# Characterizing weights in the measurement of multidimensional poverty: An application of data-driven approaches to Cameroonian data *

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**Abstract:**

The study seeks to compare multidimensional poverty indices in Cameroon generated by different multivariate techniques. After carefully exploring the theoretical and empirical review of the statistical methods of setting weights in the measurement of multidimensional poverty, the study employs three different statistical or data-driven methods - principal components analysis, multiple correspondence analysis, and fuzzy set approach to set weights in the aggregation procedure. Use is made of the 2001 Cameroonian household survey data to estimate the models. The poverty distributions obtained from the three approaches are submitted to stochastic dominance tests to investigate the sensitivity of the resultant poverty index rankings to changes in the weighting characterization. It comes out of the empirical analysis that the principal components analysis index distribution unambiguous shows less poverty than the multiple correspondence analysis and fuzzy set composite indices, while comparison of the two latter index distributions shows no clear dominance.

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## 1. INTRODUCTION

The notion that poverty should be measured on the basis of a large number of variables has enjoyed an increasing support in the recent years. For a long time, particularly since the introduction of the economic concept of poverty by Booth (1892) and Rowntree (1901), the reference indicator has often been income or expenditure per capita. But while these indicators act as reasonably accurate and useful measures of economic performance, they have been subjected to severe criticisms by several authors, among them Townsend (1993), Ravaillon (1996) and Tsui (2002). This has engendered attempts to find suitable multidimensional indicators which can capture the different facets of poverty. Since the work of Townsend (1979) it has increasingly been recognised that other aspects of human life not necessarily related to income do impact on human development. These include access to public goods, health, education, housing conditions, life satisfaction and so on. Also contributing to this increased interest in multidimensional poverty measures is the evolution in conceptual thinking on poverty towards functionnings and capabilities as initiated by Sen (1993).

The consequence of this conceptual revolution is a broadened notion of poverty to include vulnerability, exposure to risks, voicelessness and powerlessness (World Bank, 2001). Today, poverty is no longer confined to the lack of the ability of individuals/households to command sufficient resources to satisfy their basic needs (Townsend, 1993) nor considered as a mere economic and monetary dimension, but rather increasingly considered as human deprivation in various life domains. This deprivation from the multidimensional perspective includes both quantitative and qualitative measures such as the joy of choices, opportunities and others which are most basic to human development and can paint quite different pictures of the poverty situation in any given country (Alkire, 2002).

On the empirical side, the past few decades has witnessed a tremendous search for suitable approaches of measuring multidimensional poverty. These approaches include the social exclusion approach of Rene Lenoir (1974)[1], the multidimensional axiomatic approach and the UNDP (1997) human poverty index (HPI). The HPI combines life expectancy, education and health. This index, though widely used, has come under

[1] This was cited from Evans et al. (1995)

increasing criticism for leaving out an income dimension and for attributing arbitrary equal weights to each dimension[2]. Again, the choice of what variables should be included in the HPI is somehow arbitrary and may not reflect peoples' preferences and realities of the country under study (see Booysen, 2002). The multidimensional axiomatic approach begins with the specification of a general function of the form $P(x, z) = F[\pi(x_i, z)]$ where F and $\pi(.)$ are based on some axioms that stipulate how poverty indicators can be assessed (Bourguignon and Chakravarty, 2002 and Bibi, 2005). However, the specification of the functional form of the equation is quite arbitrary and subjective.

The main objective of this paper is to employ three multivariate statistical methods-notably principal components analysis, multiple correspondence analysis, and fuzzy set theory - that allow the available data to speak for themselves in determining the relevant variables and optimal weights assigned to each variable in the construction composite indices, rather than making a priori assumptions. The second goal of the paper is to apply the statistical techniques to Cameroonian data and hence investigate how composite poverty index comparisons are sensitive to changes in the aggregation and weighting schemes.

This paper is structured as follows. Section 2 briefly reviews the meaning and measurement of multidimensional poverty paying particular attention to weighting schemes. Section 3 and 4 present the methodology of statistical methods and data used for the analysis respectively. Section 5 presents the results emerging from the estimation of the statistical models and uses a stochastic dominance method to test the sensitivity of the index-based poverty rankings. Finally, Section 6 concludes with some policy implementation remarks.

## 2. THE MEANING AND MEASUREMENT OF POVERTY

### 2.1 THEORETICAL BACKGROUND

Many theoretical works and empirical research have addressed the issue of defining and measuring poverty. Different approaches can be distinguished on the basis of the variables taken into account: income/consumption, access to goods and services or the

[2] See UNDP (2004) Technical note 1

capability to obtain them. Empirical research on poverty shows that different approaches provide different results about its magnitude and evolution.

Traditional approaches to the measurement of poverty are one-dimensional, since they are based on a single indicator, generally income or expenditure per capita, to show the level of deprivation. These money-metric measures separate the population between poor and non-poor on the basis of poverty lines which can be *absolute* or *relative*. According to the absolute approach, thresholds are defined on the basis of the amount of money needed to secure a minimum standard of living (Nolan and Whelan, 1996). Conversely, relative income measures set the threshold at a certain percentage of median or mean income (usually 50 or 60%), assuming that those falling below such threshold are unlikely to be able to fully participate in the life of the society. Although money-metric measures have some advantages, in term of easy of computation and comparability across countries, they also present some drawbacks that are well documented in the literature (see Sahn and Stifel, 2003).

Building on these shortcomings, the traditional one-dimensional approaches have been questioned and alternative multidimensional approaches have been put forward. Multidimensional methods allow the researchers to consider various aspects of both monetary and non-monetary in explaining poverty and living conditions. According to the more recent literature, poverty is widely conceptualized in terms of exclusion from the life of society because of a lack of resources, while exclusion means experiencing various forms of what society considers as serious deprivation (Nolan and Whelan, 1996). Consequently, poverty should be best treated as multidimensional and non-monetary indicators should complement monetary ones in order to better identify the poor.

Despite these advantages, poverty measures which incorporate information from many variables have also some drawbacks, mainly concerning difficulties in coping with the multidimensionality and the use of non-monetary variables. When trying to make operational a multidimensional poverty concept, many theoretical and methodological challenges must be faced. Since all these choices significantly affect the resultant multidimensional index, it is important to clarify them.

## 2.2 COMPOSITE INDEXING

When poverty is conceptualized as a multidimensional construct, it should be measured through the aggregation of the different deprivation variables experienced by the individuals. Accordingly, measuring multidimensional poverty usually involves the incorporation of information provided by several variables into a composite poverty index. The general procedure in the estimation of composite indices involves the:

- choice of the variables to be considered;
- definition of a weighting scheme for each item or individual;
- aggregation of the variables and,
- identification of a threshold which separates poor and non-poor individuals.

All of these issues must be carefully addressed in the construction of a multidimensional poverty index. We only briefly review each of them.

