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Counting and Multidimensional Poverty Measurement

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Abstract

This paper proposes a new methodology for multidimensional poverty measurement consisting of an identification method ρ_k that extends the traditional intersection and union approaches, and a class of poverty measures M_α. Our identification step employs two forms of cutoff: one within each dimension to determine whether a person is deprived in that dimension, and a second across dimensions that identifies the poor by 'counting' the dimensions in which a person is deprived. The aggregation step employs the FGT measures, appropriately adjusted to account for multidimensionality. The axioms are presented as joint restrictions on identification and the measures, and the methodology satisfies a range of desirable properties including decomposability. The identification method is particularly well suited for use with ordinal data, as is the first of our measures, the adjusted headcount ratio. We present some dominance results and an interpretation of the adjusted headcount ratio as a measure of unfreedom. Examples from the US and Indonesia illustrate our methodology.

Keywords: multidimensional poverty measurement, capability approach, identification, FGT measures, decomposability, ordinal variables.

JEL classification: I3, I32, D63, O1, H1.

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The Oxford Poverty and Human Development Initiative (OPHI) is a research centre within the Oxford Department of International Development, Queen Elizabeth House, at the University of Oxford. Led by Sabina Alkire, OPHI aspires to build and advance a more systematic methodological and economic framework for reducing multidimensional poverty, grounded in people's experiences and values.

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1. INTRODUCTION

MULTIDIMENSIONAL POVERTY has captured the attention of researchers and policymakers alike due, in part, to the compelling conceptual writings of Amartya Sen and the unprecedented availability of relevant data.[1] A key direction for research has been the development of a coherent framework for measuring poverty in the multidimensional environment that is analogous to the set of techniques developed in unidimensional space. Recent efforts have identified several classes of multidimensional poverty measures, discussed their properties, and raised important issues for future work.[2]

This literature, however, has two significant challenges that discourage the empirical use of these conceptually attractive measures. First, the measurement methods are largely dependent on the assumption that variables are cardinal, when, in fact, many dimensions of interest are ordinal or categorical.[3] Second, the method for identifying the poor remains understudied: most presentations either leave identification unspecified or select criteria that seem reasonable over two dimensions, but become less tenable when additional dimensions are used. These challenges are especially pertinent given that many countries are actively seeking multidimensional poverty measures to supplement or replace official income poverty measures.

The goal of this paper is to present a new methodology that addresses these substantive issues. In recent work, Atkinson (2003) discussed an intuitive 'counting' approach to multidimensional poverty measurement that has a long history of empirical

[1] See, for example, Sen 1980, 1985a, 1985b, 1987, 1992, 1993.
[2] Anand and Sen 1997, Brandolini and D'Alessio 1998, Atkinson 2003, Deutch and Silber 2005 and Thorbecke 2008 identify cross-cutting issues. The main approaches to multidimensional poverty measures are axiomatic (Chakravarty 1998, Tsui 2002, Bourguignon and Chakravarty 2003, Chakravarty and Silber 2008); information theoretic (Maasoumi and Lugo 2008), fuzzy set (Cerioli and Zani 1990, Chiappero-Martinetti 1994, 2000, Lemmi and Betti 2006) and latent variable (Kakwani and Silber 2008b).
[3] See Atkinson 2003, Duclos et al 2007 for related discussions.

implementation but thus far has largely been disconnected from the aforementioned literature.[4] Our approach effectively melds these two approaches: We use a 'counting' based method to identify the poor, and propose 'adjusted FGT' measures that reflect the breadth, depth and severity of multidimensional poverty.[5]

In particular, we introduce an intuitive approach to identifying the poor that uses two forms of cutoffs. The first is the traditional dimension-specific line or cutoff, which identifies whether a person is deprived with respect to that dimension. The second delineates how widely deprived a person must be in order to be considered poor.[6] Our benchmark procedure uses a counting methodology in which the second cutoff is a minimum number of dimensions of deprivation; the procedure readily generalizes to situations in which dimensions have differential weights. This 'dual cutoff' identification system gives clear priority to those suffering multiple deprivations and works well in situations with many dimensions.

Our adjusted FGT measures are easy to interpret and directly generalize the traditional FGT measures. The 'adjusted headcount' measure applies to ordinal data and provides information on the breadth of multiple deprivations of the poor. It has a natural interpretation as a measure of 'unfreedom' and generates a partial ordering that lies between first and second order dominance.

The overall methodology satisfies useful properties including decomposability. It can be readily applied to existing data and can be unpacked to reveal the dimensional deprivations contributing most to poverty (a property not available to the standard headcount ratio). It embodies Sen's (1993) view of poverty as capability deprivation

[4] Examples from this literature include Mack and Lansley 1985, Erikson 1993, and Gordon 2003. Closely related papers include Chakravarty and D'Ambrosio 2006, Bossert et al 2007, Jayaraj and Subramanian 2007, and Calvo 2008.
[5] The unidimensional FGT poverty measures were introduced in Foster, Greer, and Thorbecke, 1984.
[6] In this paper we use the term 'deprived' to indicate that a person's achievement in a given dimension falls below its cutoff. If a person meets the multidimensional identification criterion, then the person is considered to be 'poor', and the condition is called 'poverty'.

and is motivated by Atkinson's (2003) discussion of counting methods for measuring deprivations.[7]

An important consideration in developing a new methodology for measuring poverty is that it can be employed using real data to obtain meaningful results. We provide illustrative examples using data from the US and Indonesia. Our results suggest that the methodology we propose is intuitive, satisfies useful properties, and can be applied to good effect with real world data.

We begin with some basic definitions and notation for multidimensional poverty in section 2, and then section 3 introduces our dual cutoff identification approach. The adjusted FGT family of poverty measures is presented in section 4, while section 5 introduces general weights. Section 6 provides a list of axioms satisfied by the combined methodology, section 7 focuses on the special properties of the adjusted headcount ratio, and section 8 discusses the choice of cutoffs. Empirical applications are presented in section 9 while a final section offers some closing observations.

2. NOTATION

Let n represent the number of persons and let $d\geq 2$ be the number of dimensions under consideration. Let $y=[y_{ij}]$ denote the $n\times d$ matrix of achievements, where the typical entry $y_{ij}\geq 0$ is the achievement of individual $i=1,2,\ldots,n$ in dimension $j=1,2,\ldots,d$. Each row vector y_i lists individual i's achievements, while each column vector y_{*j} gives the distribution of dimension j achievements across the set of individuals. In what follows we assume that d is fixed and given, while n is allowed to range across all positive integers; this allows poverty comparisons to be made across populations of different sizes. Thus the domain of matrices under consideration is given by $Y=\{y \in R_+^{nd} : n\geq 1\}$. For concreteness, we have assumed that individual achievements can be

[7] The question of how to select capabilities or dimensions for evaluation is relevant but not addressed here: see Sen 1992, 1993, 2004a, 2004b, Atkinson *et al* 2002, Robeyns 2005 and Alkire 2008.

any non-negative real; our approach can easily accommodate larger or smaller domains where appropriate. Let $z_j>0$ denote the cutoff below which a person is considered to be deprived in dimension j, and let z be the row vector of dimension-specific cutoffs. For any vector or matrix v, we use the expression $|v|$ to denote the sum of all of its elements, while $\mu(v)$ represents the mean of v, or $|v|$ divided by the total number of elements in v.

A methodology $\mathcal{M}$ for measuring multidimensional poverty is made up of an identification method and an aggregate measure (Sen 1976). Following Bourguignon and Chakravarty (2003) we represent the former using an *identification function* $\rho: R^d_+ \times R^d_{++} \to \{0,1\}$, which maps from person i's achievement vector $y_i \in R^d_+$ and cutoff vector z in R^d_{++} to an indicator variable in such a way that $\rho(y_i;z)=1$ if person i is poor and $\rho(y_i;z)=0$ if person i is not poor.[8] Applying ρ to each individual achievement vector in y yields the set $Z\subseteq\{1,\ldots,n\}$ of persons who are poor in y given z. The aggregation step then takes ρ as given and associates with the matrix y and the cutoff vector z an overall level $M(y;z)$ of multidimensional poverty. The resulting functional relationship $M: Y\times R^d_{++} \to \mathrm{R}$ is called an *index*, or *measure*, of multidimensional poverty.[9] A methodology is then given by $\mathcal{M}=(\rho,M)$.

In what follows, it will prove useful to express the data in terms of deprivations rather than achievements. For any given y, let $g^0=[g^{\mathbf{0}}_{ij}]$ denote the 0-1 *matrix of deprivations* associated with y, whose typical element $g^{\mathbf{0}}_{ij}$ is defined by $g^{\mathbf{0}}_{ij}=1$ when $y_{ij}<z_j$, while $g^{\mathbf{0}}_{ij}=0$ otherwise. Clearly, g^0 is an $n\times d$ matrix whose ij^{th} entry is 1 when person i is deprived in the j^{th} dimension, and 0 when the person is not. The i^{th} row

[8] Note that this representation assumes that the underlying identification method is individualistic (in that i's poverty status depends on y_i) and symmetric (in that it uses the same criterion for all persons). It would be interesting to explore a more general identification function which abstracts from these assumptions.

[9] A 'poverty focus axiom' ensures coherence between the identification function and the poverty measure; see section 6 below.

vector of g^0, denoted $g_i^{\mathbf{0}}$, is person i's *deprivation vector*. From the matrix g^0 we can construct a column vector c of *deprivation counts*, whose i^{th} entry $c_i=|g_i^{\mathbf{0}}|$ represents the number of deprivations suffered by person i. The vector c will be especially helpful in describing our method of identification. Notice that even when the variables in y are only ordinally significant, g^0 and c are still well defined. In other words, g^0 and c are identical for all monotonic transformations of y_{ij} and z_j.

