.7%	-0.065 ***	-12.2% ***	-0.9% **
India	**100%**	**0.300**	**56.8%**	**52.9%**	**100%**	**0.251**	**48.5%**	**51.7%**	**-0.050 *****	**-8.3% *****	**-1.2% *****

Note: *** statistically significant at $\omega=0.01$, ** statistically significant at $\omega=0.05$, * statistically significant at $\omega=0.10$
Source: Alkire and Seth (2013).

To supplement the above analysis it is useful to explore the contribution of population subgroups to the overall reduction in poverty, which not only depends on the changes in subgroups' poverty but also on changes in the population composition. This can be seen by presenting the overall change in M_0 between two periods (t^1, t^2) as

$$\Delta M_0 = \sum_{\ell=1}^{m} \left(v^{\ell, t^2} M_0(X_{t^2}^{\ell}) - v^{\ell, t^1} M_0(X_{t^1}^{\ell}) \right). \qquad \textbf{(9.14)}$$

Note that the overall change depends both on the changes in subgroup M_0's and the changes in population shares of the subgroups.

9.3   Changes Over Time by Dynamic Subgroups

The overall changes in $M_0, H,$ and A discussed thus far could have been generated in many ways. It might be desirable for policy purposes to monitor how poverty changed. In particular, one may wish to pinpoint the extent to which poverty reduction occurred due to people leaving poverty vs. a reduction of intensity among those who remained poor, and also to know the precise dimensional changes which drove each.

For example, a decrease in the headcount ratio by 10% could have been generated by an exit of 10% of the population who had been poor in the first period. Alternatively, it could have been generated by a 20% decrease in the population who had been poor,

accompanied by an influx of 10% of the population who became newly poor. Furthermore, the people who exited poverty could have had high deprivation scores in the first period – that is, been among the poorest – or they could have been only barely poor. The deprivation scores of those entering and leaving poverty will affect the overall change in intensity ΔA as will changes among those who stay poor. In addition these entries into and exits from poverty could have been precipitated by diferent possible increases or decreases in the dimensional deprivations people experienced in the first period, which will then be reflected in the changes in uncensored and censored headcount ratios.

This section introduces more precisely these dynamics of change. We first show what can be captured with panel data, then show empirical strategies to address this situation with repeated cross-sectional data. Finally we present two approaches related to Shapley decompositions which decompose changes precisely, but rely on some crucial assumptions so their empirical accuracy is questionable.

9.3.1 Exits, Entries, and the Ongoing Poor: A Two-Period Panel

Let us consider a fixed set of population of size n across two periods, t^1 and t^2. The achievement matrices of these periods are denoted by X_{t^1} and X_{t^2}. The population can be mutually exclusively and collectively exhaustively categorized into four groups that we refer to as **dynamic subgroups** as follows:

Subgroup N　　　　Contains n^N people who are *non-poor* in *both* periods t^1 and t^2

Subgroup O　　　　Contains n^O people who are *poor* in *both* periods t^1 and t^2 (*ongoing poor*)

Subgroup E^-　　　Contains n^{E^-} people who are poor in period t^1 but *exit poverty* in period t^2

Subgroup E^+　　　Contains n^{E^+} people who are not poor in period t^1 but *enter poverty* in period t^2

We denote the achievement matrices of these four subgroups in period t by X_t^N, X_t^O, $X_t^{E^-}$ and $X_t^{E^+}$ for all $t = t^1, t^2$. The proportion of multidimensionally poor populationin period t^1 is $H(X_{t^1}) = (n^O + n^{E^-})/n$ and that in period t^2 is $H(X_{t^2}) = (n^O + n^{E^+})/n$. The change in the proportion of poor people between these two periods is $\Delta H = H(X_{t^2}) - H(X_{t^1}) = (n^{E^+} - n^{E^-})/n = H(X_{t^2}^{E^+}) - H(X_{t^1}^{E^-})$. In other words, the change in the overall multidimensional headcount ratio is the difference between the proportion of poor entering and the proportion of poor exiting poverty. Note that, by construction, no person is poor in $X_{t^1}^N, X_{t^2}^N, X_{t^2}^{E^-}$, and $X_{t^1}^{E^+}$ and thus $H(X_{t^1}^N) = H(X_{t^2}^N) = H(X_{t^2}^{E^-}) =$

$H\left(X_{t1}^{E^+}\right) = 0$. This also implies, $M_0\left(X_{t1}^N\right) = M_0\left(X_{t2}^N\right) = M_0\left(X_{t2}^{E^-}\right) = M_0\left(X_{t1}^{E^+}\right) = 0$.

On the other hand, all persons in $X_{t1}^{E^-}$, $X_{t2}^{E^+}$, X_{t1}^O, and X_{t2}^O are poor and thus $H\left(X_{t1}^O\right) = H\left(X_{t2}^O\right) = H\left(X_{t1}^{E^-}\right) = H\left(X_{t2}^{E^+}\right) = 1$. Therefore, the M_0 of each of these four subgroups is equal to its intensity of poverty.

In a fixed population, the overall population and the population share of each dynamic group remains unchanged across two time periods.[15] The change in the overall M_0 can be decomposed using equation (9.14) as

$$\Delta M_0 = \frac{n^O}{n}\left(M_0\left(X_{t2}^O\right) - M_0\left(X_{t1}^O\right)\right) - \frac{n^{E^-}}{n}M_0\left(X_{t1}^{E^-}\right) + \frac{n^{E^+}}{n}M_0\left(X_{t2}^{E^+}\right). \qquad \textbf{(9.15)}$$

Thus, the right-hand side of equation (9.15) has three components. The first component$\Delta M_0^O = \frac{n^O}{n}\left(M_0\left(X_{t2}^O\right) - M_0\left(X_{t1}^O\right)\right)$ is due to the change in the intensity of those who remain poor in both periods—the ongoing poor—weighted by the size of this dynamic subgroup. The second component $\Delta M_0^{E^-} = \frac{n^{E^-}}{n}M_0\left(X_{t1}^{E^-}\right)$ is due to the change in the intensity of those who exit poverty (weighted by the size of this subgroup) and the third component $\Delta M_0^{E^+} = \frac{n^{E^+}}{n}M_0\left(X_{t2}^{E^+}\right)$ is due to the population-weighted change in the intensity of those who enter poverty. Together $\Delta M_0 = \Delta M_0^O - \Delta M_0^{E^-} + \Delta M_0^{E^+}$.

From this point there are many interesting possible avenues for analyses. Each group can be studied separately or in different combinations. For policy, it could be interesting to know who exited poverty, and their intensity in the previous period, to see if the poorest of the poor moved out of poverty. The intensity of those who entered poverty shows whether they dipped into the barely poor group, or catapulted into high-intensity poverty, perhaps due to some shock or crisis or (if the population is not fixed) migration. Intensity changes among the ongoing poor show whether their deprivations are declining, even though they have not yet exited poverty. Dimensional analyses of changes for each dynamic subgroup, which are not covered in this book but are straightforward extensions of this material, are also both illuminating and policy relevant.