The first step in the building of a summary measure of poverty concerns the selection of the appropriate indicators. Obviously, the choice depends on the data availability, but the variables considered affect the resultant index. The selection of elementary variables heavily relies on the arbitrary choices of researchers that must face a trade-off between possible redundancies caused by overlapping information and the risk of losing information (Perez-Mayo, 2005). A partial solution to such arbitrariness is provided by the use of multivariate statistical tools (e.g. principal component analysis), which allows the researchers to reveal the underlying correlation between basic items and to retain only the sub-set that best summarizes the available information.

Once a preliminary set of variables has been selected, their aggregation into a composite index implies choosing an appropriate weighting structure. A number of different weighting techniques have been used in the literature. First, some studies apply equal weighting for each variable (Townsend, 1979; UNDP, 1997 and Nolan and Whelan, 1996), thereby avoiding the need for attaching different importance to the various dimensions. Second, in an attempt to move away from purely arbitrary weights, in the construction of the composite poverty indices, variables have been combined using weights determined by a form of consultative process among poverty experts and policy analysts. Although this approach is an improvement on the first solution, it still involves subjective decisions regarding the welfare value of each component. Third, weights may be applied to reflect the underlying data quality of the variables thus giving less weight to those variables where data problems exist or with large amounts of missing values (Rowena et al. 2004). The reliability of a composite poverty index can be improved if it gives more weight to good quality data. However, this may as a result give more emphasis to variables which are easier to measure and readily available rather than more important welfare issues which may be more problematic to identify with good data. Fourth, variables have also been weighted using the judgment of individuals based on survey methods to elicit their preferences (Smith, 2002). The difficulty encountered here relates to whose preferences will be used in the application of the weights,, whether it be the preferences of policymakers, households or the public. Firth, a more objective approach is to impose a set of weights using the prices of various items. However, this is only possible if prices are available for all goods and services. Unfortunately, this is not the case. Again many respondents are unable to value their goods realistically and responses are likely to contain a large amount of error. This is further compounded in situations with significant regional price variation and high inflation. Other studies develop composite indices by aggregating the variables on the basis of their relative frequencies or relying on multivariate statistical methods to generate weights (Perez-Mayo, 2005). This approach, followed also in our work, will be discussed in greater detail in the next paragraphs.

Finally, the identification of poor or deprived households/individuals requires the definition of a threshold, an issue that raises several theoretical and empirical problems. Independently of the particular choice about the threshold, the identification of those to be considered poor implies always some degree of arbitrariness.

## 3. METHODOLOGY OF MULTIVARIATE STATISTICAL APPROACHES

As we stated in Section 2, the aggregation of variables in order to construct a multidimensional poverty index can be achieved in many ways. Statistical approaches provide alternative solutions to select and aggregate variables in index form without a priori assumptions in the weighting scheme. Only those features of each approach that are relevant to our context, namely the construction of a composite poverty index, are presented in this section, directing the reader to the appendix and/or related references for further statistical details.

## 3.1 PRINCIPAL COMPONENT ANALYSIS (PCA)

### 3.1.1 Definition and Goal of PCA

A principal components analysis (PCA) is concerned with explaining the variance-covariance structure of a set of variables through a few linear combinations of these variables (Krishnakumar and Nagar, 2007). Its general objectives are (1) data reduction and (2) interpretation.

Although $p$ components are required to reproduce the total system variability, often much of this variability can be accounted for by a smaller number $k$ of the principal components (PCs). If this is so, then there is as much information in the $k$ components as there is in the original $p$ variables. An analysis of PCs often reveals relationships that were not previously suspected and thereby allows interpretations that would not ordinally result. PCA has been applied by Klasen (2000) in South Africa and Nagar and Basu (2001) in India.

### 3.1.2 Methodological Choice

The applicability of classical factorial techniques is generally limited by the kind of data available. Specifically, standard PCA can in principle be applied only if all the variables are numeric (i.e the variables are either quantitative or continuous) and the relationships between variables are assumed to be linear (Gifi, 1990 and Kamanou, 2005). But, the variables available in our dataset are categorical, measured at nominal and ordinal level. Accordingly, linear or classical PCA would not be the most appropriate method. The problem is that ordinal variables do not have an origin or a unit of measurement and therefore means, variances and co-variances have no real meaning. As PCA relies on estimating the co-variance (correlation) matrix, the standard PCA model is no longer appropriate.

Another undesirable feature of the standard PCA is the fact that the analysis is based on z-scores which have unit variance and therefore have equal weights on the first PC (Kamanou, 2005). The variables are standardised by subtracting the sample mean and dividing by the standard deviation. For instance in the case of integer valued variables with skewed distributions, regular PCA will give large weights to variables that are most skewed, because skewness is associated with small standard deviations. To illustrate this, consider a variable that has two modalities 1 and 2, and suppose that about 90% of the data are concentrated at modality 1. Then this variable explains very little of the variability in the data and would have a very small standard deviation. When the variable is standardised by dividing by the standard deviation, the value of the variable would be magnified and it would get a large but undeserving weight in the PCA.

Therefore, to avoid limitations of standard PCA, we propose to adopt an alternative approach, allowing us to treat ordinal and binary variables. Kolenikov and Angeles (2004) have very recently described a technique, called polychoric PCA, which improves on the regular PCA. The polychoric PCA technique is especially appropriate for discrete data (binary and ordinal). For this purpose, it assumes that a latent continuous variable underlies every ordinal variable. For example, if the observed variable is health status measured on a three-point scale (good, fair, poor) it can be reasonably assumed that an underlying continuous variable exist. A respondent makes his choice on the scale depending on an implicit threshold observational rule, e.g if his health status is worse than a certain threshold $h^1$, it is poor; if it is worse than $h^2$ but better than $h^1$, it is fair and if it is better than $h^2$ it is good. These thresholds can be estimated and based on these and a distributional assumption about the underlying variable, correlation coefficients of the underlying continuous variables can be estimated (see Kuklys, 2004).

### 3.1.2 PCA Model Specification and Weighting Scheme

To specify the polychoric PCA model, we follow Kolenikov and Angeles (2004). If x is a random variable of dimension p with finite p x p variance-covariance matrix V[x] = Σ, principal components analysis solves the problem of finding directions of the greatest variance of the linear combinations of x's. In other words, the principal components ($y_j$) of the variables $x_1,\ldots,x_p$ are linear combinations $a_1^{'}x,......,a_p^{'}x$ such that

$y_j = a_j^{'}x$ j= 1, …, k ……………………………………………………….. ……. (1)

The motivation behind this problem is that the directions of greatest variability give most information about the configuration of the data in a multidimensional space. The first PC will have the greatest variance and extract the largest amount of information from the data, the second component will be orthogonal to the first one, and extract the greatest information in that sub-space; and so on. Also, the PCs minimize the sum of squared deviations of the residuals from the projections onto linear sub-spaces. The first PC gives a line such that the projections of the data onto this line have the smallest sum of squared deviations among all possible lines.