When all variables in y are cardinal, the associated matrix of (normalised) gaps or shortfalls can provide additional information for poverty evaluation. For any y, let g^1 be the matrix of *normalised gaps*, where the typical element is defined by $g_{ij}^{\mathbf{1}}=g_{ij}^{\mathbf{0}}(z_j-y_{ij})/z_j$. Clearly, g^1 is an $n\times d$ matrix whose entries are nonnegative numbers less than or equal to 1, with $g_{ij}^{\mathbf{1}}$ being a measure of the extent to which that person i is deprived in dimension j. In general, for any $\alpha>0$, define the matrix g^α by raising each entry of g^1 to the power α; e.g. when $\alpha=2$, the entry is $g_{ij}^2=(g_{ij}^1)^2$. This notation will be useful below in defining our generalisation of the FGT measures to the multidimensional environment.

3. IDENTIFYING THE POOR

Who is poor and who is not? Bourguignon and Chakravarty (2003) contend that "a multidimensional approach to poverty defines poverty as a shortfall from a threshold on each dimension of an individual's well being".[10] Hence a reasonable starting place is to compare each individual's achievements against the respective dimension-specific cutoffs, and we follow that general strategy here. But dimension-specific cutoffs alone do not suffice to identify who is poor; we must consider additional criteria that look *across* dimensions to arrive at a complete specification of an identification method. We

[10] See also Chakravarty et al 1998 and Tsui 2002 on this point.

now examine some potential candidates for ρ, using equal weights for clarity of presentation.

The 'unidimensional' method aggregates all achievements into a single cardinal variable of 'well-being' or 'income' and uses an aggregate cutoff to determine who is poor. So, for example, if y_i is a vector of commodities with market price vector p, one might define $\rho_p(y_i;z)=1$ whenever $py_i<pz$, and $\rho_p(y_i;z)=0$ otherwise. In this case, a person is poor if the monetary value of the achievement bundle is below the cost of the target bundle z. More generally, one might invoke a strictly increasing aggregator function u such that $\rho_u(y_i;z)=1$ whenever $u(y_i)<u(z)$, and $\rho_u(y_i;z)=0$ otherwise. However, the unidimensional form of identification entails a host of assumptions that restrict its applicability in practice, and its desirability in principle.[11] From the perspective of the capability approach, a key conceptual drawback of viewing multidimensional poverty through a unidimensional lens is the loss of information on dimension-specific shortfalls: indeed, aggregation before identification converts dimensional achievements into one another without regard to dimension-specific cutoffs. If, as argued above, dimensions are independently valued and dimensional deprivations are inherently undesirable, then there are good reasons to look beyond a unidimensional approach to identification methods that focus on dimensional shortfalls.

The most commonly used identification criterion of this type is called the *union* method of identification.[12] In this approach, a person i is said to be multidimensionally poor if there is at least one dimension in which the person is deprived (i.e., $\rho(y_i;z)=1$ if

[11] One common assumption is that prices exist and are adequate normative weights for the dimensions; however, as noted by Tsui 2002, this assumption is questionable. Prices may be adjusted to reflect externalities, but exchange values do not and 'indeed cannot give…*interpersonal comparisons* of welfare or advantage' (Sen 1997, p.208). Subjective poverty lines cannot replace prices for all attributes, and markets may be missing or imperfect (Bourguignon and Chakravarty 2003, Tsui 2002). In practice, income may not be translated into basic needs (Ruggeri-Laderchi, Saith and Stewart 2003, Sen 1980). Finally, aggregating across dimensions entails strong assumptions regarding cardinality and comparability, which are impractical when data are ordinal (Sen 1997).

[12] Atkinson 2003 first applied the terms 'union' and 'intersection' in the context of multidimensional poverty.

and only if $c_i \geq 1$). If sufficiency in every dimension were truly essential for avoiding poverty, this approach would be quite intuitive and straightforward to apply. However, when the number of dimensions is large, the union approach will often identify most of the population as being poor, including persons whom many would not consider to be poor. For example, deprivation in certain single dimensions may be reflective of something other than poverty. Consequently, a union based poverty methodology may not be helpful for distinguishing and targeting the most extensively deprived. For these reasons, the union method, though commonly used, is not appropriate in all circumstances.

The other multidimensional identification method is the *intersection* approach, which identifies person i as being poor only if the person is deprived in all dimensions (i.e., $\rho(y_i;z)=1$ if and only if $c_i=d$). This criterion would accurately identify the poor if sufficiency in any single dimension were enough to prevent poverty; indeed, it successfully identifies as poor a group of especially deprived persons. However, it inevitably misses persons who are experiencing extensive, but not universal, deprivation (for example, a person with insufficiency in every other dimension who happens to be healthy). This creates a different tension–that of considering persons to be non-poor who evidently suffer considerable multiple deprivations.

A natural alternative is to use an intermediate cutoff level for c_i that lies somewhere between the two extremes of 1 and d. For $k=1,\ldots,d$, let ρ_k be the identification method defined by $\rho_k(y_i;z)=1$ whenever $c_i \geq k$, and $\rho_k(y_i;z)=0$ whenever $c_i<k$. In other words, ρ_k identifies person i as poor when the number of dimensions in which i is deprived is at least k; otherwise, if the number of deprived dimensions falls below the cutoff k, then i is not poor according to ρ_k. Since ρ_k is dependent on both the *within dimension* cutoffs z_j and the *across dimension* cutoff k, we will refer to ρ_k as the

dual cutoff method of identification.[13] Notice that ρ_k includes the union and intersection methods as special cases where k=1 and k=d. Identification could also be defined using a strict inequality (so that $\rho_k(y_i;z)=1$ if and only if $c_i>k$), yielding the same set Z at a slightly smaller k.

Similar methods of identification can be found in the literature, albeit with different motivations. For example, Mack and Lansley *Poor Britain* (1985) identified people as poor if they were poor in 3 or more out of 26 deprivations. The UNICEF *Child Poverty Report 2003* identified any child who was deprived with respect to two or more dimensions as being in extreme poverty (Gordon, *et al.*, 2003). However, as a general methodology for identifying the poor, the dual cutoff approach has not been explicitly formulated in the literature, nor have its implications for multidimensional poverty measures – or their axioms – been explored.[14]

The dual cutoff method has a number of characteristics that deserve mention. First, it is 'poverty focused' in that an increase in an achievement level y_{ij} of a non-poor person leaves its value unchanged. Second, it is 'deprivation focused' in that an increase in any non-deprived achievement $y_{ij}\geq z_j$ leaves the value of the identification function unchanged; in words, a person's poverty status is not affected by changes in the levels of non-deprived achievements. This latter property separates ρ_k from the unidimensional method ρ_u, which allows a higher level of a non-deprived achievement to compensate for other dimensional deprivations in deciding who is poor or non-poor. Finally, the dual cutoff identification method can be meaningfully used with ordinal data, since a person's poverty status is unchanged when a monotonic transformation is

[13] See section 8 on the choice of k (and z).

[14] An analogous approach has been used in the measurement of chronic poverty, with duration in that context corresponding to breadth in the present case. See Foster 2007.

applied to an achievement level and its associated cutoff.[15] This rules out the typical unidimensional ρ_u, which aggregates dimensions *before* identifying the poor, and thus can be altered by monotonic transformations.

In the next section, we introduce multidimensional poverty measures based on the FGT class that use the ρ_k identification method and its associated set $Z_k=\{i : \rho_k(y_i;z)=1\}$ of poor people. Accordingly, we will make use of some additional notation that censors the data of non-poor persons. Let $g^\alpha(k)$ be the matrix obtained from g^α by replacing its i^{th} row g_{ij}^α with a vector of zeros whenever $\rho_k(y_i;z)=0$, so that $g_{ij}^\alpha(k) = g_{ij}^\alpha\rho_k(y_i;z)$. As the cutoff k rises from 1 to d, the number of nonzero entries in the associated matrix $g^\alpha(k)$ falls, reflecting the progressive censoring of data from persons who are not meeting the dimensional poverty requirement presented by ρ_k. It is clear that the union specification $k=1$ does not alter the original matrix at all; consequently, $g^\alpha(1)=g^\alpha$. The intersection specification $k=d$ removes the data of any person who is not deprived in all d dimensions; in other words, when the matrix $g^\alpha(d)$ is used, a person deprived in just a single dimension is indistinguishable from a person deprived in d–1 dimensions. When $k=2,\ldots,d-1$, the dual cutoff approach provides an intermediate option between the union and intersection methods as reflected in the matrix $g^\alpha(k)$.