In the case of panel data with a fixed population we are able to estimate these precisely. We can thus monitor the extent to which the change in M_0 is due to movement into and

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[15] Suitable adjustments can be made for demographic shifts when the population is not fixed across two periods.

out of poverty, and the extent to which it is due to a change in intensity among the ongoing poor population. The example in Box 9.1 may clarify.

Box 9.1 Decomposing the Change in M_0 across Dynamic Subgroups: An Illustration

Consider the following six-person, six-dimension g^0 matrices, in which people enter and exit poverty, and intensity among the poor also increase and decreases.

g^0 Period 1

$$\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 \\ 1 & 1 & 1 & \mathbf{1} & 0 & 0 \\ 1 & 1 & \mathbf{1} & 0 & 0 & 0 \\ 1 & 1 & \mathbf{0} & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 & 0 \end{bmatrix}$$

g^0 Period 2

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 \\ 1 & 1 & 1 & \mathbf{0} & 0 & 0 \\ 1 & 1 & \mathbf{0} & 0 & 0 & 0 \\ 1 & 1 & \mathbf{1} & 0 & 0 & 0 \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 & 0 \end{bmatrix}$$

Let us use a poverty cutoff of 33% or two out of six dimensions. Increases and decreases are depicted in bold. Below we summarize $M_0, H,$ and A in two periods and their changes across two periods.

	t^1	t^2	Δ (Change)
M_0	5/9	11/36	**−1/4**
H	5/6	2/3	**−1/6**
A	2/3	11/24	**−5/24**

So in period two there are four kinds of changes affecting the dynamic subgroups as follows:

1) E^-: persons 1 and 2 become non-poor (move out of or exit poverty)
2) E^+: person 6 enters poverty.
3) O : two kinds of changes occur
    a. deprivations of ongoing poor persons 3 and 4 **reduce** by one deprivation each
    b. deprivations of ongoing poor person 5 **increases** by one deprivation.

The descriptions and the decompositions of M_0 for the changes are in the following table.

Subgroup Decompositions	E^-	E^+	O	Δ
$n^{subgroup}/n$	2/6	1/6	3/6	$\Delta H = \left(-\dfrac{2}{6}\right) + \dfrac{1}{6} = -\dfrac{\mathbf{1}}{\mathbf{6}}$
ΔA	−11/12	3/6	−1/18	
ΔM_0	$-\dfrac{11}{36}$	$\dfrac{1}{12}$	$-\dfrac{1}{36}$	$\Delta M_0 = -\dfrac{11}{36} + \dfrac{3}{36} - \dfrac{1}{36} = -\dfrac{\mathbf{1}}{\mathbf{4}}$
	E		O	
	89%		11%	100%
	E^-	E^+	O	
$\Delta M_0 (\%)$	−122%	+33%	−11%	−100%

What is particularly interesting for policy is that we can notice that, in this example, 11% of the reduction in poverty was due to changes in intensity among the 50% of the population who stayed poor, that poverty was effectively increased 33% by the new entrant, but that this was more than compensated by those who exited poverty (–122%), because they initially had very high intensities. In this dramatic example, the poorest of the poor exited poverty, while the less poor experienced smaller reductions.

9.3.2 Decomposition by Incidence and Intensity for Cross-Sectional Data

The previous section explained the changes for a fixed population over time. To estimate that empirically requires *panel data* with data on the same persons in both periods which can be used to track their movement in and out of poverty. Yet analyses over time are

often based on *repeated cross-sectional data* having independent samples that are statistically representative of the population under study, but that do not to track each specific observation over time. This section examines the decomposition of changes in M_0 for cross-sectional data.

With cross-sectional data, we cannot distinguish between the three groups identified above, nor can we isolate the intensity of those who move into or out of poverty. Observed values are only available for:

$H(X_{t^1}), H(X_{t^2}), \Delta H, A(X_{t^1}), A(X_{t^2}), \Delta A, M_0(X_{t^1}), M_0(X_{t^2})$, and ΔM_0. Using these, it is categorically impossible to decompose ΔM_0 with the empirical precision that panel data permits.

Nonetheless, if required one can move forward with some simplifications. Instead of three groups $(E^-, E^+, and\ O)$ let us consider just two, which might be referred to (somewhat roughly) as *movers* and *stayers*. We define movers as the ΔH people who reflect the net change in poverty levels across the two periods. Stayers are ongoing poor plus the proportion of previously poor people who were replaced by 'new poor', and total those who are poor in period 2 $H(X_{t^2})$. In considering only the 'net' change in headcount, one effectively permits the larger of $E^-\ or\ E^+$ to dominate: if poverty rose nationally, it is the group who entered poverty who dominate; if poverty fell nationally, the group who exited poverty. The subordinate third group is allocated among the ongoing poor and the dominant group. For the remainder of this section we presume that both M_0 and H decreased overall. In this case, $E^- > E^+$. So $\Delta H = (H^{E^-} - H^{E^+})$, and $H(X_{t^2}) = (H^O + H^{E^+})$. As is evident, this simplification is performed because empirical data exist in repeated cross-sections for ΔH and $H(X_{t^2})$.

Example: Suppose that 37% of people are ongoing poor, 3% enter poverty, 13% exit poverty, and 47% remain non-poor. Suppose the overall headcount ratio decreased by 10 percentage points, and the headcount ratio in period 2 is 40%, whereas in period 1 it was 50% (37%+13%). We now primarily consider two numbers: the headcount ratio in period 2 of 40% (interpreted broadly as ongoing poverty) and the change in headcount ratio of 10% (interpreted broadly as moving into/out of poverty). In doing so we are effectively permitting the 'new poverty entrants' to be considered as among the group in ongoing poverty in period 2 (37% + 3% = 40%). To balance this, we effectively replace 3% of those who exited poverty (13% − 3% = 10% = ΔH), and consider this slightly

reduced group to be those who moved out of poverty. If poverty had increased overall, the swaps would be in the other direction.

If poverty has reduced and there has not been a large influx of people into poverty, that is, if $H^{E^+} = n^{E^+}/n$ is presumed to be relatively small empirically, then this strategy would be likely to shed light on the relative intensity levels of those who moved out of poverty $H^{E^-} = n^{E^-}/n$, and the changes in intensity among those who remained poor $H^O = n^O/n$. If empirically H^{E^+} is expected (from other sources of information) to be large, or if their intensity is expected to differ greatly from the average, this strategy is not advised.[16]

Consider the intensity of the net population who exited poverty – under these simplifying assumptions reflected by the net change in headcount, denoted $A^{\hat{E}}$ – and the intensity change of the net ongoing poor, whom we will presume to be $H(X_{t^2})$, denoted $\Delta A^{\hat{O}}$. The ΔM_0 can be decomposed according to these two groups. These decompositions can be interpreted as showing percentage of the change in M_0 that can be attributed to those who moved out of poverty, versus the percentage of change which was mainly caused by a decrease in intensity among those who stayed poor. We use the terms movers and stayers to refer to these less precise dynamic subgroups in cross-sectional data analysis.

$$\Delta M_0 = \underbrace{\Delta H \times A^{\hat{E}}}_{\substack{\text{Movers} \\ \text{effect}}} + \underbrace{H(X_{t^2}) \times \Delta A^{\hat{O}}}_{\substack{\text{Stayers} \\ \text{effect}}}. \qquad \text{(9.16)}$$