The solution to equation (1) is found by solving the eigenproblem for the co-correlation matrix Σ. This consists of finding $\lambda$ and $a$ such that:

$$\Sigma a = \lambda a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The solution to the eigenproblem (2) for the correlation matrix gives the set of principal components weights *a* (also called factor loadings), the linear combinations a'x (referred to as factor scores) and eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots\ldots\ldots \geq \lambda_p$. It is easy to establish that V[a'x] = $\lambda_k$ so that the eigenvalues[3] are the variances of the linear combinations (see Technical notes at Appendix).

Total variance = $\lambda_1 + \lambda_2 + \ldots\ldots\ldots + \lambda_p$ and consequently the proportion of total variance explained by the k-th PC = $\dfrac{\lambda_k}{\lambda_1 + \lambda_2 + \ldots\ldots + \lambda_p}$

Note that since the variables in our model are binary and ordinal, the matrix on which the PCA is based is the polychoric correlation matrix, and not the standard Pearson correlation matrix. Polychoric correlations are those correlations between ordinal variables and the latent continuous variables underlying each of the ordinal variables. They can be interpreted just as the standard Pearson correlation coefficients. Solving the eigenproblem of the polychoric correlation matrix is obtained by the bivariate information maximum likelihood procedure (Joreskog, 2004).

PCA is an appealing method for combining variables because the component loadings[4] or weights generated have a fairly intuitive interpretation. The magnitude of the coefficient on any one variable measures the importance of that variable to the PC, irrespective of the other variables. That is, they only measure the univariate contribution

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[3] The eigenvalue for a given factor or component measures the variance in all the variables which is accounted for by that factor.

[4] The factor loadings, also called component loadings in PCA, are the correlation coefficients between the variables (rows) and factors (columns).

of an individual variable to the PC, and do not provide information about the other variables. In welfare analysis the first PC explains most of the variance in the original data set and is often considered to represent the composite poverty index. The index is a weighted average of the variable scores with weights equal to the loadings of the first PC. Analytically, the composite index C takes the following form:

$$C_i = \sum_{i=1}^{n} w_i x_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where $C_i$ is the composite welfare index, n is the number of variables, $w_i$ is the weight attached to variable *i*, and $x_i$ the score on variable *i*.

## 3.2 MULTIPLE CORRESPONDENCE ANALYSIS (MCA)

### 3.2.1 When to use MCA

MCA allows one to analyze the pattern of relationships of several categorical dependent variables (Asselin, 2002). As such, MCA is used when the variables to be analyzed are categorical (nominal) instead of quantitative. Each nominal variable comprises several levels, and each of these levels is coded as a binary variable. MCA can also accommodate quantitative variables by recoding them as nominal observations. Studies based on MCA to generate composite poverty indices include the works of Asselin and Vu Tuan (2005) in Vietnam; Ki et al. (2005) in Senegal; Ningaye and Ndjanyou (2006) and Njong (2007) in the case of Cameroon.

### 3.2.2 The MCA Model

Technically MCA is obtained by using a standard correspondence analysis on an indicator matrix (*i.e.,* a matrix whose entries are 0 or 1) (see Technical notes at the Appendix). The principle of the MCA is to extract a first factor which retains maximum information contained in this matrix. The ultimate aim of MCA (in addition to data reduction) is to generate a composite indicator for each household.

For the construction of a CPI from K ordinal categorical indicators, the monotonicity axiom must be respected (Asselin, 2002). The axiom just means that if a household i improves its situation for a given variable, then its composite poverty index value $CPI_i$ increases: its poverty level decreases (larger values mean less poverty or equivalently, welfare improvement). The monotonicity axiom translates into the First Axis Ordering Consistency (FAOC) principle (Asselin, 2002). This means that the first axis must have growing factorial scores indicating a movement from poor to non-poor situation. For each of the ordinal variables, the MCA calculates a discrimination measure on each of the factorial axes. It is the variance of the factorial scores of all the modalities of the variable on the axis and measures the intensity with which the variable explains the axis.

The weights given by MCA correspond to the standardized scores on the first factorial axis. When all the variable modalities have been transformed into a dichotomous nature coded 0/1, giving a total of P binary indicators, the CPI for a given household i can be written as (see Asselin, 2002):

$$CPI_i = \frac{1}{K}\left(W_1 I_{i1} + W_2 I_{i2} + ... + W_P I_{iP}\right), \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ldots(4)$$

Where $W_p$ = the weight (score of first standardized axis, (score/$\sqrt{\lambda_1}$) ) of category p

$I_{p,}$ = binary indicator 0/1, which takes on the value 1 when the household has the modality, and 0 otherwise. The CPI value reflects the average global welfare level of a household.

The CPI constructed using MCA has a tendency of being negative in its lowest part. This would make interpretation difficult. However, it can be made positive by a translation using the absolute value of the average $C_{min}$ of the minimal categorical weight $W^k_{\min}$ of each indicator. Asselin (2002) expresses this average minimal weight as:

$$C_{\min} = \frac{\sum_{k=1}^{K} W^k_{\min}}{K}$$

The absolute value of $C_{min}$ can then be added to the CPI of each household to obtain the new positive CPI scores

### 3.3 THE FUZZY SET APPROACH TO POVERTY

In the poverty literature, a poverty threshold is often established below which households/individuals are considered to be poor. Unfortunately, the choice of a poverty threshold is an arbitrary one (Filippone *et al.* 2001) in that it establishes an artificial dichotomy between poor and non-poor. As pointed out by Cerioli and Zani (1990) and Cheli *et al*. (1994), the problem is that a sharp division of households between poor and non-poor is unrealistic. This has led Qizilbash (2001) to characterise poverty as a vague concept since there seams to be no clear cut-line between the poor and the non-poor. This calls for a mathematical framework capable of modelling vague concepts such as poverty. The fuzzy set theory seems particularly appropriate. Cerioli and Zani (1990) were the first to apply the concept of fuzzy sets to the measurement of poverty. Apiah-Kubi *et al*. (2007) have used the fuzzy set approach to study multidimensional poverty in Ghana.

### 3.3.1 Exposition of the Fuzzy Set Method

For a brief mathematical exposition of the fuzzy set theory, we follow Dagum and Costa (2004), and Apiah-Kubi et al. (2007) to proceed as follows: let *X* be a set and *x* an element of *X*. A fuzzy subset *P* of *X* can therefore be defined as follows:

$$P = \{x, F_P(x)\}, \text{ for all } x \in X$$

where, $F_P$, is a membership function which takes its values in the closed interval [0, 1]. In other words, the fuzzy sub-set P of X is characterized by a membership function $F_P(x)$ associating a real number in the interval [0, 1] to each point of X. The value $F_P$ represents the degree of belonging to P. That is, each value $F_P(x)$ is the degree of membership of x to P.