4. MEASURING POVERTY

Suppose, then, that a particular identification function ρ_k has been selected. Which multidimensional poverty measure $M(y;z)$ should be used with it to form a methodology $\mathcal{M}$? A natural place to begin is with the percentage of the population that is poor. The headcount ratio $H=H(y;z)$ is defined by $H=q/n$, where $q=q(y;z)=\Sigma_{i=1}^n\rho_k(y_i,z)$ is number

[15] In other words, $\rho_k(y_i;z)=\rho_k(y_i';z')$ where for each $j=1,\ldots,d$ we have $y'_{ij}=f_j(y_{ij})$ and $z_j'=f_j(z_j)$ for some increasing function f_j. It would be interesting to characterize the identification methods ρ satisfying the above three properties.

of persons in the set Z_k, and hence the number of the poor identified using the dual cutoff approach. The resulting methodology (ρ_k,H) is entirely analogous to the income headcount ratio and inherits the virtue of being easy to compute and understand, and the weakness of being a crude, or partial, index of poverty.[16] Notice, though, that an additional problem emerges in the multidimensional setting. If a poor person becomes deprived in a new dimension, H remains unchanged. This violates what we will call 'dimensional monotonicity' below, which says that if poor person i becomes newly deprived in an additional dimension, then overall poverty should increase. Also, H cannot be broken down to show how much each dimension contributes to poverty.

To reflect these concerns, we can include additional information on the breadth of deprivation experienced by the poor. Define the *censored vector of deprivation counts* $c(k)$ by $c_i(k)=\rho_k(y_i;z)c_i$ for $i=1,\ldots,n$. Notice that $c_i(k)/d$ represents the share of possible deprivations experienced by a poor person i, and hence the *average deprivation share* across the poor is given by $A=|c(k)|/(qd)$. This partial index conveys relevant information about multidimensional poverty, namely, the fraction of possible dimensions d in which the average poor person endures deprivation. Consider the following multidimensional poverty measure $M_0(y;z)$ which combines information on the prevalence of poverty and the average extent of a poor person's deprivation.

DEFINITION 1: The *adjusted headcount ratio* is given by $M_0=HA=\mu(g^0(k))$.

As a simple product of the two partial indices H and A, the measure M_0 is sensitive to the frequency and the breadth of multidimensional poverty. In particular, the methodology (ρ_k,M_0) clearly satisfies dimensional monotonicity, since if a poor person becomes deprived in an additional dimension, then A rises and so does M_0. The equivalent definition $M_0=\mu(g^0(k))$ interprets M_0 as the total number of deprivations

[16] A partial index provides information on only one aspect of poverty. See Foster and Sen 1997.

experienced by the poor, or $|c(k)|=|g^0(k)|$, divided by the maximum number of deprivations that could possibly be experienced by all people, or nd. The adjusted headcount ratio can be used with purely ordinal data, which arises frequently in multidimensional approaches based on capabilities. This important characteristic of the measure will be discussed at some length in a separate section below.

The methodology (ρ_k,M_0) is based on a dichotomisation of data into deprived and non-deprived states, and so it does not make use of any dimension-specific information on the depth of deprivation. Consequently it will not satisfy the traditional monotonicity requirement that poverty should increase as a poor person becomes more deprived in any given dimension. To develop a methodology that is sensitive to the depth of deprivation (when data are cardinal), we return to the censored matrix of normalised gaps $g^1(k)$. Let G be the *average poverty gap* across all instances in which poor persons are deprived, given by $G=|g^1(k)|/|g^0(k)|$. Consider the following multidimensional poverty measure $M_1(y;z)$ which combines information on the prevalence of poverty, the average range of deprivations and the average depth across deprived dimensions.

DEFINITION 2: The *adjusted poverty gap* is given by $M_1=HAG=\mu(g^1(k))$.

The adjusted poverty gap is thus the product of the adjusted headcount ratio M_0 and the average poverty gap G. The equivalent definition $M_1=\mu(g^1(k))$ says that the adjusted poverty gap is the sum of the normalised gaps of the poor, or $|g^1(k)|$ divided by the highest possible sum of normalised gaps, or nd. Under methodology (ρ_k,M_1) if the deprivation of a poor person deepens in any dimension, then the respective $g^1_{ij}(k)$ will rise and hence so will M_1. Consequently, (ρ_k,M_1) satisfies the monotonicity axiom (as defined below). However, it is also true that the increase in a deprivation has the same impact no matter whether the person is very slightly deprived or acutely deprived in that dimension. One might argue that the impact should be larger in the latter case.

Consider the censored matrix $g^2(k)$ of squared normalised shortfalls. This matrix provides information on the severity of deprivations of the poor (as measured by the square of their normalised shortfalls). Rather than using the matrix $g^1(k)$ to supplement the information of M_0 (as was done in M_1), we can use the matrix $g^2(k)$ which suppresses the smaller gaps and emphasises the larger ones. The *average severity* of deprivations, across all instances in which poor persons are deprived, is given by $S=|g^2(k)|/|g^0(k)|$. The following multidimensional poverty measure $M_2(y;z)$ combines information on the prevalence of poverty and the range and severity of deprivations.

DEFINITION 3: The *adjusted* FGT *measure* is given by $M_2=HAS$.

M_2 is thus the product of the adjusted headcount ratio M_0 and the average severity index S. Its alternative definition $M_2=\mu(g^2(k))$ indicates that M_2 is the sum of the squared normalised gaps of the poor, or $|g^2(k)|$, divided by the highest possible sum of the squared normalised gaps, or nd. Under (ρ_k,M_2), a given-sized increase in a deprivation of a poor person will have a greater impact the larger the initial level of deprivation. Consequently, the methodology satisfies the transfer property (as defined below), and is sensitive to the inequality with which deprivations are distributed among the poor, and not just their average level. Indeed, $M_2=(M_1)^2 + V$, where V is the variance across all normalised gaps.[17]

We generalise M_0, M_1, and M_2 to a class $M_\alpha(y;z)$ of multidimensional poverty measures associated with the unidimensional FGT class.

[17] In other words, $V=\Sigma_i\Sigma_j(\mu(g^1) - g^1_{ij})^2/(nd)$. The formula can also be expressed as $M_2=(M_1)^2[1 + C^2]$, where $C^2=V/(\mu(g^1))^2$ is the squared coefficient of variation inequality measure. This is analogous to a well-known formula for the FGT measure P_2.

DEFINITION 4: The *adjusted* FGT *class* of multidimensional poverty measures are given by $M_\alpha=\mu(g^\alpha(k))$ for $\alpha\geq 0$.

In other words, M_α is the sum of the α powers of the normalised gaps of the poor, or $|g^\alpha(k)|$, divided by the highest possible value for this sum, or *nd*. The methodology employing the dual cutoff function ρ_k and an associated FGT measure M_α will be denoted by $\mathcal{M}_{k\alpha}=(\rho_k,M_\alpha)$.[18]

5. GENERAL WEIGHTS

By defining a poverty measurement methodology based on deprivation counts and simple averages, we have implicitly assigned an equal weight of $w_j=1$ to each dimension j. This is appropriate when the dimensions have been chosen to be of relatively equal importance. As Atkinson *et al* observe, equal weighting has an intuitive appeal: "the interpretation of the set of indicators is greatly eased where the individual components have degrees of importance that, while not necessarily exactly equal, are not grossly different" (2002, p. 25; see also Atkinson 2003 p. 58).

Yet in other settings there may be good arguments for using general weights. Indeed, the choice of dimensional weights may be seen as a value judgement which should be open to public debate and scrutiny: "It is not so much a question of holding a referendum on the values to be used, but the need to make sure that the weights – or ranges of weights – used remain open to criticism and chastisement, and nevertheless enjoy reasonable public acceptance" (Foster and Sen, 1997). We now show how weighted versions of ρ_k and M_α can be defined for a given set of dimensional weights.[19]

[18] Note that each choice of ρ_k gives rise to a potentially different functional form for M_α (or H), and hence a more precise notation would be $M_{k\alpha}$ (or H_k) for the measures and $(\rho_k,M_{k\alpha})$ (or (ρ_k,H_k)) for the methodologies.

[19] Techniques for setting weights across dimensions include statistical, survey-based, normative-participatory, or frequency-based, or a combination of these. See Sen 1996, Brandolini and D'Alessio

Let w be a d dimensional vector of positive numbers summing to d, whose j^{th} coordinate w_j is viewed as the weight associated with dimension j. Define $g^{\alpha}=[g_{ij}^{\alpha}]$ to be the $n\times d$ matrix whose typical element is $g_{ij}^{\alpha}=w_j((z_j-y_{ij})/z_j)^{\alpha}$ whenever $y_{ij}<z_j$, and $g_{ij}^{\alpha}=0$ otherwise. The identification step uses the rows g_i^0 of the weighted deprivation matrix g^0 to construct the vector c of weighted deprivation counts, whose i^{th} entry $c_i=|g_i^0|$ is the sum of weights for the dimensions in which i is deprived. Each c_i varies between 0 and d, and so the associated dimensional cutoff is taken to be a real number k satisfying $0<k \leq d$. The dual cutoff identification method ρ_k associated with w is defined by $\rho_k(y_i;z)=1$ whenever $c_i\geq k$, and $\rho_k(y_i;z)=0$ otherwise. For $k\leq\min_j w_j$, we obtain the union identification case, and for $k=d$, the intersection; thus ρ_k includes both of these methods given any w.

Notice that the specification $w_j=1$ for $j=1,\ldots,d$ corresponds to the previous case where each dimension has equal weight and the dimensional cutoff k is an integer. The alternative specification $w_1=d/2$ and $w_2=\ldots=w_d=d/(2(d-1))$ is an example of a *nested* weighting structure, in which the overall weight is first split equally between dimension 1 and the remaining (d-1) dimensions, and then the weight allotted the second group is allocated equally across the (d-1) dimensions. A cutoff of $k=d/2$, for example, would then identify as poor anyone who is either deprived in dimension 1 or in all the remaining dimensions, while a slightly higher value of k would require deprivation in the first dimension and in one other.