Cross sectional data does not provide the intensity of either of those who stayed poor or of those who moved out of poverty. One way forward is to estimate these using existing data. First, identify the $\Delta H \times n$ poor persons having the lowest intensity in the dataset (sampling weights applied), and use the average of these scores for $A^{\hat{E}}$ then solve for $A^{\hat{O}}$.[17] Subsequently, identify the $\Delta H \times n$ poor persons having the highest intensity in

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[16]The corresponding considerations apply if poverty has increased and H^{E^-} is expected to be small

[17] Naturally it is also possible to create estimates for $A^{\hat{O}}$ where the upper bound was the overall change in intensity, and the lower bound was zero, and solve for $A^{\hat{E}}$. However this would not permit an increase in intensity (which would happen if the barely poor left poverty and the others stayed the same, for example), nor for an even stronger reduction in intensity. For example, in the example in Table 9.2, Nepal's **A**

that dataset, and repeat the procedure. This will generate upper and lower estimates for $A^{\hat{E}}$ and $A^{\hat{O}}$ in a given dataset, which will provide an idea of the degree of uncertainty that different assumptions introduce. To estimate stricter upper and lower bounds it could be assumed that those moved out of poverty had an intensity score of the value of k (the theoretically minimum possible), and subsequently assume that their intensity was 100% (the theoretically maximum possible).[18]

Table 9.6 provides the empirical estimations for the upper and lower bounds for the same four countries discussed above plus Ethiopia. At the upper bound those who moved out of poverty could have had average intensities ranging from 59% in Peru (the least poor country) to 99% in Ethiopia or 100% in Senegal, according to the datasets. This in itself is interesting, as it shows that Rwanda—which is the poorest country of the four—had 'movers' with lower average deprivations than Ethiopa. Those who stayed poor would have had, in this case, small if any increases or decreases in intensity—less than four percentage points. At the lower bound, those who moved out of poverty could have had intensities from 33% in Peru and Senegal to 38% in Nepal, and intensity reductions among the ongoing poor could have ranged from 2% in Senegal to 13% in Nepal. At the upper bound (where we assume the poorest of the poor moved out of poverty), for Nepal, Rwanda and Peru, over 100% of the poverty reduction was due to the movers, because intensity among the ongoing poor would have had to increase (to create the observed ΔM_0). At the lower bound, where the least poor people moved out of poverty, movers contributed 47–67% to ΔM_0. Senegal did not have a statistically significant reduction in poverty. Ethiopia provides a different example where the upper and lower bound are closer together and reductions in intensity among the ongoing poor would have contributed 31% to 73%.

Table 9.6 Decomposing the Change in M_0 by Dynamic Subgroups

Panel A

Country	M_0			H		A		ΔH
	t^1	t^2	ΔM_0	t^1	t^2	t^1	t^2	
Ethiopia 2005-2011	0.605	0.523	-0.081	89.7	84.1	67.4	62.3	5.66
Nepal 2006-2011	0.350	0.217	-0.133	64.7	44.2	54.0	49.0	20.55
Peru 2005-2008	0.085	0.066	-0.019	19.5	15.7	43.6	42.2	3.78
Rwanda 2005-2010	0.460	0.330	-0.130	82.9	66.1	55.5	49.9	16.75

reduced by five percentage points, whereas in our upper bound, intensity among the 'stayers' increased by 4% and in the lower bound it decreased by 13%.

[18] These bounds are theoretically possible lower and upper bounds. Further research using panel datasets is required to investigate the likelihood of these bounds.

| Senegal 2005-2010/11 | 0.440 | 0.423 | -0.017 | | 71.3 | 70.8 | | 61.7 | 59.7 | 0.46 | |

Panel B

| | Upper Bound | | | | Lower Bound | | | | Shapley Decomposition | |
Country	*A* Movers	Δ*A* Stayers	Movers Effect	Stayers Effect	*A* Movers	Δ*A* Stayers	Movers Effect	Stayers Effect	Incidence Effect *H*	Intensity Effect *A*
Ethiopia 2005-2011	0.99	-0.03	**68.7%**	**31.3%**	0.38	-0.07	**26.6%**	**73.4%**	45%	55%
Nepal 2006-2011	0.74	0.04	**113.4%**	**-13.4%**	0.38	-0.13	**58.2%**	**41.8%**	79%	21%
Peru 2005-2008	0.59	0.02	**119.5%**	**-19.5%**	0.33	-0.04	**67.1%**	**32.9%**	86%	14%
Rwanda 2005-2010	0.78	0	**101.4%**	**-1.4%**	0.36	-0.1	**47.1%**	**52.9%**	68%	32%
Senegal 2005-2010/11	1	-0.02	**26.8%**	**73.2%**	0.33	-0.02	**8.9%**	**91.1%**	16%	84%

Source: Alkire, Roche and Vaz (2014)

This empirical investigation shows that, when implemented with the mild assumptions that are required for cross-sectional data, the upper and lower bounds according to each country's dataset are very wide apart. In reality, the relative contribution, of the 'movers' and 'stayers' to overall poverty reduction could vary anywhere in this range.

As the example shows, the empirical upper and lower bounds vary greatly across countries. In the case of Ethiopia, movers explain 27% to 69% of the changes in poverty, and stayers account for 31% to 73%. These boundaries do not permit us to assess whether the actual contribution from 'movers' was greater than or less than that of 'stayers'. In Nepal and Peru the 'movers' probably contributed more than 'stayers' to poverty reduction, as in all cases their lowest effect is above 50%. Given these wide-ranging upper and lower bounds, empirically we are unable to answer questions such as whether the intensity of the ongoing poor decreased, or whether it was the barely poor or the deeply poor who moved out of poverty. While this can seem disappointing, for policy purposes, as Sen stresses, it may be better to be 'vaguely right than precisely wrong', and repeated cross-sectional data simply do not permit us, at this time, to bound ahead with precision.

9.3.3 Theoretical Incidence-Intensity Decompositions

Whereas monitoring and policy inputs must be based on empirical analyses, some research topics utilize theoretical analyses. This section introduces two theory-based approaches to decomposing changes in repeated cross-sectional data according to what we call 'incidence' and 'intensity'. In each approach assumptions are made regarding the intensity of those who exit or remain poor. As we have already noted, the task implies some challenges because the empirical accuracy of the underlying assumptions is completely unknown, and as Table 9.6 showed, the actual range may be quite large. These techniques are thus offered in the spirit of academic inquiry.