In a simple application to poverty measurement, we can let X be a set of n households or individuals (i=1, 2, 3, …, n) and P, a fuzzy subset of X, the set of poor people. In the fuzzy approach $F_P(x)$, the membership function of the poor set (household/individual i) is defined as :

- $x_{ij}$= 0, if household i is absolutely non-poor
- $x_{ij}$=1, if household i completely belongs to the poor set, and (5)
- $0<x_{ij}<1$, if the household reveals a partial membership to the poor set

### 3.3.2 Constructing the Multidimensional Deprivation Index

To aggregate of the various variables into a composite index we may proceed in two operational stages which consist of (i) specifying the household membership function for each of these variables and (ii) specifying the weighting structure and aggregating the membership functions.

**3.3.2.1 Estimation of Membership Function**

The determination of the individual membership function $F_P(x_i)$ depends on the type variable. Since the variables considered in this study are discrete we restrict the construction of membership functions to binary/dichotomous and ordinal variables.

- **Dichotomous variables**

The typical case of dichotomous variables is the possession or non-possession of durable goods. But there are also some questions about subjective feelings that are dichotomous, that is, answered by yes or no. The 'have' attribute is assumed to have a low risk of deprivation, while the 'have not' has a high risk of deprivation. The two attributes have the values of 0 and 1 in the closed set [0, 1], whereby 0 takes the low risk of deprivation and 1 takes on the high risk of deprivation. Following Costa (2002) we can define the *degree of membership* to the fuzzy set P of the $a_i^{th}$ household (i=1, 2, …,n) with respect to the $j^{th}$ attribute (j=1,……,m), as follows:

$$F_P(X_j(a_i)) = x_{ij}, \qquad 0 \le x_{ij} \le 1 \quad \text{.........................................} (6)$$

In other words, $X_j(a_i)$ represents an m-order vector of socio-economic attributes which will result in the state of poverty of a household $a_i$ if partially or not possessed by the household.

In this case:

- $x_{ij}=1$, iff the $a_i^{th}$ household does not possess the $j^{th}$ attribute (it completely belongs to the poor set)
- $x_{ij}=0$, iff the $a_i^{th}$ household possesses the $j^{th}$ attribute (it is absolutely non-poor).

Thus the deprivation index of the $a_i^{th}$ household, $F_P(a_i)$ (i.e. the degree of membership of the $a_i^{th}$ household to the fuzzy set P) can be defined as the weighted average of $x_{ij}$:

$$F_P(a_i) = \frac{\sum_{j=1}^{m} x_{ij} w_j}{\sum_{j=1}^{m} w_j} \quad \text{……………………………………………………………….. (7)}$$

where $w_j$ is the weight attached to the $j^{th}$ attribute. It is an inverse function of the degree of deprivation of this attribute by the population of households. In other words, the lower the frequency of poverty in terms of a given variable, the greater the weight this indicator will receive. In order to reduce the arbitrariness involved in the estimation of weights Cerioli and Zani (1990) propose a logarithmic function represented by the following expression:

$$w_j = \log\left[\frac{n}{\sum_{i=1}^{n} x_{ij} n_i}\right] \geq 0 \quad \text{…………………………………………………………… (8)}$$

$n_i$ represents the weight attached to each household $a_i$. Note that $n_i$ is equivalent to n times the relative frequency of household $a_i$ in the total population. It follows that $\sum_{i=1}^{n} n_i = n$.

- **Ordinal variables**

Ordinal or categorical discrete variables are those that present several modalities (more than two values). The variable presents *m* modalities ranked from the modality with a high risk of poverty to the one with a lower risk (or the reverse). We assign a score $c_j$ to each modality corresponding to the value (integer) of that modality. If we represent the lowest modality as $c_{inf,j}$ and the highest modality as $c_{sup,j}$ , then we follow Costa (2002), Cerioli and Zani (1990) and Dagum and Costa (2004) to express the membership function of the $a_i^{th}$ household as:

$F_P(a_i) = 1$ if $0 < c_{ij} \leq c_{inf,j}$

$$F_P(a_i) = \frac{c_{sup,j} - c_{ij}}{c_{sup,j} - c_{inf,j}} \quad \text{if } c_{inf,j} < c_{ij} < c_{sup,j} \qquad (9)$$

$F_P(a_i) = 0$ if $c_{ij} \geq c_{sup,j}$

We observe that $c_{inf,j}$ and $c_{sup,j}$ stand for the two threshold (or extreme) values. Since the values are arranged in order of deprivation, $c_{inf,j}$ is threshold below which the household is poor and $c_{sup,j}$ is the threshold above which the household is not poor relative to the $j^{th}$ attribute. If $c_{ij}$ is between these two thresholds then the household is partially deprived in the attribute. We assume that the modalities in the data set are equally spaced.

### 3.3.3 Aggregation Procedure

Having computed for each $a_i^{th}$ household/individual the value of his membership function, that is, his "degree of belonging to the set of poor" we can compute the multidimensional deprivation index of the population by aggregating the values of the membership functions determined above. Costa (2002) specifies the fuzzy poverty index of the *population* as a weighted average of the poverty ratio/index of the $a_i^{th}$ household which we represent as follows:

$$F_P = \frac{\sum_{i=1}^{n} F_P(a_i) n_i}{\sum_{i=1}^{n} n_i} = \frac{1}{n} \sum_{i=1}^{n} F_P(a_i) n_i \quad \ldots\ldots (10)$$

In a further refinement Costa (2002) defines another technique of constructing the population multidimensional deprivation index by aggregating the one-dimensional poverty indices for each of the *j* attributes considered. Equation (10) expresses the degree of deprivation of the $j^{th}$ attribute for the entire population of *n* households.

$$F_P(X_j) = \frac{\sum_{i=1}^{n} x_{ij} n_i}{\sum_{i=1}^{n} n_i} \quad \ldots\ldots (11)$$

From equation (11) one can express the multidimensional poverty index of the population $F_P$ as a weighted average of $F_P(X_j)$ with the weight $w_j$ as defined in equation 8 (see Dagum and Costa, 2004).

$$F_P = \frac{\sum_{j=1}^{m} F_P(X_j) w_j}{\sum_{j=1}^{m} w_j} \quad \text{……………………………………………………..………… (12)}$$

We observe that the multidimensional deprivation index of the population is obtained by aggregating across either households or across poverty attributes. It should be noted that the composite deprivation index $F_p$ is a monotonic increasing function of the degree of deprivation or poverty of each household. In this case a deterioration of the living conditions of the population, *ceteris paribus*, results in an increase in the composite poverty index $F_p$.