The weighted multidimensional poverty indices M_{α} are defined in a similar fashion as before. The censored matrices $g^{\alpha}(k)$ are obtained by replacing the entries of non-poor persons with 0, and the family of adjusted FGT measures associated with w is defined

1998, Decanq and Lugo 2008, and Alkire and Clark 2009. In practical applications involving weights, it may be desirable to perform robustness tests. See Foster et al 2009.

by $M_\alpha=\mu(g^\alpha(k))$ for $\alpha\geq 0$.[20] The overall weighted methodology is then denoted by $\mathcal{M}_{k\alpha}=(\rho_k,M_\alpha)$. The weighted headcount ratio is $H=q/n$, where $q=\Sigma_{i=1}^{n}\rho_k(y_i,z)$ is the number of poor persons identified by the weighted ρ_k, and the associated weighted methodology is (ρ_k,H).

6. PROPERTIES

We now evaluate our new methodologies using axioms for multidimensional poverty measurement.[21] The axiomatic framework for multidimensional measurement draws heavily upon its unidimensional counterpart. However, there is one key distinction: in the multidimensional context, the identification step is no longer elementary, and axioms must be viewed as restrictions on the overall methodology $\mathcal{M}=(\rho,M)$. This is less important for certain axioms such as 'symmetry' given below, which are satisfied by $\mathcal{M}=(\rho,M)$ for any given ρ whenever M has a requisite characteristic. However, other axioms such as 'poverty focus' given below, make explicit use of ρ in their definition, and could be satisfied for $\mathcal{M}=(\rho,M)$ and violated for an alternative methodology $\mathcal{M}'=(\rho',M)$ with the same measure M. This point has not been emphasized in the previous measurement literature, which has focused on the union identification approach and defined axioms relative to this particular specification. In contrast, our axioms can be used to evaluate *any* methodology, including ones that employ a dual cutoff identification approach.

A key first property for $\mathcal{M}$ is 'decomposability' which requires overall poverty to be the weighted average of subgroup poverty levels, where weights are subgroup population shares. In symbols, let x and y be two data matrices and let (x,y) be the

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[21] This discussion builds upon Chakravarty et al 1998, Tsui 2002, Atkinson 2003, Bourguignon and Chakravarty 2003, Duclos, Sahn and Younger 2006, Chakravarty and Silber 2008, and Maasoumi and Lugo 2008.

matrix obtained by merging the two populations. Denote by $n(x)$ the number of persons in x (and similarly for $n(y)$ and $n(x,y)$).

DECOMPOSABILITY: For any two data matrices x and y we have

$$M(x,y;z)=\frac{n(x)}{n(x,y)}M(x;z)+\frac{n(y)}{n(x,y)}M(y;z).$$

Repeated application of this property shows that the decomposition holds for any number of subgroups, making it an extremely useful property for generating profiles of poverty and targeting high poverty populations.[22] If we apply a decomposable methodology to a *replication* x of y, which has the form $x=(y,y,\ldots,y)$, it follows that x has the same poverty level as y, and hence the following axiom must hold.

REPLICATION INVARIANCE: If x is obtained from y by a replication, then $M(x;z)=M(y;z)$.

This property ensures that poverty is evaluated relative to the population size, so as to allow meaningful comparisons across different sized populations.

Now let x be obtained from y by a *permutation*, by which it is meant that $x=\Pi y$, where Π is some $n\times n$ permutation matrix.[23] This has the effect of reshuffling the vectors of achievements across people.

SYMMETRY: If x is obtained from y by a permutation, then $M(x;z)=M(y;z)$.

According to symmetry, if two or more persons switch achievements, measured poverty is unaffected. This ensures that $\mathcal{M}$ does not place greater emphasis on any person or group of persons.

[22] Any decomposable methodology also satisfies 'subgroup consistency' which requires overall poverty to increase when poverty rises in the first subgroup and does not fall in the second (given fixed population sizes). As discussed in Foster, Greer and Thorbecke 1984 and Foster and Sen 1997, it is this property that allows the coordination of local and national poverty alleviation policies.

[23] A permutation matrix Π is square matrix with a single '1' in each row and each column, and the rest '0's.

The traditional focus axiom requires a poverty measure to be independent of the data of the non-poor, which in the unidimensional or income poverty case is simply all incomes at or above the single poverty line.[24] In a multidimensional setting, a non-poor person could be deprived in several dimensions while a poor person might not be deprived in all dimensions. There are two forms of multidimensional focus axioms, one concerning the poor, and the other pertaining to deprived dimensions. We say that x is obtained from y by a *simple increment* if $x_{ij}>y_{ij}$ for some pair $(i, j)=(i', j')$ and $x_{ij}=y_{ij}$ for every other pair $(i, j) \neq (i', j')$. We say it is a simple increment *among the non-poor* if i' is not in Z for y (whether or not i' is deprived in j'); it is a simple increment *among the nondeprived* if $y_{ij}\geq z_j$ for $(i, j)=(i', j')$, whether or not i' happens to be poor.

POVERTY FOCUS: If x is obtained from y by a simple increment among the non-poor, then $M(x;z)=M(y;z)$.

DEPRIVATION FOCUS: If x is obtained from y by a simple increment among the nondeprived, then $M(x;z)=M(y;z)$.

In the poverty focus axiom, the set Z of the poor is identified using ρ, and M is required to be unchanged when anyone outside of Z experiences a simple increment. This is a basic requirement that ensures that M measures poverty in a way that is consistent with the identification method ρ. In the deprivation focus axiom, the simple increment is defined independently of the particular identification method employed and is applicable to all nondeprived entries in y – poor and non-poor alike.

It is possible for a multidimensional poverty methodology to follow the poverty focus axiom without satisfying the deprivation focus axiom. Consider, for example, a unidimensional approach that, say, adds dimensional achievements to create an aggregate variable, identifies the poor using an aggregate cutoff, and employs a

[24] An alternative definition would consider persons strictly *above* the cutoff to be non-poor. See Foster and Sen 1997 p.175.

standard income poverty measure on the aggregate variable. Given the assumed tradeoffs across dimensions, it is possible for a poor person to be lifted out of poverty as a result of an increment in a nondeprived dimension, thus lowering the measured level of poverty and violating deprivation focus. Conversely, the deprivation focus axiom may be satisfied without accepting the poverty focus axiom: for example if the average gap $\mu(g^1)$ over all deprivations (poor or non-poor) is taken to be the measure and yet an intersection approach to identification is used.

The two forms of focus axioms are related in certain cases. When union identification is used, it can be shown that the deprivation focus axiom implies the poverty focus axiom. When an intersection approach is used, the poverty focus axiom implies the deprivation version. Bourguignon and Chakravarty (2003), for example, assume the deprivation focus axiom (their 'strong focus axiom') along with union identification, and so their methodology automatically satisfies the poverty focus axiom.

The next set of properties ensures that methodology $\mathcal{M}$ has the proper orientation. Consider the following extensions to the definition of a simple increment: We say that x is obtained from y by a *deprived increment among the poor* if in addition to being a simple increment we have $z_{j'}>y_{i'j'}$ for $i' \in Z$; it is a *dimensional increment among the poor* if it satisfies $x_{i'j'}\geq z_{j'}>y_{i'j'}$ for $i' \in Z$. In other words, a deprived increment among the poor improves a deprived achievement of a poor person, while a dimensional increment among the poor completely removes the deprivation. Consider the following properties.

WEAK MONOTONICITY: If x is obtained from y by a simple increment, then $M(x;z)\leq M(y;z)$.

MONOTONICITY: $\mathcal{M}$ satisfies weak monotonicity and the following: if x is obtained from y by a deprived increment among the poor then $M(x;z)<M(y;z)$.

DIMENSIONAL MONOTONICITY: If x is obtained from y by a dimensional increment among the poor, then $M(x;z)<M(y;z)$.

Weak monotonicity ensures that poverty does not increase when there is an unambiguous improvement in achievements. Monotonicity additionally requires poverty to fall if the improvement occurs in a deprived dimension of a poor person. Dimensional monotonicity specifies that poverty should fall when the improvement removes the deprivation entirely; it is clearly implied by monotonicity.

The weak monotonicity and focus axioms ensure that a measure M achieves its highest value at x^0 in which all achievements are 0 (and hence each person is maximally deprived), while it achieves its lowest value at any x^z in which all achievements reach or exceed the respective deprivation cutoffs given in z (and hence no one is deprived). 'Nontriviality' ensures that these maximum and minimum values are distinct, while 'normalisation' goes further and assigns a value of 1 to x^0 and a value of 0 to each x^z.

NONTRIVIALITY: M achieves at least two distinct values.

NORMALISATION: M achieves a minimum value of 0 and a maximum value of 1.

One can also explore how a methodology regards changes in inequality among the poor. The first axiom of this sort is based on an 'averaging' of the achievement vectors of two poor persons i and i', in which person i receives $\lambda>0$ of the first vector and $1-\lambda>0$ of the second with the shares reversed for person i'. Following Kolm (1977) these d many 'progressive transfers' between the poor represent an unambiguous decrease in inequality, which some would argue should be reflected in a lower or equal value of multidimensional poverty. In general, we say that x is obtained from y by an *averaging* *of achievements among the poor* if $x=By$ for some $n\times n$ bistochastic matrix[25] B satisfying $b_{ii}=1$ for every non-poor person i in y. Note that the requirement $b_{ii}=1$ ensures that all the non-poor columns in y are unaltered in x, while the fact that B is bistochastic ensures that the poor columns in x are obtained as a convex combination of the poor columns in y, and hence inequality has fallen or remained the same. Consider the following property.