For simplicity of notation, in this subsection, we denote the M_0, H, and A for period t^1 by $M_0^{t^1}, H^{t^1}$; and A^{t^1} and that for period t^2 by $M_0^{t^2}, H^{t^2}$, and A^{t^2}. The first approach consists in the additive decomposition proposed by Apablaza and Yalonetzky (2013), which is illustrated in Panel A of Figure 9.2. Since $M_0 = H \times A$, they propose to decompose the change in M_0 by changes in its partial indices as follows

$$\Delta M_0 = A^{t^2}\left(H^{t^2} - H^{t^1}\right) + H^{t^2}\left(A^{t^2} - A^{t^1}\right) - \left(\left(H^{t^2} - H^{t^1}\right)\left(A^{t^2} - A^{t^1}\right)\right) \qquad \textbf{(9.17)}$$

$$\underbrace{\phantom{A^{t^2}\left(H^{t^2} - H^{t^1}\right)}}_{\substack{\text{Poverty effect} \\ \text{from entry and exit}}} \underbrace{\phantom{H^{t^2}\left(A^{t^2} - A^{t^1}\right)}}_{\substack{\text{Poverty effect} \\ \text{among ongoing poor}}} \underbrace{\phantom{\left(\left(H^{t^2} - H^{t^1}\right)\left(A^{t^2} - A^{t^1}\right)\right)}}_{\text{Interaction effect}}$$
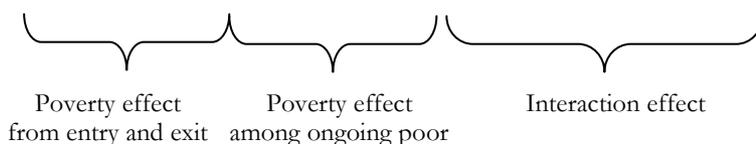
Note that the illustration in Figure 9.2 assumes reductions in M_0, H, and A over time, but the graph can be adjusted to incorporate situations where they do not necessarily fall. This approach involves two assumptions. First, the intensity among those who left poverty is assumed to be the same as the average intensity in period t^2. Second, the intensity change among the ongoing poor is assumed to equal the simple difference in intensities of the poor across the two periods. The decomposition is completed using an interaction term, as depicted in Panel A of Figure 9.2, below. This is indeed an additive decomposition of changes in the Adjusted Headcount Ratio (M_0). Apablaza and Yalonetzky interpret these changes as reflecting: (1) changes in the incidence of poverty (H), (2) changes in the intensity of poverty (A), and (3) a joint effect that is due to interaction between incidence and intensity ($\Delta H \times \Delta A$).

Figure 9.2 Theoretical Decompositions
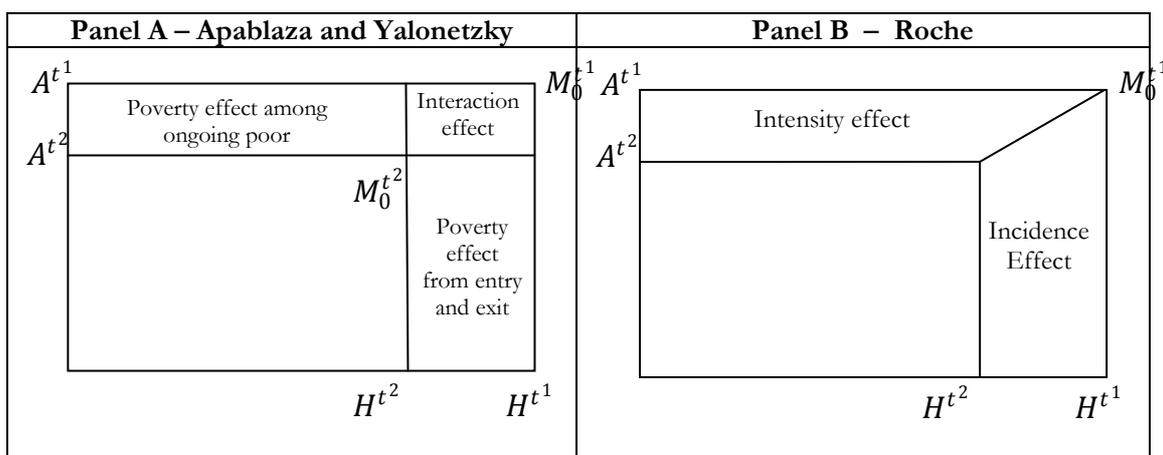
A second theoretical approach corresponds to a Shapley decomposition proposed by Roche (2013). This builds on Apablaza and Yalonetzky (2013) and performs a Shapley

value decomposition following Shorrocks (1999).[19] It provides the marginal effect of changes in incidence and intensity as follows:

$$\Delta M_0 = \underbrace{\frac{A^{t^2} + A^{t^1}}{2}\left(H^{t^2} - H^{t^1}\right)}_{\substack{\text{Incidence of} \\ \text{Poverty effect}}} + \underbrace{\frac{H^{t^2} + H^{t^1}}{2}\left(A^{t^2} - A^{t^1}\right)}_{\substack{\text{Intensity of} \\ \text{Poverty effect}}} \quad \textbf{(9.18)}$$

Panel B of Figure 9.2 illustrates Roche's application of Shapley decompositions, which focuses on the marginal effect without the interaction effect. Roche's proposal assumes that the intensity of those who exited poverty (our terms) is the average intensity of the two periods $\frac{A^{t^2}+A^{t^1}}{2}\left(H^{t^2} - H^{t^1}\right)$, and calls this the 'incidence effect'. He takes the other group as comprising the average headcount ratio between the two periods, and their change in intensity as the simple difference in intensities across the periods $\frac{H^{t^2}+H^{t^1}}{2}\left(A^{t^2} - A^{t^1}\right)$, and describes this as the 'intensity effect'.

Roche's masterly presentation systematically applies Shapley decompositions to each step of dynamic analysis using the AF method. For example, if the underlying assumptions are transparently stated and accepted, the theoretically derived marginal contribution of changes in incidence and marginal contribution of changes in intensity can be expressed as a percentage of the overall change in M_0 so they both add to 100% and can be written as follows

$$\Phi_H^0 = \frac{\left(\frac{A^{t^2} + A^{t^1}}{2}\left(H^{t^2} - H^{t^1}\right)\right) \times 100}{\Delta M_0} \qquad \textbf{(9.19)}$$

$$\Phi_A^0 = \frac{\left(\frac{H^{t^2} + H^{t^1}}{2}\left(A^{t^2} - A^{t^1}\right)\right) \times 100}{\Delta M_0} \qquad \textbf{(9.20)}$$

To address demographic shifts Roche follows a similar decomposition of change as that used in FGT unidimensional poverty measures (Ravallion and Huppi, 1991) and Shapely decompositions (Duclos and Araar 2006, Shorrocks 1999). This approach, presented in

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[19] The Shapley value decomposition was initially applied to decomposition of income inequality by Chantreuil and Trannoy (2011, 2013) and Morduch and Sinclair (1998). Shorrocks (1999) showed that it can be applied to any function under certain assumptions.

Equation (9.21), attribute demographic effects to the average population shares and subgroup M_0's across time. Roche argues that, if the underlying assumptions are accepted, the overall change in poverty level can be broken down in two components: 1) changes due to intra-sectoral or within-group poverty effect, 2) changes due to demographic or inter-sectoral effect. So the overall change in the adjusted headcount between two periods respectively (t^2, t^1), could be expressed as follows

$$\Delta M_0 = \sum_{\ell=1}^{m} \left(\frac{v^{\ell,t^2} + v^{\ell,t^1}}{2} \right) \left(M_0^{\ell,t^2} - M_0^{\ell,t^1} \right) + \sum_{\ell=1}^{m} \left(\frac{M_0^{\ell,t^2} + M_0^{\ell,t^1}}{2} \right) \left(v^{\ell,t^2} - v^{\ell,t^1} \right) \qquad \textbf{(9.21)}$$

Within-group poverty effect   Demographic or sectoral effect

It is common to express the contribution of each factor as a proportion of the overall change, in which case equation (9.21) is divided throughout by ΔM_0.