**4. SOURCE AND NATURE OF DATA**

This study uses data from the 2001 Cameroon Household Survey (ECAM II) data because of its detailed multi-variate nature which was designed to measure poverty and living conditions of the population. ECAM II data represents the most recent and, probably, the richest dataset currently available for realising multidimensional analysis of poverty and deprivation in Cameroon. ECAM II data are obtainable from the National Institute of Statistics. Since the details of the survey methods are published elsewhere we provide only a brief description here (see ECAM II, 2001).

In the data-set a range of questions relating to income /expenditure information are available. As far as non-monetary variables are concerned the questions posed covered a wide spectrum of items ranging from possession of consumer durables availability of certain basic goods and services, quality of housing, education and health status. In our analysis of multiple deprivations in this study we select 20 variables from the original ECAM II data-set that theoretically capture various dimensions of poverty. The survey actually visited 10,992 households. However, to take care of missing values we only considered those households that responded to *all* the 20 questionnaires captured in this study. This results in a reduction of the sample size to 9329 households[5].

The quantitative continuous variable retained in this study is household expenditure per capita which, following Cerioli and Zani (1990), and Appiah-Kubi et al.

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[5] Though this results in the loss of information, we are confident that all households are treated on the same basis.

(2007) we proceed to render it ordinal by categorizing the households into three modalities: (1) those with expenditure below the mean ($y_{ij} < y_{mean}$); (2) those with expenditure between the mean and 60% above the mean ($y_{mean} < y_{ij} < y_{max}$) and (3) those with expenditure greater than 60% above the mean ($y_{ij} > y_{max}$). We also transformed and recoded some of the other quantitative variables (e.g. distances/time with regard to basic infrastructure) into qualitative and ordinal ones.This ensures that all the variables in this study are discrete (binary or ordinal) in nature. To ensure comparability of the results, each of the multivariate statistical techniques considered in this study is estimated using the same 20 variables.

## 5. RESULTS AND DISCUSSION

### 5.1 ESTIMATION OF THE PCA MODEL

To estimate the PCA model, we submitted the 20 variables to a polychoric PCA. In the analysis of the polychoric correlation matrix[6] we ensured that it be positive semi-definite, and so be a proper co-variance matrix. If the matrix is not positive semi-definite, it will have negative eigenvalues. By setting negative eigenvalues to zero and reconstructing, we obtain the least-squares positive semi-definite approximation to the matrix. Estimation of the polychoric correlation matrix shows that the first PC has an eigenvalue of 10.03 and explains 50% of the total variance while the second PC has an eigenvalue of 1.4 and explains only 7.1% of the variance (see Table 1A). The leading eigenvectors from the first PC eigenvalue decomposition of the correlation matrix are presented in Table 1.

**Table 1: Principal Component Loadings**

| Variable | Loadings/Weight |
|---|---|
| 1. Sick during last two weeks | 0.0038 |
| 2. Type of health centre consulted | 0.0271 |
| 3. Can read /write a simple phrase | 0.0525 |
| 4. Level of education | 0.0524 |

[6] The polychoric correlation matrix is generated using STATA version 9.2 updated with the polychoric menu in the internet.

| | |
|---|---|
| 5. Source of water supply | 0.0595 |
| 6. Source of lighting | 0.0657 |
| 7. Energy for cooking | 0.0571 |
| 8. Type of toilet facility | 0.0602 |
| 9. Roof material | 0.0518 |
| 10. Floor material | 0.0640 |
| 11. Possession of mobile phone | 0.0603 |
| 12. Possession of TV set | 0.0629 |
| 13. Number of times deprived of water because of unpaid bills | 0.0615 |
| 14. Distance to nearest heath centre | 0.0346 |
| 15. Distance to nearest tarred road | 0.0506 |
| 16. Number of times deprived of electricity because of unpaid bills | 0.0524 |
| 17. Distance to nearest public school | 0.0213 |
| 18. Possession of refrigerator | 0.0619 |
| 19. Possession of a car | 0.0554 |
| 20. Expenditure per capita | 0.0451 |
| **Total** | **1.0000** |

Source: Computed using 2001 CHS data

The interpretation of the component loadings /weights is quite straightforward: each of them can be thought as the variable's relative contribution (see Figure 1A for the absolute component loadings plot) to the overall poverty component. Use is made of these weights to compute a household-specific composite poverty indicator based on each household's variable values as described in equation (3).

Close examination of these component loadings reveal that *Source of lighting, floor material, and possession of TV set* are the variables that account for most of the poverty component.

**5.2 ESTIMATION OF THE MCA MODEL**

The MCA[7] based on 20 variables and 59 modalities, demonstrates that the first factorial axis, explains 34% of the observed inertia (i.e the eigenvalue) while the second axis accounts for only 1.5%. To construct the CPI for each household, use is made of the functional form of the CPI expressed in equation (4).

The weights (factorial scores on first axis) attributed to the variable modalities are presented in Table 2.

**Table 2: Weights on Variable Modalities**

[7] The MCA was conducted using the 4.01 SPAD software.

| Variables | Modalities | Weights |
|---|---|---|
| 1. Sick during last two weeks | Yes | -0.04 |
| | No | 0.03 |
| 2. Type of health centre consulted | Tradi-practitioners | -0.52 |
| | Modern health centre | 0.11 |
| 3. Can read /write a simple phrase | No | -0.82 |
| | Yes | 0.33 |
| 4. Level of education | No Level | -0.81 |
| | Primary level | -0.20 |
| | Secondary | 0.43 |
| | Higher education | 1.55 |
| 5. Source of water supply | Streams/others | -0.89 |
| | Spring/wells | -0.38 |
| | Public tap | 0.24 |
| | Individual taps | 1.72 |
| 6. Source of lighting | Wood/others | -1.17 |
| | Kerosene lamp | -0.84 |
| | Generator | 0.05 |
| | Electricity | 0.61 |
| 7. Energy for cooking | Firewood | -0.40 |
| | Charcoal/sawdust | 0.32 |
| | Kerosene | 0.35 |
| | Gaz | 1.32 |
| | Electricity | 1.38 |
| 8. Type of toilet facility | No toilet | -1.24 |
| | Unconstructed latrine | -0.59 |
| | Constructed latrine | 0.27 |
| | Flush toilet | 1.90 |
| 9. Roof material | Thatches/mats | -1.26 |
| | Zinc sheets | 0.16 |
| | Cement/tiles | 1.05 |
| 10. Floor material | Mud/wood/others | -0.86 |
| | Cement | 0.44 |
| | Tiles | 1.97 |
| 11. Possession of mobile phone | No | -0.17 |
| | Yes | 1.72 |
| 12. Possession of TV set | No | -0.39 |
| | Yes | 1.16 |
| 13. Number of times deprived of water because of unpaid bills | deprived of water>3 times | -0.24 |
| | deprived of water <3 times | 1.71 |
| 14. Distance to nearest heath centre | distance>3 km | -.85 |
| | 1km<distance<3km | 0.03 |
| | 500m<distanced<1km | 0.26 |
| | distance<500m | 0.44 |
| 15. Distance to nearest tarred road | distance>10km | -0.82 |
| | 1km< distance<10km | -0.30 |
| | distance<500m | 0.59 |
| 16. Number of times deprived of electricity | deprived >2 times | -0.42 |

| | | |
|---|---|---|
| because of unpaid bills | 1< deprived <2 | 0.59 |
| | Never | 0.96 |
| 17. Distance to nearest public school | distance>3km | -0.85 |
| | 1km< distance<3km | 0.08 |
| | distance<1km | 0.18 |
| 18. Possession of refrigerator | No | -0.23 |
| | Yes | 1.61 |
| 19. Possession of a car | No | -0.10 |
| | Yes | 1.92 |
| 20. Expenditure per capita ($y_{ij}$) | $y_{ij} < y_{mean}$ | -0.33 |
| | $y_{mean} < y_{ij} < y_{max}$ | 0.43 |
| | $y_{ij} > y_{max}$ | 1.06 |