WEAK TRANSFER: If x is obtained from y by an averaging of achievements among the poor, then $M(x;z)\leq M(y;z)$.

This axiom ensures that an averaging of achievements among the poor generates a poverty level that is less than or equal to the original poverty level.

A second axiom relating poverty to inequality has its origins in the work of Atkinson and Bourguignon (1982). The concept is based on a different sort of 'averaging' across two poor persons, whereby one person begins with weakly more of each achievement than a second person, but then switches one or more achievement levels with the second person so that this ranking no longer holds. Motivated by Boland and Proschan (1988), we say x is obtained from y by a *simple rearrangement among the poor* if there are two persons i and i' who are poor in y, such that for each j either $(x_{ij},x_{i'j})=(y_{ij},y_{i'j})$ or $(x_{ij},x_{i'j})=(y_{i'j},y_{ij})$, and for every other person $i''\neq i,i'$ we have $x_{i''j}=y_{i''j}$. In other words, a simple rearrangement among the poor reallocates the achievements of the two poor persons but leaves the achievements of everyone else unchanged. We say x is obtained from y by an *association decreasing rearrangement among the poor* if, in addition, the achievement vectors of i and i' are comparable by vector dominance in y but are not comparable in x. The following property ensures that

[25] A bistochastic matrix is a nonnegative square matrix having the property that the sum of the elements in each row (or column) is 1.

reducing inequality in this way generates a poverty level that is less than or equal to the original level.

WEAK REARRANGEMENT: If x is obtained from y by an association decreasing rearrangement among the poor, then $M(x;z)\leq M(y;z)$.

The following result establishes the axiomatic characteristics of our methodologies.

Theorem 1 For any given weighting vector and cutoffs, the methodology $\mathcal{M}_{k\alpha}=(\rho_k, M_\alpha)$ satisfies: decomposability, replication invariance, symmetry, poverty and deprivation focus, weak and dimensional monotonicity, nontriviality, normalisation, and weak rearrangement for $\alpha\geq 0$; monotonicity for $\alpha>0$; and weak transfer for $\alpha\geq 1$.

Proof In the Appendix

Note that if the number of dimensions were set to d=1, methodology $\mathcal{M}_{k\alpha}$ would naturally reduce to the single cutoff identification method and the standard single dimensional P_α measures; the poverty and deprivation focus axioms would become the usual focus axiom; weak rearrangement would be trivially satisfied; and the conclusions of Theorem 1 would reduce to the standard list of axioms satisfied by the P_α measures.

The following formulas for $\mathcal{M}_{k\alpha}$ are also helpful in empirical applications:

(1a) $$M_\alpha(y;z)=\Sigma_i\,\mu(g_i^\alpha(k))/n$$

(1b) $$M_\alpha(y;z)=\Sigma_j\, w_j\,\mu(g_{*j}^\alpha(k))$$

where $g_i^\alpha(k)$ is the i^{th} row, and $g_{*j}^\alpha(k)$ is the j^{th} column, of the censored matrix $g^\alpha(k)$.

Since $\mu(g_i^\alpha(k))=M_\alpha(y_i;z)$ is person i's poverty level, (1a) says that overall poverty is the average of the individual poverty levels. It is the natural extension of decomposability to singleton subgroups. Expression (1b) provides an analogous breakdown across dimensions, but with an important difference. The term $\mu(g^{\alpha}_{*j}(k))$ depends on all dimensions, not just on j, and so (1b) is not a complete decomposition of M_α by dimension. However, once the identification step is complete, (1b) allows total poverty to be viewed as a weighted average of dimensional values $\mu(g^{\alpha}_{*j}(k))$, and we can interpret $w_j\mu(g^{\alpha}_{*j}(k))/M_\alpha(y;z)$ as the (post-identification) contribution of dimension j.[26]

How does the methodology $\mathcal{M}_{k\alpha}$ compare to existing approaches? Several classes of multidimensional poverty measures can be found in the literature, including those of Tsui 2002 and Bourguignon and Chakravarty 2003. However, virtually all existing measures have been defined only for union identification, and would violate the poverty focus axiom if paired with other identification functions.[27] Our methodology uses identification functions that include the union and intersection approaches but also could fall between them, and constructs new poverty measures that are specifically appropriate for our identification functions in that the resulting methodology satisfies the poverty focus axiom.

7. THE CASE OF THE ADJUSTED HEADCOUNT RATIO

The methodology $\mathcal{M}_{k0}=(\rho_k,M_0)$ has several characteristics that merit special attention. First, it can accommodate the ordinal (and even categorical) data that commonly arise in multidimensional settings. This means that the methodology delivers identical conclusions when monotonic transformations are applied to both

[26] In the presence of union identification, formula (1b) becomes a true 'factor decomposability' as defined by Chakravarty et al 1998; in general, though, (1b) does not account for dimensional contributions to poverty via the identification step. It may well be possible to account for total contributions using other methods, such as a Shapley-based approach; however, this is a topic for future research.

[27] Union based versions of certain members of M_α can be obtained from classes given by Brandolini and D'Alessio 1998, Bourguignon and Chakravarty 2003, Deutsch and Silber 2005, and Chakravarty and Silber 2008. A related measure of social exclusion is found in Chakravarty and D'Ambrosia 2006.

variables and cutoffs. In symbols, let f_j denote the strictly increasing function on variable $j=1,\ldots,d$. Then where $f(y_i)$ is the vector whose j^{th} entry is $f_j(y_{ij})$, and $f(z)$ is the vector whose j^{th} entry is $f_j(z_j)$, we have $\rho_k(f(y_i);f(z))=\rho_k(y_i;z)$ for $i=1,\ldots,n$. In other words, identification is unaffected by monotonic transformations, which is clearly relevant for consistent targeting using ordinal data. Similarly, where $f(y)$ is the matrix whose ij^{th} entry is $f_j(y_{ij})$ we have $M_0(f(y);f(z))=M_0(y;z)$, which indicates that the poverty value is also unchanged.[28] These characteristics of $\mathcal{M}_{k0}=(\rho_k,M_0)$ are especially relevant when poverty is viewed from the capability perspective, since many key functionings are fundamentally ordinal (or categorical) variables.

Virtually every other multidimensional methodology defined in the literature (including our (ρ_k,M_α) for $\alpha>0$) lacks one or both forms of invariance, and for most measures, the underlying *ordering* is not even preserved, i.e., $M(\mathrm{x};z)>M(\mathrm{y};z)$ and $M(f(\mathrm{x});f(z))<M(f(\mathrm{y});f(z))$ can both be true. Special care must be taken not to use measures whose poverty judgments are meaningless (i.e., reversible under monotonic transformations) when variables are ordinal. The methodology (ρ_k,H) survives this test. But it does so at the cost of violating dimensional monotonicity, among other properties. In contrast, (ρ_k,M_0) provides both meaningful comparisons and favourable axiomatic properties and is arguably a better choice when data are ordinal.

Second, the measure M_0 from the methodology $\mathcal{M}_{k0}$ conveys tangible information on the deprivations of the poor in a transparent way. Some measures have aggregate values whose meaning can only be found relative to other values. M_0 is the frequency of poverty H times the average breadth A of deprivation among the poor or, equivalently, the aggregate deprivations experienced by the poor as a share of the

[28] The set of the poor and the measured value of poverty are therefore meaningful in the sense of Roberts 1979. Note that M_0 can also be applied to categorical variables (which do not necessarily admit a unique ordering across categories), so long as achievements can be separated into deprived and nondeprived sets.

maximum possible range of deprivations across society. Its simple structure ensures that M_0 is easy to interpret and straightforward to calculate.

Breakdown (1b) becomes especially useful in this case, since the term $\mu(g^{\mathbf{0}}_{*j}(k))$ is the percentage H_j of the population that is *both* deprived in dimension *j and* poor, and $M_0(y;z)=\Sigma_j\ w_jH_j$ is the weighted average of these dimensional headcounts. Dimension j's contribution to overall M_0 poverty is disproportionately high (i.e., exceeds its baseline weight w_j) exactly when H_j exceeds its average value M_0. The headcount methodology (ρ_k,H), by comparison, has no such dimensional breakdown.

Third, the adjusted headcount methodology is fundamentally related to the axiomatic literature on freedom. In a key paper, Pattanaik and Xu (1990) explore a 'counting' approach to measuring freedom that ranks opportunity sets according to the number of (equally weighted) options they contain. Now suppose that our matrix y has been normatively constructed so that each dimension represents an equally valued functioning. Then deprivation in a given dimension is suggestive of capability deprivation, and since M_0 counts these deprivations, it can be viewed as a measure of 'unfreedom' analogous to Pattanaik and Xu. Indeed, the link between (ρ_k,M_0) and unfreedom can be made precise, yielding a result that simultaneously characterizes ρ_k and M_0 using axioms adapted from Pattanaik and Xu.[29] This general approach also has an appealing practicality: as suggested by Anand and Sen (1997), it may be more tractable to monitor a small set of deprivations than a large set of attainments.

8. CHOOSING CUTOFFS

To implement our methodology, two general forms of cutoffs must be chosen: the within dimension cutoffs z_j and the cross dimensional cutoff k. We now briefly

[29] For a fuller discussion see Alkire and Foster 2007.

discuss the practical and conceptual considerations surrounding cutoff selection, and also provide some elementary dominance results for variable k.