The last columns of Table 9.6 Panel B provide Shapley decompositions for the same five countries. We see that in all cases the Shapley decompositions lie, as anticipated, between the upper and lower bounds. The Shapley decompositions have the broad appeal of presenting point estimates that pinpoint the exact contribution of each partial index to changes in poverty, according to their underlying assumptions, and thus may be used in analyses when empirical accuracy is not required or the assumptions are independently verified. A full illustration of the Shapley decomposition methods using data on multidimensional child poverty in Bangladesh is given in Roche (2013).[20]

9.4   This Complementary Chronic Poverty with Multiple Time Periods

Panel datasets provide information on precisely the same individual or household at different periods of time. Good quality panel datasets are particularly rich and useful for analysing multidimensional poverty because their analysis provides policy-relevant insights that extend what time series data provide. For example, using panel data we can distinguish the deprivations experienced by the chronically poor from those experienced by the transitory poor and thus identify the combination of deprivations that trap people in long-term multidimensional poverty. Also, we can analyse the duration over which a person was deprived in each indicator—and the sequences by which their deprivation

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[20] The multidimensional poverty index implemented in Roche (2013) focuses on children under 5. The choice of dimensions and indicators are similar to Gordon et al. (2003).

profile evolved. As section 9.3.1 showed, we can identify precisely the contributions to poverty reduction that were generated by changes entries and exits from poverty and by the ongoing poor.

The following section very briefly presents a counting-based class of chronic multidimensional poverty measures that use a triple-cutoff method of identifying who is poor. We give prominence here to the measure that can be estimated using ordinal data. The partial indices associated with the chronic multidimensional poverty methodology include the headcount ratio and intensity, as well as new indices related to the duration of poverty and of dimensional deprivation. We also present a linked measure of transient poverty. As in other sections, we presume that interested readers will master standard empirical and statistical techniques that are appropriate for studies using panel data, and apply these in the analyses of poverty transitions and chronic poverty here described.

The closing section on poverty transitions informally sketches revealing analyses that can be undertaken without generating a chronic poverty measure. Rather, people are identified as multidimensionally poor or non-poor in each period, then population subgroups are identified that have differing sequences of multidimensional poverty. For example, one group might include non-poor people who 'fell' into multidimensional poverty, a second might include multidimensionally poor people who 'rose' out of poverty, a third might contain those who were poor in all periods, and a fourth might contain those who 'churned' in and out of poverty across periods. Naturally the number of 'dynamic subgroups' depends upon the sample design, the number of waves of data and the precise definition of each group.

9.4.1 Chronic Poverty Measurement Using Panel Data

Multiple approaches to measuring chronic poverty in one dimension exist, many of which have implications for the measurement of chronic multidimensional poverty.[21] Alongside important qualitative work, multiple methodologies for measuring chronic multidimensional poverty have also been proposed.[22] This section combines the AF

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[21] This section draws upon Alkire, Apablaza, Chakravarty, and Yalonetzky (2014). See also McKay and Lawson (2003), Dercon and Shapiro (2007), Foster (2009), Foster and Santos (2013), Jalan and Ravallion (1998), Calvo and Dercon (2013), Hoy and Zheng (2011), Gradín, Del Rio, and Canto (2011), and Bossert, Chakravarty, and D'Ambrosio (2012).

[22] Hulme and Shepherd (2003), Chakravarty and D'Ambrosio (2006), Calvo (2008), Addison, Hulme, and Kanbur (2009), Baluch and Masset (2003), Bossert, Ceriani, Chakravarty, and D'Ambrosio (2012), Nicholas and Ray (2011), Nicholas, Ray and Sinha (2013), and Porter and Quinn (2013).

methodology with the counting-based approach to chronic poverty measurement proposed in Foster (2009), which has a dual-cutoff identification structure and aggregation method that are very similar to the AF method. Foster (2009) provides a methodology for measuring unidimensional chronic poverty in which each time period t is equally weighted for all at $t = 1, ..., T$. An $n \times T$ matrix is constructed in which each entry takes a value of one if person i is identified as poor in period t and a value of 0 otherwise. A n-dimensional 'count' vector is constructed in which each entry shows the number of periods in which person i was poor. A second time cutoff τ is applied such that each person is identified as chronically poor if he or she has been poor in τ or more periods. Associated FGT indices and partial indices are then constructed from the relevant censored matrices.

9.4.1.1 Order of Aggregation

This combined chronic multidimensional poverty measure applies three sets of cutoffs: deprivation cutoffs z_j, a multidimensional poverty cutoff k, and a duration cutoff τ. It is possible to analyse multidimensional poverty using panel data by combining the AF methodology and the Foster (2009) chronic poverty methods following two different orders of aggregation, which we call **chronic deprivation** or **chronic poverty**. These alternatives effectively interchange the order in which the poverty and duration cutoffs are applied. In both cases, we first apply a fixed set of deprivation cutoffs to the achievement matrix in each period.

In the **chronic deprivation** option (τ before k), we first consider the duration of deprivation in each indicator for each person and then compute a multidimensional poverty measure which summarizes only those deprivations that have been experienced by the same person across τ or more periods. This approach aggregates all 'chronic' deprivations into a multidimensional poverty index, regardless of the period in which those deprivations were experienced. This approach would provide complementary information that could enrich analyses of multidimensional poverty, but cannot be broken down by time period, nor does it show whether the deprivations were experienced simultaneously (Alkire, Apablaza, Chakravarty, and Yalonetzky 2014).

In the **chronic multidimensional poverty** option (k before τ), we first identify each person as multidimensionally poor or non-poor in each period using the poverty cutoff k. We then count the periods in which each person experienced multidimensional poverty.

We identify as chronically multidimensionally poor those persons who have experienced multidimensional poverty in τ or more periods.

9.4.1.2 Deprivation matrices

We observe achievements across d dimensions for a set of n individuals at T different time points. Let x_{ij}^t stand for the achievement in attribute j of person i in period t, where $x_{ij}^t \geq 0 \; \forall i,j,t$. Let X_t denote a $n \times d$ matrix whose elements reflect the dimensional achievements of the population in period t. The deprivation cutoff vector z_j is fixed across periods. As before, a person is deprived with respect to deprivation j in periods t if $x_{ij}^t < z_j$ and non-deprived otherwise. By applying the deprivation cutoffs to the achievement matrix for each period, we can construct the period-specific deprivation matrices $g_{ij}^{0,t}$. For simplicity, this section uses the non-normalized or numbered weights notation across dimensions, such that $\sum_{j=1}^d w_j = d$. Time periods are equally weighted. When achievement data are cardinal we can also construct normalized gap matrices $g_{ij}^{1,t}$ and squared gap matrices $g_{ij}^{2,t}$ or more generally, powered matrices of normalized deprivation gaps $g_{ij}^{\alpha,t}$ where $\alpha \geq 0$. In a similar manner as previously, we generate the n-dimensional c_i^t column vector, which reflects the weighted sum of deprivations person i experiences in periods t.