Source: Computed by authors based on 2001 ECAM II data using SPAD software

An analysis of the signs of the weights shows that a negative sign reduces welfare, while a positive sign positively contributes to household welfare. Using these weights we compute the CPI of each household. To avoid having negative values of CPI we estimate the average of the negative values of the CPI and add the absolute value of this average to the CPI of each household to obtain the positive CPI scores.

For the construction of a CPI from categorical indicators, the monotonicity axiom must be respected. The composite poverty indicator must be monotonically increasing in each of the primary indicators (Asselin, 2002). The axiom just means that if a household improves its situation for a given primary variable, then its CPI value increases: its poverty level decreases (larger values mean less poverty or equivalently, welfare improvement). The monotonicity axiom translates into the First Axis Ordering Consistency (FAOC) principle. This means that the axis has growing factorial scores indicating a movement from poor to non-poor situation.

In Table 3 we present the discriminatory measures which indicate the relative contributions of the variables to the compsite poverty index.

**Table 3 Discriminatory Measures of Variables**

| Variable | Relative Contribution (%) |
|---|---|
| 1. Sick during last two weeks | 0.02 |
| 2. Type of health centre consulted | 0.72 |
| 3. Can read /write a simple phrase | 3.5 |
| 4. Level of education | 5.6 |
| 5. Source of water supply | 7.9 |
| 6. Source of lighting | 6.8 |
| 7. Energy for cooking | 5.8 |
| 8. Type of toilet facility | 7.5 |
| 9. Roof material | 3.2 |
| 10. Floor material | 7.1 |
| 11. Possession of mobile phone | 3.8 |
| 12. Possession of TV set | 5.9 |
| 13 Number of times deprived of water because of unpaid bills | 5.3 |
| 14. Distance to nearest heath centre | 2.9 |
| 15. Distance to nearest tarred road | 4.7 |
| 16. Number of times deprived of electricity b/c of unpaid bills | 4.6 |
| 17. Distance to nearest public school | 1.36 |
| 18. Possession of refrigerator | 4.6 |
| 19. Possession of a car | 2.4 |
| 20. Expenditure per capita | 3.4 |
| **Total** | **100** |

Computed by Authors using 2001 CHS data

Observe that *source of water supply, type of toilet facility, and floor material* contribute the most to the construction of the first axis which is the axis of poverty.

## 5.3 RESULTS IN THE FUZZY SET FRAMEWORK

The results of the estimation of the membership functions depicting the levels of deprivation for the various categories of deprivation variables, together with their weights are presented in Table 4.

**Table 4: Fuzzy Poverty Indices (Membership Functions) by Attribute.**

| Variable | $w_j$ | $F_P(X_j)$ | $w_j.(F_P(X_j))$ |
|---|---|---|---|
| Sick during last two weeks | 0.4112 | 0.3879 | 0.1595 |
| Type of health centre consulted | 0.7570 | 0.1750 | 0.1325 |
| Can read /write a simple phrase | 0.5398 | 0.2885 | 0.1558 |
| Level of education | 0.2280 | 0.5915 | 0.1349 |
| Source of water supply | 0.2232 | 0.5981 | 0.1335 |

| | | | |
|---|---|---|---|
| Source of lighting | 0.8049 | 0.1567 | 0.1261 |
| Energy for cooking | 0.0577 | 0.8757 | 0.0505 |
| Type of toilet facility | 0.3236 | 0.4747 | 0.1536 |
| Roof material | 0.2592 | 0.5506 | 0.1427 |
| Floor material | 0.1718 | 0.6733 | 0.1157 |
| Possession of mobile phone | 0.0412 | 0.9095 | 0.0375 |
| Possession of TV set | 0.1270 | 0.7465 | 0.0948 |
| Number of times deprived of water because of unpaid bills | 0.0574 | 0.8763 | 0.0503 |
| Distance to nearest heath centre | 0.3135 | 0.4859 | 0.1523 |
| Distance to nearest tarred road | 0.4142 | 0.3853 | 0.1596 |
| Number of times deprived of electricity because of unpaid bills | 0.1437 | 0.7183 | 0.1032 |
| Distance to nearest public school | 0.4174 | 0.3825 | 0.1596 |
| Possession of refrigerator | 0.0570 | 0.8769 | 0.0500 |
| Possession of a car | 0.0214 | 0.9520 | 0.0204 |
| Expenditure per capita | 0.1139 | 0.7694 | 0.0876 |
| Total | 5.4830 | ***0.4049*** | 2.2200 |

Source: Computed from ECAM II Survey Data

*Notes:* $\boldsymbol{w}_j$*= weight attached to variable j;* $F_P(X_j)$ *= fuzzy poverty index with respect to variable j;* $\boldsymbol{w_j}.(F_P(X_j))$*= weighted fuzzy poverty index, and* $F_P$ *is aggregate fuzzy poverty index.*

Our study estimates a composite deprivation degree (global fuzzy poverty index) for the whole country of 0.4049 in 2001. This means that of Cameroonian households, 40.48 percent on average registered deprivation on the various well-being indicators.

Table 4 also reports the weights attached to the attributes considered in this study. The smaller the number of households deprived in an attribute, the greater the weight attached to it.[8] Observe that the highest weight is attached to the variables; *source of lighting* and *type of health centre consulted*, indicating how strongly these variables do impact on the poverty status of Cameroonian households. Observe also that the lowest weights are attached to the possession of some goods of comfort namely; *possession of car, mobile phone, refrigerator and source of energy for cooking*. These low weights signal how these attributes are not possessed by many of the households. The low weight attached to expenditure per capita signals a low intensity in the deprivation of this

[8] The reasoning here is that if owning a radio is much more common than owning a TV, a greater weight should be given to the former indicator so that if a household does not own a radio, this rare occurrence will be taken much more into account in computing the poverty index than if some household does not own a TV, a case which is assumed to be more frequent.

indicator, meaning that monetary poverty is a common phenomenon in the Cameroon society.