The dual cutoffs in our approach are quite different from one another. Dimensional cutoffs like z_j have long been used to identify deprivations in a dimension of interest. Consequently, in many variables there is a general understanding of what a given cutoff level means and how to go about selecting it.[30] To be sure, any specific choice of z, no matter how well grounded, is somewhat arbitrary and should be subject to robustness tests – say, by evaluating poverty levels for a grid of nearby cutoffs.[31] But selecting reasonable levels for z should not be an unduly taxing exercise.

The cross-dimensional cutoff k, by comparison, may seem less tangible, since it resides in the space between dimensions rather than within a specific domain. This sense is reinforced by the relative lack of attention that has been paid to the identification step: apart from the union and intersection approaches, specific multidimensional identification procedures are not typically given in the literature. But the identification method ρ_k and its parameter k provide a concrete solution to identification that can be readily grasped, especially in the equal-weighted 'counting' case that focuses on the number of dimensions in which people are deprived. A person with a greater multiplicity of deprivations is given higher priority than someone with only one or two deprivations; setting k establishes the minimum eligibility criteria for poverty in terms of breadth of deprivation and reflects a judgement regarding the maximally acceptable multiplicity of deprivations.

The choice of k could therefore be a normative one, with k reflecting the minimum deprivation count required to be considered poor in a specific context under

[30] On the setting of poverty lines see Sen 1981, Ravallion 1994, Foster and Sen 1997, Bourguignon and Chakravarty 2003, Foster 2006.
[31] Alternatively, we might draw on the multidimensional dominance tests in the literature (Duclos et al 2007).

consideration. As noted by Tsui, "In the final analysis, how reasonable the identification rule is depends, *inter alia,* on the attributes included and how imperative these attributes are to leading a meaningful life." (2002 p. 74). If, for example, deprivation in each dimension meant a terrible human rights abuse and data were highly reliable, then k could be set at the minimal union level to reflect the fact that human rights are each essential, have equal status, and cannot be positioned in a hierarchical order. There may also be a role for empirical evidence in the setting of k. If studies were to reveal that persons enjoying six functionings tended not to value a seventh, this might suggest setting a cutoff at a k of two or more dimensions rather than the union level of one.

The choice of k could also be chosen to reflect specific policy goals and priorities. For example, in order to focus on the multidimensionally poorest decile of the population, one could select a k cutoff whose resulting headcount was closest to 10%. By changing k one might be able to 'zoom in' to analyse a smaller group with greater multiplicity of deprivations or 'zoom out' to consider a wider population with fewer. Relatedly, if a budget constraint restricted coverage to a certain number of persons, then one could use k to target those suffering the greatest breadth of deprivation. Thus the choice of k could be a useful policy tool.

No matter which technique is finally employed in selecting the parameter k, it clearly makes sense to check robustness for values near the original cutoff, or even to opt for dominance tests that cover *all* possible values of k. Suppose that weights w and dimensional cutoff z have been selected, and let y and y' be any two data matrices. We say that *y M_α dominates y'*, written $y\boldsymbol{M_\alpha}y'$, if multidimensional poverty in y' is at least as high as that in y according to all methodologies (ρ_k, M_α) with $k \in (0,d]$, and is strictly higher for some k. We define $y\boldsymbol{H}y'$ in an analogous way. The easiest way to empirically implement these partial orderings is to calculate poverty levels at an appropriately defined grid of k values (namely, $k=\Sigma_{j\in J}\, w_j$ for every nonempty $J\subseteq\{1,\ldots,d\}$, which for equal weights reduces to $k=1,\ldots,d$). However, in the cases of (ρ_k,M_0) and (ρ_k,H), one can obtain useful characterization results for the associated partial orderings.

Let $c=(c_1,\ldots,c_d)$ be the vector of weighted deprivation counts for the matrix y. Then the associated vector of weighted *attainments*, denoted by $a=(a_1,\ldots,a_n)$, can be defined by $a_i=(d-c_i)$ for each $i=1,\ldots,n$. Clearly a is a unidimensional distribution and as such has a cumulative distribution function F_a. Let ***FD*** and ***SD*** denote the usual first order and second order stochastic dominance partial orderings over attainment vectors.[32] We have the following result.

Theorem 2 Where a and a' are the respective attainment vectors for y and y' in Y, we have:

(i) $y\boldsymbol{H}y' \Leftrightarrow a\boldsymbol{FD}a'$

(ii) $a\boldsymbol{FD}a' \Rightarrow y\boldsymbol{M_0}y' \Rightarrow a\boldsymbol{SD}a'$, and the converse does not hold.

Proof In the Appendix.

This result shows that first order dominance over attainment vectors ensures that multidimensional poverty as evaluated by the methodology (ρ_k,H) is lower (or no higher) for all possible values of the cross dimensional cutoff k – and the converse is true as well. This result is reminiscent of an analogous result for the unidimensional headcount ratio given in Foster and Shorrocks (1988). The second result shows that $\boldsymbol{M_0}$ is implied by first order dominance, and implies second order, in turn. Consequently, the $\boldsymbol{M_0}$ partial ordering is more complete than the $\boldsymbol{H}$ partial ordering, and is able to make more comparisons independently of the selection of cutoff k.

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[32] Let a and a' be two attainment distributions with associated cumulative distribution functions F_a and $F_{a'}$. We say that *a first order dominates a'*, written $a\boldsymbol{FD}a'$, if $F_a(s)\leq F_{a'}(s)$ for all $s\geq 0$, with < for some s. Similarly, *a second order dominates a'*, written $a\boldsymbol{SD}a'$, if $\int_0^s F_a(r)dr \leq \int_0^s F_{a'}(r)dr$ for all $s\geq 0$, with < for some s.

9. ILLUSTRATIVE EXAMPLES

We now illustrate the measurement methodology and its variations using data from the United States and Indonesia.

9.1 *United States*

To estimate multidimensional poverty in the US we use data from the 2004 National Health Interview Survey[33] on adults aged 19 and above (n=45,884). We draw on four variables: (1) income measured in poverty line increments and grouped into 15 categories, (2) self-reported health, (3) health insurance, and (4) years of schooling. For this illustration, we assume that all variables are ordinal and therefore restrict consideration to H and M_0. The dimensional cutoffs are as follows: if a person (1) lives in a household falling below the standard income poverty line, (2) reports 'fair' or 'poor' health, (3) lacks health insurance, or (4) lacks a high school diploma, then the person is considered to be deprived in the respective dimension.[34] The population is partitioned into four groups: Hispanic/Latino, (Non-Hispanic) White, (Non-Hispanic) African American/Black and Other.

Table 1 presents the traditional income poverty headcount (the share of the population below the income cutoff), and the multidimensional measures H and M_0, where the latter are evaluated using k=2 and equal weights. Column 3 gives the population share in each group while Column 5 presents the share of all income poor people found in each group. Comparing these two columns, we see that the incidence of income poverty is disproportionately high for the Hispanic and African-American populations. Moving now to the multidimensional headcount ratio H, column 7 gives the percentage of all multidimensionally poor people who fall within each group. The

[33] US National Center for Health Statistics 2004b.
[34] Precise definitions of the indicators and their respective cutoffs appear in Alkire and Foster 2007.

percentage of the multidimensionally poor who are Hispanic is much higher than the respective figure in column 5, while the percentage who are African-American is significantly lower, illustrating how our multidimensional approach to identifying the poor can alter the traditional, income-based poverty profile. Whereas column 7 gives the distribution of poor people across the groups, column 9 lists the distribution of *deprivations* experienced by the poor people in each group. The resulting figures for M_0 further reveal the disproportionate Hispanic contribution to poverty that is evident in this dataset.

Table 1: Profile of US Poverty by Ethnic/Racial Group (*k*=2)

1	2	3	4	5	6	7	8	9
Group	**Population**	**Percentage Contrib.**	**Income Poverty Headcount**	**Percentage Contrib.**	***H***	**Percentage Contrib.**	M_0	**Percentage Contrib.**
Hispanic	9100	19.8%	0.23	37.5%	0.39	46.6%	0.229	47.8%
White	29184	63.6%	0.07	39.1%	0.09	34.4%	0.050	33.3%
African-American	5742	12.5%	0.19	20.0%	0.21	16.0%	0.122	16.1%
Others	1858	4.1%	0.10	3.5%	0.12	3.0%	0.067	2.8%
Total	**45884**	**100.0%**	**0.12**	**100.0%**	**0.16**	**100.0%**	**0.09**	**100.0%**

Why does multidimensional poverty paint such a different picture? In Table 2, we use our methodology to identify the dimension-specific changes driving the variations in M_0. The final column of Table 2 reproduces group poverty levels found in Column 8 of Table 1, while the rows break these poverty levels down by dimension. We use formula (1b), which in the present case becomes $M_0=\Sigma_j H_j/d$, where H_j is the share of the respective population that is both poor and deprived in dimension j. The first row gives the decomposition for the Hispanic population, with column 2 reporting that 20% of Hispanics are both multidimensionally poor and deprived in income. Column 6 has the overall M_0 for Hispanics, which is simply the average of H_1 through H_4. The second row expresses the same data in percentage terms, with column 2 providing the per cent contribution of the income dimension to the Hispanic level of M_0 or, alternatively, the percentage of all deprivations experienced by the Hispanic poor population that are income deprivations. Notice that for Hispanics, the contribution from health insurance and schooling is quite high, whereas the contribution of income is relatively low. In contrast, the contribution of income for African-Americans is relatively high. This explains why, in comparison to traditional income based poverty, the percentage of overall multidimensional poverty originating in the Hispanic population rises, while the contribution for African-Americans is lower. The example shows how the measure M_0 can be readily broken down by population subgroup and dimension to help explain its aggregate level.[35]