9.4.1.3 Identification

To identify who is chronically multidimensionally poor we first construct an identification matrix. The same matrix can be used to identify the transient poor in each period and to create subgroups of those who exhibit distinct patterns of multidimensional poverty (for analysis of poverty transitions).

Identification Matrix

Let $Q(k)$ be an $n \times T$ identification matrix whose typical element $Q_{it}(k)$ is one if person i is identified as multidimensionally poor in period t using the AF methodology, that is, using a poverty cutoff k, which is fixed across periods, and 0 otherwise.

The typical column $Q_{\cdot t}(k)$ reflects the identification status $\rho_i^t(k)$ for the ith person in period t, whereas the typical row $Q_{i\cdot}(k)$ displays the periods in which person i has been

identified as multidimensionally poor (signified by an entry of 1) or non-poor (0). Thus we might equivalently consider each column of $Q(k)$ to be an identification column vector for period t such that $\rho_i^t(k) = 1$, if and only if person i is multidimensionally poor in period t according to the deprivation cutoffs z_j, weights w_j, and poverty cutoff k; and $\rho_i^t(k) = 0$ otherwise.

Episodes of Poverty Count Vector e_i

From the $Q(k)$ matrix we construct the n-dimensional column vector $e(k)$ whose i^{th} element $e_i(k) = \sum_{t=1}^{T} Q_{it}(k)$ sums the elements of the corresponding row vector of $Q(k)$ and provides the total number of periods in which person i is poor, or the total episodes of poverty, as identified by poverty cutoff k. Naturally, $0 \leq e_i(k) \leq T$, that is, each person may have from 0 episodes of poverty to T episodes, the latter indicating that a person was poor in each of the t periods.

Chronic Multidimensional Poverty: Identification and Censoring

We apply the duration cutoff τ where $0 < \tau \leq T$ to the $e_i(k)$ vector in order identify the status of each person as chronically multidimensionally poor or not. We identify a person to be chronically multidimensionally poor if $e_i(k) \geq \tau$. That is, if they have experienced τ or more periods in multidimensional poverty. A person is considered non-chronically poor if $0 \leq e_i(k) < \tau$. We doubly censor the $e_i(k)$ vector such that it takes the value of 0 (non-chronically poor) if $0 \leq e_i(k) < \tau$ and takes the value of $e_i(k)$ otherwise. The notation $e_i(k, \tau)$ indicates the censored vector of poverty episodes—just as the notation $c_i(k)$ indicated the censored deprivation count vector. Positive entries reflect the number of periods in which chronically poor people experienced poverty; entries of 0 mean that the person is not identified as chronically poor.

Among the non-chronically poor, we could (as we will elaborate) identify two subgroups: the non-poor and the transient poor. A person is considered transiently poor if $0 < e_i(k) < \tau$. And naturally a person for whom $e_i = 0$, that is, who is non-poor in all periods, is considered non-poor.

An alternative but useful notation for the identification of chronic multidimensional poverty uses the identification function: we apply the identification function $\rho(\tau)$ to censor the $Q(k)$ matrix and the $e_i(k)$ vector. The doubly censored Q matrix reflects

solely those periods in poverty that are experienced by the chronically poor (censoring all periods of transient poverty) and is denoted by $Q(k, \tau)$. After censoring, the typical element $e_i(k, \tau)$ is defined by $\rho_i(\tau) = \mathbb{I}(e_i(k) \geq \tau)$. The entry takes a value of $e_i(k)$ if person i is chronically multidimensionally poor and 0 otherwise.

Censored (k, τ) Deprivation Matrices and Count Vector

To identify the censored headcounts, as well as the dimensional composition of poverty in each period, we censor the T sets of $n \times d$ deprivation matrices by applying the twin identification functions $\rho^t(k)$ and $\rho(\tau)$. We denote the censored matrices by $g^{0,t}(k, \tau)$ and the censored deprivation count vectors for each period $c_i^t(k, \tau)$.

Deprivation Duration Matrix

Finally, to summarize the overall dimensional deprivations of the poor, as well as the duration of these deprivations, it will be useful to create a duration matrix based on the censored deprivation matrices $g_0(k, \tau)$. Let L be an $n \times d$ matrix whose typical element L_{ij} provides the number of periods in which is chronically poor and is deprived in dimension j. Note that $0 \leq L_{ij} \leq T$. We can use the duration matrix to obtain the deprivation-specific duration indices, which show the percentage of periods in which, on average, poor people were deprived in each indicator.

9.4.1.4 Aggregation

The measure of chronic multidimensional poverty when some data are ordinal may be written as follows:

$$M_0^C(X; z) = H^C \times A^C \times D^C = \frac{1}{ndT} \sum_{i=1}^{n} \sum_{j=1}^{d} \sum_{t=1}^{T} w_j \, g_{ij}^{0,t}(k, \tau). \qquad \text{(9.22)}$$

Thus the Adjusted Headcount Ratio of chronic multidimensional poverty M_0^C is the mean of the set of T deprivation matrices $(g^{0,t}(k, \tau))$ that have been censored of all deprivations of persons who are not chronically multidimensionally poor. Alternative notation for this measure can be found in Alkire, Apablaza, Chakravarty, and Yalonetzky (2014).

When data are cardinal, the M_α^C class of measures are, like the AF class, the means of the respective powered matrices of normalized gaps.

$$M_\alpha^C(X;z) = \frac{1}{ndT} \sum_{i=1}^{n} \sum_{j=1}^{d} \sum_{t=1}^{T} w_j \, g_{ij}^\alpha(k,\tau). \tag{9.23}$$

9.4.2 Properties

For chronic multidimensional poverty, as for multidimensional poverty, the specification of axioms is, in some cases, a joint restriction on the (triple-cutoff) identification and aggregation strategies and, hence, on the overall poverty methodology. The properties are now defined with respect to the chronically multidimensionally poor population. The class of measures present respects the key properties that were highlighted as providing policy relevance and practicality to the AF measures, such as subgroup consistency and decomposability, dimensional monotonicity, dimensional breakdown, and ordinality. In addition, this class of measures satisfies a form of **time monotonicity** as highlighted in Foster (2009) in the unidimensional case. The intuition is that if a person who is chronically poor becomes poor in an additional period, poverty rises.

A full definition of the properties that this chronic multidimensional poverty measure fulfils is provided in Alkire, Apablaza, Chakravarty, and Yalonetzky (2014). The methodology of multidimensional chronic poverty measurement fulfils the appropriately stated properties of anonymity, time anonymity, population replication invariance, chronic poverty focus, time focus, chronic normalization, chronic dimensional monotonicity, chronic weak monotonicity, time monotonicity, chronic monotonicity in thresholds, monotonicity in multidimensional poverty identifier, chronic duration monotonicity, chronic weak transfer, non-increasing chronic poverty under association-decreasing switch, and additive subgroup decomposability for all $\alpha \geq 0$. The class of measures also satisfies chronic strong monotonicity for $\alpha > 0$ and chronic strong transfer when $\alpha \geq 1$.