## 5.4 COMPARISON OF POVERTY INDEX DISTRIBUTIONS

### 5.4.1 Descriptive Statistics of the Distributions

To be able to compare the fuzzy poverty index with the poverty indices derived from PCA and MCA analysis, we proceed to compute the complement of the household-specific fuzzy poverty index. This is achieved by performing the following logical transformation; $F^c_{pi} = 1 - F_{pi}$ where $F^c_{pi}$ is the complement of the fuzzy poverty index for household i. In the fuzzy set framework, the closer the index to 1, the more deprived is the household. On the contrary, the closer the conventional poverty indices to unity, the less poor or the better off is the household. By computing the complement of the fuzzy deprivation index we make a transition from the ordinary fuzzy set framework to the conventional welfare measure framework. Such a transition is to facilitate interpretation by ensuring that we follow the conventional requirement in distributive analysis that more is better than less. It should be noted that such a transformation of indices will not change the information provided, since the order of household rank is maintained although the distribution average changes. Table 5 gives basic descriptive statistics for the composite indices of well-being derived from the three methods.

**Table 5: Descriptive Statistics for the Index Distributions**

| Composite Index | No. of Observations | Mean | Standard Deviation | Minimum | Maximum |
|---|---|---|---|---|---|
| PCA | 9326 | 1.809 | .406 | 1 | 2.997 |
| MCA | 9326 | .717 | .341 | .093 | 1.830 |
| Fuzzy Set | 9326 | .595 | .168 | 0.001 | .976 |

Source: Computed by authors

The analysis of Table 5 shows that whether measured by the mean or the standard deviation, the household composite indices generated by PCA are higher than MCA indices, which on their part are higher than the fuzzy set composite indices. In other words national head count poverty measured by PCA index is lower than that measured by the MCA index which is lower than that measured by the fuzzy set method. Since these are only simply summary statistics, we need to check the robustness of the rankings.

**5.4.2 Sensitivity Tests**

To check the sensitivity of the orderings of the three multivariate statistical approaches in terms of poverty measurement, we resort to stochastic dominance tests. The theoretical foundations on poverty dominance have been developed by Duclos et al. (2003, 2006), Consider the case where we intend to investigate whether poverty is lower for method 1 than for method 2. The traditional procedure to address this question is to establish a poverty threshold, choose a poverty index (e.g the FGT index), and calculate poverty based on the two different methods and compare. The basic drawback of this procedure is that it depends on the poverty line or measure that is chosen. Setting the poverty line is an arbitrary process, and it is possible that choosing a different poverty line/measure will reverse the poverty rankings (Njong, 2007).The dominance approach to poverty analysis aims to avoid these problems by making poverty comparisons that are robust to the poverty threshold or measure selected. Suppose that we have two distributions with cumulative density functions (cdfs) F(x) and G(x) derived from methods 1 and 2 respectively. These particular cdfs are also called poverty incidence curves because each point on the curve gives the proportion of the population below the poverty line. Then, F(x) first-order stochastically dominates G(x) if $G(x) \geq F(x)$. In other words, G(x) is everywhere above distribution F(x). In this case, the head count poverty index ($P_0$) will always be higher for the first distribution than the second. Duclos et al.(2003, 2006) have shown that this sort of poverty comparison is robust to the choice of the poverty threshold and for all poverty measures. However, the mere visual inspection of cdfs overlooks the issue of sampling variation and may be untrustworthy. Since the cdfs are based on samples, there’s the possibility that observed differences merely reflect sampling variation and are not significant in the statistical sense.

Practically, to test for differences between the distributions we follow the approach of Bishop et al. (1991), Duclos et al. (2006) and Araar (2006). They suggest that when testing for dominance we calculate *test statistics* for a number of points (e.g. 10x10 grids of points) within the relevant interval. If this difference is always of the same sign (positive or negative) and statistically significant, then dominance holds for all poverty lines and measures. Two distributions are ranked as equivalent if there are no significant differences, while if the differences in the set of ordinates change signs within the interval, rankings are ambiguous and we may proceed to second-order dominance test.

Table 6 gives the dominance test results for pair-wise poverty index comparisons. We used DASP version 1.3 software to conduct the dominance tests.[9]

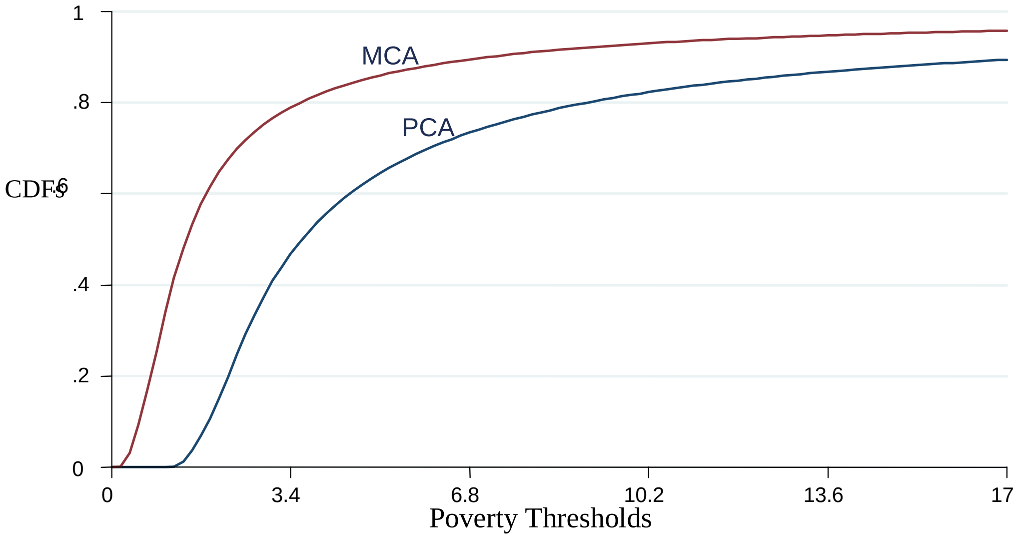
**Table 6: First-order Poverty Dominance Results**

| Index Distribution | | Observation |
|---|---|---|
| PCA | MCA | No intersection found. PCA dominates MCA |
| PCA | Fuzzy Set | No intersection found. PCA dominates Fuzzy set |
| MCA | Fuzzy Set | Intersection found. Dominance is ambiguous |

Source: Computed by authors

We observe from Table 6 that PCA index distribution dominates both MCA and Fuzzy set index distributions. This finding is confirmed by Figures 1 through 2, which reveal