Table 2: Contribution of Dimensions to Group M_0

1	2	3	4	5	6
Group	H_1 *Income*	H_2 *Health*	H_3 *H. Insurance*	H_4 *Schooling*	M_0
Hispanic	0.200	0.116	0.274	0.324	0.229
Percentage Contrib.	21.8%	12.7%	30.0%	35.5%	100%
White	0.045	0.053	0.043	0.057	0.050
Percentage Contrib.	22.9%	26.9%	21.5%	28.7%	100%
African-American	0.142	0.112	0.095	0.138	0.122
Percentage Contrib.	29.1%	23.0%	19.5%	28.4%	100%

The results above were reported for the value of k =2. In particular, we saw that Hispanics had higher M_0 poverty than White, and Whites than African-Americans. A natural question is whether the results would change for different k cutoffs. Table 3 and Figure 1 below report M_0 levels for k=1,2,3,4, and show that the ranking of the three key population sub-groups is robust. Indeed, it can be shown that the multidimensional headcount ratio likewise yields dominance across the three groups, which according to Theorem 2 would ensure that M_0 dominance also holds. It might

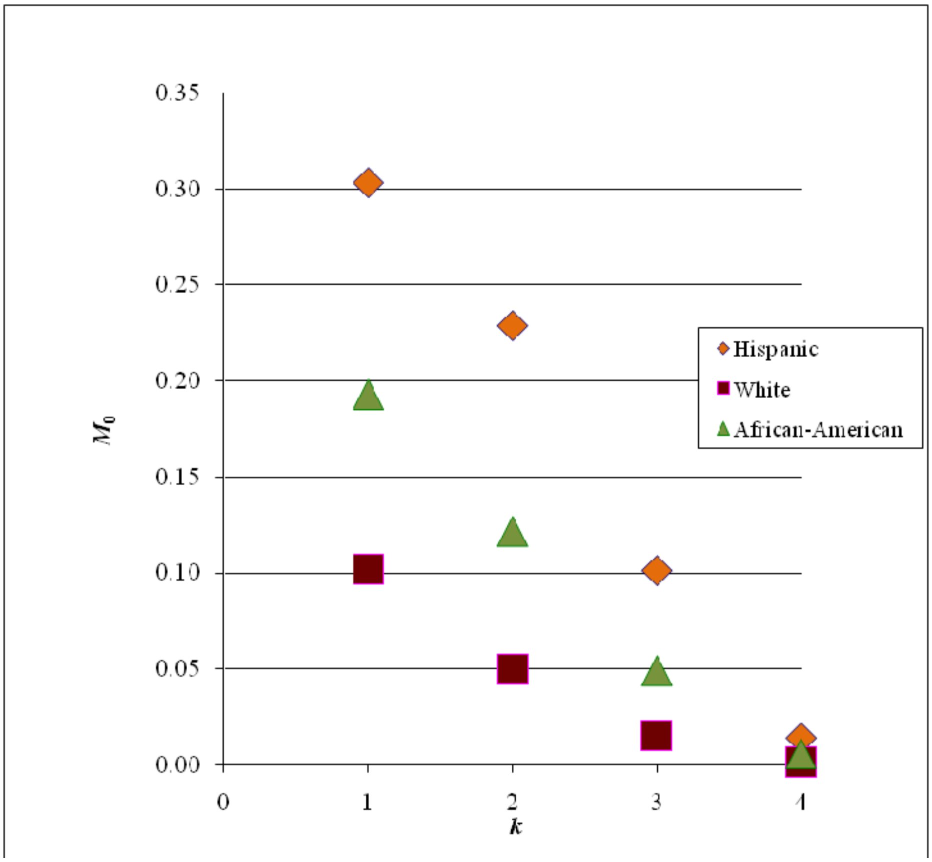
[35] In addition, a government who targets education, for example, would be able to see this directly reflected in the overall level of poverty (rather than having to wait until the effects show up much later in income) and could break the total down to understand the relationship between dimensional policies and overall poverty impacts. We are grateful to Karla Hoff for pointing out this useful characteristic of the measure for policy discussions.

also be useful to test whether the differences between groups were statistically significant.

Table 3: M_0 values

	k=1	*k=2*	*k=3*	*k=4*
Hispanic	0.3031	0.2287	0.1012	0.0141
White	0.1019	0.0496	0.0151	0.0014
African-American	0.1931	0.1217	0.0489	0.0056

Figure 1: M_0 Dominance



9.2 *Indonesia*

The data for this example are drawn from the Rand Corporation's 2000 Indonesian Family Life Survey (Strauss, *et. al.*, 2004). Our sample consists of all adults aged 19 years and above (n=19,752). We use d=3 dimensions: (1) expenditure, (2) health measured as body mass index BMI, and (3) years of schooling. We assume that the three variables are cardinally measurable.[36] The dimensional cutoffs are as follows: if a person (1) lives in a household with expenditures below 150,000 Rupiah, (2) has a BMI of less than 18.5 kg/m^2, or (3) has fewer than six years of schooling then the person is deprived in the respective dimension.[37] The respective headcounts of deprivation are 30.1 per cent for expenditure, 17.5 per cent for BMI, and 36.4 per cent for schooling.

Table 4 presents M_α poverty levels for all relevant values of k for two weighting structures.

Table 4: Multidimensional Poverty Measures: Cardinal Variables

Equal Weights				
Measure	k=1 (Union)	k=2		k=3 (Intersection)
H	0.577	0.225		0.039
M_0	0.280	0.163		0.039
M_1	0.123	0.071		0.016
M_2	0.088	0.051		0.011
General Weights				
Measure	k = 0.75 (Union)	k = 1.5	k = 2.25	k = 3 (Intersection)
H	0.577	0.346	0.180	0.039
M_0	0.285	0.228	0.145	0.039
M_1	0.114	0.084	0.058	0.015
M_2	0.075	0.051	0.036	0.010

In the equally weighted case, we see from Table 3 that when k=2, the headcount ratio is 22.5%, and the value of $M_0=HA$ is 0.163; M_0 departs from H according to the level of A. In the present case, A=0.72, because 83% of the poor are deprived in exactly two

[36] To be more precise, we assume variables to be measureable on a ratio scale, which means that they have a natural zero and are unique up to multiplication by a positive constant. Let Λ be the $d\times d$ diagonal matrix having $\lambda_j>0$ as its j^{th} diagonal element. Matrix multiplying Λ by y and z has the effect of rescaling the dimension j achievements and cutoff by λ_j, which is precisely the transformation allowable for ratio scale variables. Indeed, it is an easy matter to show that $M_\alpha(y\Lambda;z\Lambda)=M_\alpha(y;z)$ and hence the poverty values rendered by the adjusted *FGT* indices are meaningful when achievements are measured as ratio scale variables.

[37] In Indonesia primary school is completed in six years. Precise definitions and justifications of variables and cutoffs are presented in Alkire and Foster 2007.

dimensions, while the remaining 17% are deprived in all three. Note that M_0 and H coincide when all poor persons are deprived in d dimensions, as always occurs with the intersection method. Moving to $M_1=HAG$, the relevant factor is the average gap, which is G=0.44 in the present case. This indicates that the average achievement of a poor person in a deprived state is 56% of the respective cutoff – quite a large gap. If all deprived achievements were 0, hence G were 1, then M_1 and M_0 would have the same value. $M_2=HAS$ shows a further decrease from M_1 (0.051 rather than 0.071), and reflects the severity of poverty S. If all normalized gaps were identical, we would expect S to equal G^2 (or 0.19 in this case). Instead, S=0.31, and this larger value means that there is inequality among deprived states of the poor.

In the second case we use a weight of w_1=1.5 for expenditure, a weight of $w_2=w_3=0.75$ for the other two dimensions. This results in a relative weight of 50 per cent for expenditure, and 25 per cent for the other two dimensions. The new weighting structure clearly affects identification and the meaning of k. Union now is reached when k is equal to the lowest weight applied, or 0.75; Intersection is k=3 as before, and two additional intermediate cases are relevant. When k=1.5 every poor person is at least deprived in either expenditure *or* in both other dimensions. When k=2.25 each poor person is deprived in income and at least one additional dimension.

While the union headcount is, naturally, the same for equal and general weights, the value of M_0 increases in general weights, because the expenditure deprivation headcount exceeds the BMI deprivation headcount. However M_1 is smaller, which indicates that the normalized gap in expenditure deprivation is less than the normalized gap in at least one other dimension. Thus this section provides an example in which variables are cardinal, and illustrates how straightforward it is to calculate our methodology for the case of equal weights and of general weights.

10. CONCLUDING REMARKS

This paper proposes a methodology $\mathcal{M}_{k\alpha}$ for measuring multidimensional poverty whose 'dual cutoff' identification function ρ_k is a natural generalization of the traditional union and intersection identifications, and whose aggregation method M_α appropriately extends the FGT measures for the given ρ_k. We show that $\mathcal{M}_{k\alpha}$ satisfies a range of desirable properties including population decomposability; it also exhibits a useful breakdown by dimension once the identification step has been completed. The adjusted headcount methodology $\mathcal{M}_{k0}=(\rho_k,M_0)$ is particularly well-suited for use with ordinal data and is an intuitive measure of the breadth of multidimensional poverty. Basic dominance results for variable k are obtained for the methodologies (ρ_k,H) and (ρ_k,M_0). An empirical example illustrates our methods and, in particular, shows how multidimensional evaluation differs from an income based approach.