9.4.3 Consistent Partial Indices

Like M_0, the chronic multidimensional poverty measure M_0^C is the product of intuitive partial indices that convey meaningful information on different features of a society's experience of chronic multidimensional poverty. In particular, $M_0^C(X;z) = H^C \times A^C \times D^C$ where:

- H^C is the headcount ratio of chronic multidimensional poverty—the percentage of the population who are chronically multidimensionally poor according to k and τ.
- A^C is the average intensity of poverty among the chronically multidimensionally poor—the average share of weighted deprivations that chronically poor people experience in those periods in which they are multidimensionally poor.
- D^C reflects the average duration of poverty among the chronically poor — the average share of T periods in which they experience multidimensional poverty.

These partial indices can also be calculated directly. In particular,

$$H^C = \frac{1}{n} \sum_{i=1}^{n} \rho_i(k;\tau) = \frac{q^C}{n}. \tag{9.24}$$

That is, the headcount ratio of chronic multidimensional poverty is the number of people who have been identified as chronically multidimensionally poor divided by the total population. We denote the number of chronically multidimensionally poor people by q^C.

The intensity of chronic multidimensional poverty is the sum of the weighted deprivation scores of all poor people over all time periods, divided by the number of dimensions times the total number of people who are poor in each period summed across periods. Note that $\frac{k}{d} \leq A^C \leq 1$.

$$A^C = \frac{\sum_{i=1}^{n} \sum_{t=1}^{T} c_i^t(k,\tau)}{d \times \sum_{i=1}^{n} \sum_{t=1}^{T} Q(k,\tau)}. \tag{9.25}$$

The average duration of chronic multidimensional poverty—the percentage of periods on average in which the chronically person was poor—can be easily assessed using the $e_i(k,\tau)$ vector.

$$D^C = \frac{\sum_{i=1}^{n} e_i(k,\tau)}{q^C \times T}. \tag{9.26}$$

The duration is the sum of the total number of periods in which the chronically poor experience multidimensional poverty, divided by the number of periods and the number of chronically poor. Note that $\frac{\tau}{T} \leq D^C \leq 1$. Box 9.2 illustrates these with a simple example.

Box 9.2 Computing Incidence and Duration of Chronic Poverty

Consider three people and four periods, with $\tau = 2$.

> Person 1 is multidimensionally poor in period 1
> Person 2 is multidimensionally poor in periods 2, 3, and 4
> Person 3 is multidimensionally poor in periods 1, 2, 3, and 4

Two people are chronically poor because they experience multidimensional poverty in $\tau = 2$ or more periods. So the percentage of people identified as chronically poor H^C is 67%.

In this case, the vector $e = (0,3,4)$; $q^C = 2$ and our duration index is $\frac{(3+4)}{2 \times 4} = \frac{7}{8} = 87.5\%$. That is, on average, chronically poor persons are poor during 87.5% of the time periods.

9.4.3.1 Dimensional Indices

For chronic multidimensional poverty, it is possible and useful to generate the standard dimensional indicators for each period: the censored headcount and percentage contribution. It is also possible and useful to generate the period-specific Adjusted Headcount Ratio (M_0), headcount ratio (H), and intensity (A) figures, which are different from, but can be consistently related to, the chronic poverty headcount and intensity values presented in 9.4.3. Finally, and of tremendous use, it is possible to present the average duration of deprivation in each dimension and to relate this directly to the overall duration of chronic poverty. Box 9.3 presents the intuition of this set of consistent indices; for their precise definition see Alkire, Apablaza, Chakravarty, and Yalonetzky (2014).

Box 9.3 Computing Incidence and Duration of Chronic Poverty

Cross-period indices reflecting chronic poverty:

M_0^C: Adjusted Headcount Ratio of chronic multidimensional poverty

H^C: Headcount ratio, showing the percentage of the population who are chronically poor

A^C: Intensity, showing the average percentage of deprivations experienced by the chronically multidimensionally poor in those periods in which they are poor

D^C: Average duration of chronic poverty, expressed as a percentage of time periods

$h_j^C(k, \tau)$: Average censored headcount of dimension j among the chronically poor in all periods in which they are poor and are deprived in dimension j

D_j: Average duration of deprivation in dimension j among the chronically poor, expressed as a percentage of time periods

$\phi_j^C(k, \tau)$: Percentage contribution of dimension j to the deprivations of the chronically poor.

Single-period indices reflecting the profiles of the chronic poor in that particular period of poverty:

H^t: Headcount ratio, showing the percentage of the population who are chronically poor in period t

A^t: Intensity, showing the average percentage of deprivations experienced by the chronically multidimensionally poor in period t

$h_j^t(k, \tau)$: Censored headcount of dimension j among the chronically poor in period t

$\phi_j^t(k, \tau)$: Percentage contribution of dimension j to the deprivations of the chronically poor in period t.

Cross-period averages of the unidimensional indices can also be constructed, such as $\bar{H}, \bar{A}$ and $\bar{h}_j^t(k, \tau)$ and analysed in conjunction with the relevant duration measure.

9.4.3.2 Censored Headcount Ratios

The censored headcount ratios for each period t are constructed as the mean of the dimensional column vector for each period and represent the proportion of people who are chronically poor in time period t and are deprived in dimension j:

$$h_j^t(k, \tau) = \mu\left(g_j^{0,t}(k, \tau)\right). \tag{9.27}$$

We can also describe the average censored headcount ratios of chronic multidimensional poverty across T periods in each dimension as simply the mean of the censored headcounts in each period:

$$h_j^C(k, \tau) = \frac{1}{T}\sum_{t=1}^{T} h_j^t(k, \tau). \tag{9.28}$$

The Adjusted Headcount Ratio of chronic multidimensional poverty across all periods is simply the mean of the average weighted censored headcount ratios:

$$M_0^C = \frac{1}{d}\sum_{j=1}^{d} w_j\, h_j^C(k,\tau) = \frac{1}{Td}\sum_{j=1}^{d}\sum_{t=1}^{T} w_j\, h_j^t(k,\tau). \qquad \textbf{(9.29)}$$

9.4.3.3 Percentage Contributions of Dimension

The percentage contributions show the (weighted) composition of chronic multidimensional poverty in each period and across periods.

We may seek an overview of the dimensional composition of poverty across all periods. The total percentage contribution of each dimension to chronic poverty across all periods is given by

$$\phi_j^C(k,\tau) = \frac{h_j^C(k,\tau)}{d \times M_0^C}. \qquad \textbf{(9.30)}$$

We may also be interested in analysing the percentage contributions of each dimension across various periods and thus in comparing the percentage contributions of dimensions across periods. The total percentage contribution in period t is

$$\phi_j^t(k,\tau) = \frac{h_j^t(k,\tau)}{d \times M_0^t}. \qquad \textbf{(9.31)}$$

9.4.3.4 Censored Dimensional Duration

We are also able to construct a new set of statistics that provide more detail regarding the duration of dimensional deprivations among the chronically poor. We use the $n \times d$ deprivation duration matrix L, constructed in 9.4.1.3, in which each entry reflects the number of periods in which person i was chronically poor (by k and τ) and was deprived in dimension j. Recall that for the chronically poor, $0 \le L_{ij} \le T$ in each dimension. The value of L_{ij} is, naturally, 0 for non-poor persons in all dimensions. Thus the matrix will have a positive entry for q^C persons and an entry of 0 for all persons who were never chronically poor.