Several other aspects of our measurement methodology warrant further study. First, the identification method is based on cutoffs and is sensitive to certain changes, but insensitive to others. Small changes in individual achievements around a z-cutoff can lead to a change in the poverty status of an individual, and moreover can cause the poverty level to vary discontinuously in achievements.[38] Note though, that this characteristic is also exhibited by the standard income based headcount ratio – arguably the most frequently used poverty measure. Hence a violation of continuity at cutoffs need not preclude the use of a technology in practice. Even so, it would be interesting to explore this question further and to see whether natural methods exist for 'smoothing' the discontinuities.

[38] For example, using the intersection method of identification, if any given achievement rises above its cutoff, then the person will no longer be poor. Consequently, the multidimensional headcount will fall by $1/n$, while the change in M_α will be no larger than the change in H, and is weakly decreasing in α.

At the same time, the poverty status of a person is unaffected by certain other changes in achievements. For example, a poor person can never rise out of poverty by increasing the level of a non-deprived achievement, while a non-poor person will never become poor as a result of decrease in the level of a deprived achievement. This insensitivity is perhaps not unexpected, given our interest in applying the method to ordinal data and in avoiding aggregation before identification. However, there are tensions here that could be evaluated as part of a more systematic investigation of identification methods.[39]

Second, unlike other recent contributions, our presentation has not emphasized the potential interrelationships among dimensions that can exist when variables are cardinal. To be sure, the identification method ρ_k takes into account a rather crude form of linkage across dimensions, since a person must be deprived in k dimensions in order to be considered as poor. However, for $\alpha>0$, the aggregation method M_α is 'neutral' in that individual i's poverty level $M_\alpha(y_i;z)$ has a vanishing cross partial derivative for any pair of dimensions in which i is deprived. It is sometimes argued that the cross partials should be positive, reflecting a form of complementarity across dimensions; alternatively, they might be negative so as to yield a form of substitutability. Since M_α is neutral, it is a trivial matter to convert M_α into a measure that satisfies one or the other requirement: replace the individual poverty function $M_\alpha(y_i;z)$ with $[M_\alpha(y_i;z)]^\gamma$ for some $\gamma>0$ and average across persons.[40] The resulting poverty index regards all pairs of dimensions as substitutes when $\gamma<1$, and as complements when $\gamma>1$, with $\gamma=1$ being our basic neutral case. Of course, when there are more than two dimensions, it might be natural to expect some pairs of dimensions to be complements and others to be substitutes, and with varying degrees and strengths. The γ transformation requires

[39] For example it would be interesting to see whether a measure that reflects the depth of dimensional deprivations can be crafted for ordinal data

[40] Bourguignon and Chakarvarty 2003 present poverty indices of this kind.

dimensions to be *all* substitutes or *all* complements, and with a strength that is uniform across all pairs and for all people. This seems unduly restrictive, but is reflective of the state of the literature.

There are additional problems that need to be faced when considering interrelationships among dimensions. At a theoretical level, there are several definitions of substitutes and complements that could be applied, and the leading candidate – the Auspitz-Lieben-Edgeworth-Pareto (ALEP) definition – has certain difficulties (Kannai 1980). On the empirical side, there does not seem to be a standard procedure for determining the extent of substitutability and complementarity across dimensions of poverty. Moveover, it is not entirely clear that any interrelationships across variables must be incorporated into the overarching methodology for evaluating multidimensional poverty. Instead, the interconnections might plausibly be the subject of separate empirical investigations that supplement, but are not necessarily part of, the underlying poverty measure. Our methodology provides a neutral foundation upon which more refined accounts of the interconnection between dimensions could be built. We hope that the methodology developed in this paper will be a useful touchstone for future research efforts.

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Appendix

Proof of Theorem 1 Verification is immediate for all properties except for weak rearrangement and weak transfer. For weak rearrangement, is clear that the rearrangement does not change the set of the poor nor the collection of achievements among the poor. Hence, M_α is unaffected by the rearrangement and just satisfies the axiom for all $\alpha \geq 0$.

As for the weak transfer property, let $\alpha \geq 1$ and consider any x and y such that x is obtained from y by an averaging of achievements among the poor. Then where q is the number of poor persons in y, let y' be the matrix obtained from y by replacing each of the n-q non-poor rows of y with the vector z. Similarly, let x' be the matrix obtained from x by replacing the same n-q rows with z. Clearly $M_\alpha(y;z)=M_\alpha(y';z)$ and $M_\alpha(x;z)=M_\alpha(x';z)$. For any data matrix v, let $g^\alpha(v)$ denote the matrix of α powers of normalised gaps (or shortfalls) associated with v, and notice that $\mu(g^\alpha(v))$ is a convex function of v for $\alpha \geq 1$. Since $x'=By'$ for some bistochastic matrix B, it follows that $\mu(g^\alpha(x')) \leq \mu(g^\alpha(y'))$. But $M_\alpha(y';z)=\mu(g^\alpha(y'))$ by the construction of y', and if the number of poor in x is q, then $M_\alpha(x';z)=\mu(g^\alpha(x'))$ and we would be done. However, it is also possible that the number of poor in x is less than q; in other words the smoothing process has moved at least one person from being poor to being non-poor. Then it follows that the associated rows in $g^\alpha(x')$ will need to be censored in measuring $M_\alpha(x';z)$, implying that $M_\alpha(x';z) \leq \mu(g^\alpha(x'))$. Either way, it follows that $M_\alpha(x;z) \leq M_\alpha(y;z)$ and hence M_α satisfies the weak transfer axiom for $\alpha \geq 1$.

Proof of Theorem 2

(i) It is clear that for $k \in (0,d]$ we have $H(y;z)=F_a(s)$ for $s=d-k$, where $s \in [0,d)$. Consequently, $y\boldsymbol{H}y'$ implies $F_a(s) \leq F_{a'}(s)$ for all $s \in [0,\mathrm{d})$ with < for some s. Note that for any $s \geq d$ we have $F_a(s)=F_{a'}(s)=1$, and hence $a\mathbf{FD}a'$. The converse holds by analogous arguments.

(*ii*) It is clear that for $k\in(0,d]$ we have $M_0(y;z)= [\int_0^s F_a(t)dt + sF_a(s)]/d$ where $s=d-k$, and therefore $y\boldsymbol{M_0}y'$ is equivalent to

(A1) $\int_0^s (F_{a'}(t)-F_a(t))dt + s(F_{a'}(s)-F_a(s)) \geq 0$ for all $s\in[0,d)$, with > for some s.

Suppose that $a\mathbf{FD}a'$. Then $\int_0^s F_a(t)dt \leq \int_0^s F_{a'}(t)dt$ and $sF_a(s) \leq sF_{a'}(s)$ hold for all $s\in[0,d)$ while each inequality must be strict for some s. By (A1), then, we have $y\boldsymbol{M_0}y'$. Now suppose that $y\boldsymbol{M_0}y'$. We want to show that $a\mathbf{SD}a'$, which since $F_{a'}(s)=F_a(s)$ for $s\geq d$, becomes

(A2) $\int_0^s (F_{a'}(t)-F_a(t))dt \geq 0$ for all $s\in[0,d)$, with > for some s.

Pick any $s_1\in[0,d)$. If $F_{a'}(s_1)-F_a(s_1) \leq 0$, then by (A1),

$$\int_0^{s_1} (F_{a'}(t)-F_a(t))dt \geq \int_0^{s_1} (F_{a'}(t)-F_a(t))dt + s_1(F_{a'}(s_1)-F_a(s_1)) \geq 0,$$

as required by (A2). Alternatively, suppose that $F_{a'}(s_1)-F_a(s_1) > 0$. If $F_{a'}(s)-F_a(s) \geq 0$ for all $s<s_1$, then clearly $\int_0^{s_1} (F_{a'}(t)-F_a(t))dt \geq 0$ once again as required by (A2). On the other hand, if it is not true that $F_{a'}(s)-F_a(s) \geq 0$ for all $s<s_1$, then the set $S=\{s \in [0,s_1): F_{a'}(s)-F_a(s) < 0\}$ must be nonempty. Since each $s\in S$ satisfies $F_{a'}(s)-F_a(s) < 0$, it follows that $\int_0^s (F_{a'}(t)-F_a(t))dt \geq 0$ for each $s\in S$, again by (A1). Let $s_0 \leq s_1$ be the least upper bound of S. Then it follows from the continuity of $\int_0^s (F_{a'}(t)-F_a(t))dt$ in s that $\int_0^{s_0} (F_{a'}(t)-F_a(t))dt \geq 0$. Moreover, $F_{a'}(s)-F_a(s) \geq 0$ for all $s \in (s_0,s_1)$ and so we conclude that $\int_0^{s_1} (F_{a'}(t)-F_a(t))dt \geq 0$ again. Since s_1 can be any element of $[0,d)$, it follows that $\int_0^s (F_{a'}(t)-F_a(t))dt \geq 0$ for all $s\in[0,d)$. If it were the case that

$\int_0^s (F_{a'}(t) - F_a(t))dt = 0$ for all $s \in [0,d)$, then this would imply that $F_{a'}(s)$ and $F_a(s)$ are identical, which is ruled out by the strict portion of (A1). Hence (A2) is true and $\boldsymbol{a}\mathbf{SD}\boldsymbol{a}'$.