For each dimension we can then define a dimensional duration index for dimension j as follows

$$D_j = \frac{1}{q^C \times T}\sum_{i=1}^{n} L_{ij}. \qquad \textbf{(9.32)}$$

The value of D_j provides the percentage of periods in which the chronically poor were deprived in dimension j on average.

The relationship between the mean across all D_j and the chronic multidimensional poverty figure provided earlier is also elementary

$$M_0^C = H^C \sum_{j=1}^{d} w_j D_j, \tag{9.33}$$

and

$$\sum_{j=1}^{d} w_j D_j = A^C \times D^C. \tag{9.34}$$

9.4.3.5 Period-Specific Partial Indices

From the $n \times T$ censored identification matrix $Q(k, \tau)$, we can also compute the period-specific headcounts of chronic multidimensional poverty. The headcount H^t for period t is the mean of the column vector of $Q(k, \tau)$ for period t. The average headcount across all periods is $\bar{H} = (1/T) \sum_{t=1}^{T} H^t$. The average headcount across all periods and the chronic multidimensional poverty headcount are related by the average duration of poverty thus:

$$H^C = \frac{\bar{H}}{D^C}. \tag{9.35}$$

Similarly,

$$A^C = \frac{\bar{A}}{D^C}, \tag{9.36}$$

and

$$h_j^C(k, \tau) = \frac{\bar{h}_j(k, \tau)}{D_j}. \tag{9.37}$$

9.4.3.6 Illustration using Chilean CASEN

We present an example in Table 9.7 using three variables: schooling, overcrowding, and income in Chile's CASEN (Encuesta de Caracterización Socioeconómica Nacional) dataset for three periods: 1996, 2001, and 2006. The table reports the Adjusted Headcount Ratio of chronic multidimensional poverty (M_0^C) and its three partial indices:

the headcount ratio (H^C), the average chronic intensity (A^C) and the average duration (D^C) for three poverty cutoffs $k = 1/3, 2/3, 1$ and three different duration cutoffs $t = 1/3, 2/3, 1$. All dimensions and periods are equally weighted for both identification and aggregation. When $k = 1/3$ and $\tau = 1/3$, the identification follows a double union approach. In this case, 49% of people are identified as chronically multidimensionally poor. However, a double-union approach does not appear to capture people who are either chronically or multidimensionally poor in any meaningful sense.

Table 9.7 Cardinal Illustration with Relevant Values of k and τ

	$k = 1/3$			$k = 2/3$			$k = 1$		
	$\tau = 1/3$	$\tau = 2/3$	$\tau = 1$	$\tau = 1/3$	$\tau = 2/3$	$\tau = 1$	$\tau = 1/3$	$\tau = 2/3$	$\tau = 1$
M_0^c	0.124	0.095	0.049	0.053	**0.028**	0.007	0.008	0.003	0.000
H^c	**0.49**	0.27	0.10	0.16	**0.05**	0.01	0.02%	0.005	0.0005
D^c	0.58	0.79	1.00	0.46	**0.72**	1.00	0.43	70.01%	1.00
A^c	0.43	0.45	0.48	0.70	**0.72**	0.75	1.00	1.00	1.00

We thus consider the cutoffs where $k = \tau = 2/3$. In this situation, 5% of people are chronically multidimensionally poor. The average chronic intensity is 72%, meaning that people experience deprivations in 72% of dimensions in the periods in which they are poor. The average chronic duration is 72% also, meaning that the average poor person is deprived in 72% of the three periods. The overall chronic Adjusted Headcount Ratio of 0.028 shows that Chile's population experiences only 2.8% of the deprivations it could possibly experience. All possible deprivations occur if all people are multidimensionally poor in all dimensions, and in all periods.

9.4.4 Poverty Transitions Using Panel Data

Using the identification matrix and the associated doubly censored deprivation matrices that have been constructed above, it is also possible to analyse poverty transitions. Comparisons can be undertaken – for example, between subgroups experiencing different dynamic patterns of multidimensional poverty – to ascertain different policy sequences or entry points that might have greater efficacy in eradicating multidimensional poverty. This section very briefly describes the construction of dynamic subgroups and some of the descriptive analyses that can be undertaken.

9.4.4.1 Constructing Dynamic Subgroups

The chronic multidimensional poverty measures constructed previously respect the property of subgroup consistency and subgroup decomposability; thus, they can be decomposed by any population subgroup for which the data are arguably representative. In addition, it can be particularly useful to describe multidimensional poverty for what we earlier called 'dynamic subgroups' – the definition of which can be extended when panel data cover more than two periods.

By 'dynamic subgroups', we mean population subgroups that experience different patterns of multidimensional poverty over time. These include the groups mentioned in section 9.3.1 who exited poverty (1,0), entered poverty (0,1), or were in ongoing poverty (1,1). The possible patterns will vary according to the number of waves in the sample as well as the observed patterns in the dataset. With three waves, there are four basic groups: **falling**—people who were non-poor and became multidimensionally poor; **rising**—people who were multidimensionally poor and exited poverty; **churning**—people who both enter and exit multidimensional poverty in different periods, and **long-term**—people who remain multidimensionally poor continuously.[23]

The dynamic subgroups are formed by considering the $n \times T$ identification matrix $Q(k)$. Note that we use the matrix that is censored by the poverty cutoff k but we do not, in this section on poverty transitions, apply the duration cutoff. Consider a matrix of four persons and three periods in which each person experiences one of the four categories mentioned above. Recall that an entry of one indicates that person i is multidimensionally poor in period t and a 0 indicates they are non-poor.

Falling:	0 0 1	(and 0 1 1)
Rising:	1 0 0	(and 1 1 0)
Churning:	1 0 1	(and 0 1 0)
Long-term:	1 1 1	

For more than three periods, additional categories can be formed. Note that the categories can and must be mutually exhaustive. Each person who is multidimensionally poor in any period (whether chronically or transiently poor) can be categorized into one of these four groups.

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[23] See Hulme, Moore and Shepherd (2001), Hulme and Shepherd (2003), and Narayan and Petesch (2007).

9.4.4.2 Descriptive Analyses

Having decomposed the population into the non-poor and these (or additional) dynamic subgroups of the population, it can be useful to provide the standard partial indices for each subgroup, both per period and across all three periods:

- M_0, H, and A (and standard errors);
- Percentage composition of poverty by dimension (often revealing) and censored headcounts;
- Intensity profiles across the poor (or inequality among the poor—see section 9.1).

It can also be useful to provide details regarding the sequences of evolution. For example, from the $Q(k)$ matrix, isolate the subgroup of the poor who 'fell into' poverty between period 1 and period 2 (that is, whose entries are 0,1 for the respective periods t_1 and t_2). Compare their evolution with those who stayed poor (1,1) and those who stayed non-poor (0,0), in the following ways:

- At the individual level, compare the raw headcount in period 1 with the raw headcount in period 2.
- Identify the dimensions in which deprivations were (a) experienced in both periods, (b) only experienced in period 1, and (c) only experienced in period 2.
- Summarize the results, if relevant and legitimate, further decomposing the population into relevant subgroups whose compositional changes follow different patterns.
- Repeat for each adjacent pair of periods. Analyse whether the patterns are stable or differ across different adjacent periods.

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