Figure 1: PCA and MCA Poverty Incidence Curves



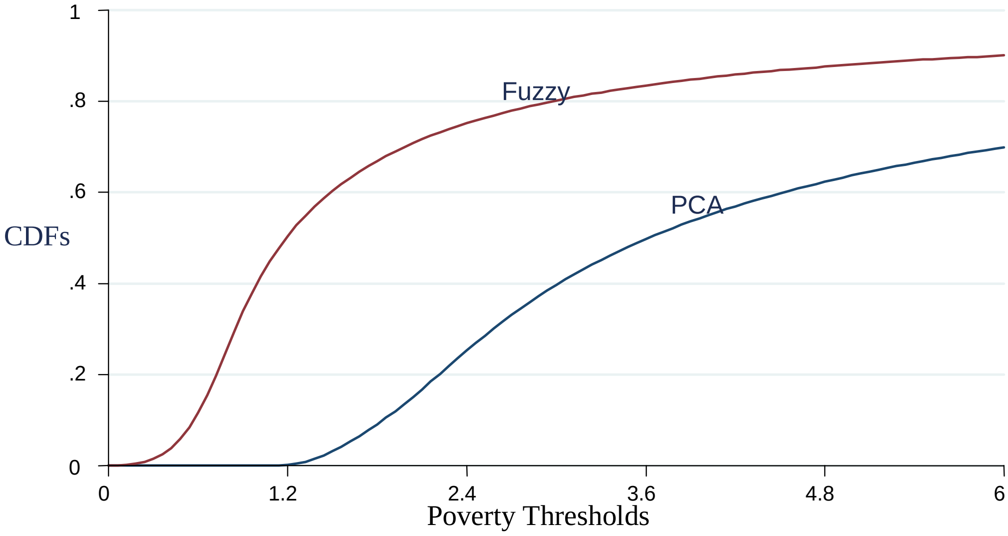

Source: Drawn by authors using DASP software

that the poverty incidence curve (cumulative distribution function) of PCA index is consistently below the other methods' index distribution functions over a wide range of interval. This indicates significant first-order poverty dominance for the PCA cdf over

[9] DASP: Distributive Analysis Stata Package by Abdelkrim Araar and Jean Yves Duclos, University of Laval, World Bank, PEP and CIRPEE, June 2007.

MCA and Fuzzy set cdfs. We may conclude with a fair degree of confidence that over all possible poverty frontiers and a broad class of poverty measures the PCA index has less poverty than any other.

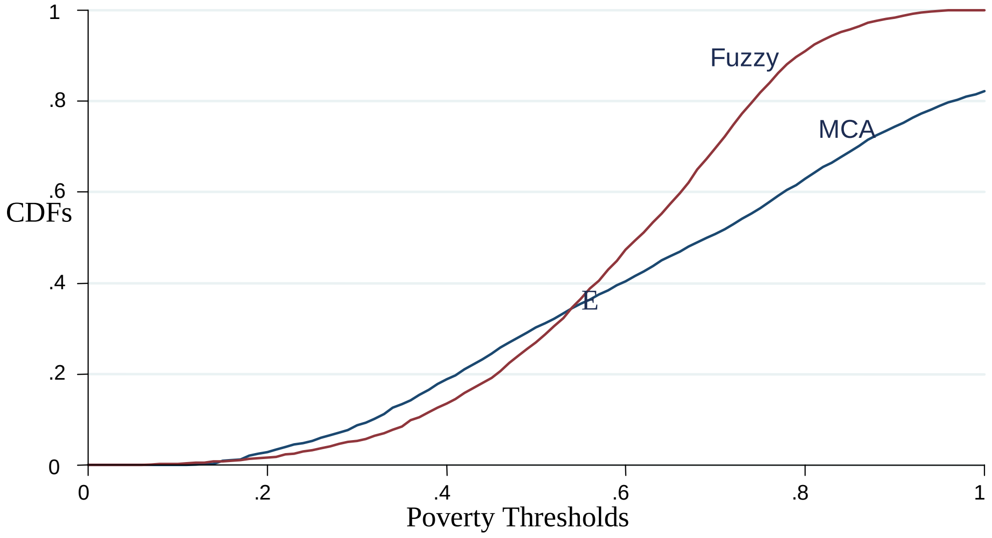
Figure 2: PCA and Fuzzy Set Poverty Incidence Curves




Source: Drawn by authors using DASP software

It comes out of the dominance analysis in Table 6 that there is no clear dominance when we compare the MCA and Fuzzy set index distributions. This is further confirmed by Figure 3 which shows that before the intersection at the point E, the Fuzzy set cdf dominates the MCA cdf and after the intersection, the reverse holds.

Figure 3: MCA and Fuzzy Set Poverty Incidence Curves




Source: Drawn by authors using DASP software

Since the cdfs in Figure 3 cross and the crossing is significant (for brevity we do not present the precise test statistic) we conclude that first-order poverty dominance is inconclusive. Given that first-order dominance is not observed, we tested for higher–order dominance (ie second- order and third-order dominance) and found that no clear dominance of one index over the other.

**6. CONCLUSION AND POLICY IMPLICATION**

In this study we have attempted to experiment a variety of weighting techniques and compare the results across the techniques before recommending one or a combination of the weighting schemes in deriving index estimates. The comparisons give an unambiguous ordering: the PCA index dominates the MCA and fuzzy set index distributions. This means that national poverty head count estimates derived from PCA weighting techniques are unambiguously lower than those obtained from MCA and fuzzy set weighting schemes. Comparison between MCA and fuzzy set indices give inconclusive dominance results. From a policy stand point, policy-makers may pay particular attention to index values from MCA and fuzzy set methods over PCA composite indexing. The intuition is that the weighting schemes of MCA and fuzzy set indexing depict more poverty than that of PCA.

Since the three index values were constructed using the same 20 ordinal variables, we may attribute differences in the dominance results to differences in the weighting and aggregation methods. Observe that in the case of PCA, variables are weighted with the proportion of the variance in the original set of variables explained by the first PC. This technique has the advantage of determining the set of weights which explains the largest variation in the original variables. MCA has the same logic as PCA, but it goes further to dichotomize and weights the variable modalities instead of the variables themselves. The aggregation weights of these two techniques are based on the co-variance or correlation matrix of the indicators. The fuzzy set weighting scheme, which is a function of the frequency of deprivation in terms of a given variable, assigns weights to the variables themselves. Thus differential weighting and conceptual issues may limit meaningful comparisons of the index values.

The statistical approaches experimented in this study have the advantage of allowing the available data to determine the optimal weights associated with each variable, rather than making value judgments. Although objective, these techniques are completely data-driven and the weights obtained are very rigid and may not necessarily be policy- appropriate for the country concerned. Despite the objectivity of the statistical methods employed in the composite indexing, some subjectivity is evident. Notably, the selection of variables to be included in the original sample of variables is ad hoc, and the scaling phase of variables to make them ordinal is subjective. Irrespective of these drawbacks, the multidimensionality of composite indices represents one of the major advantages of composite indexing. The indices represent aggregate measures of several variables that capture different complex development phenomena.

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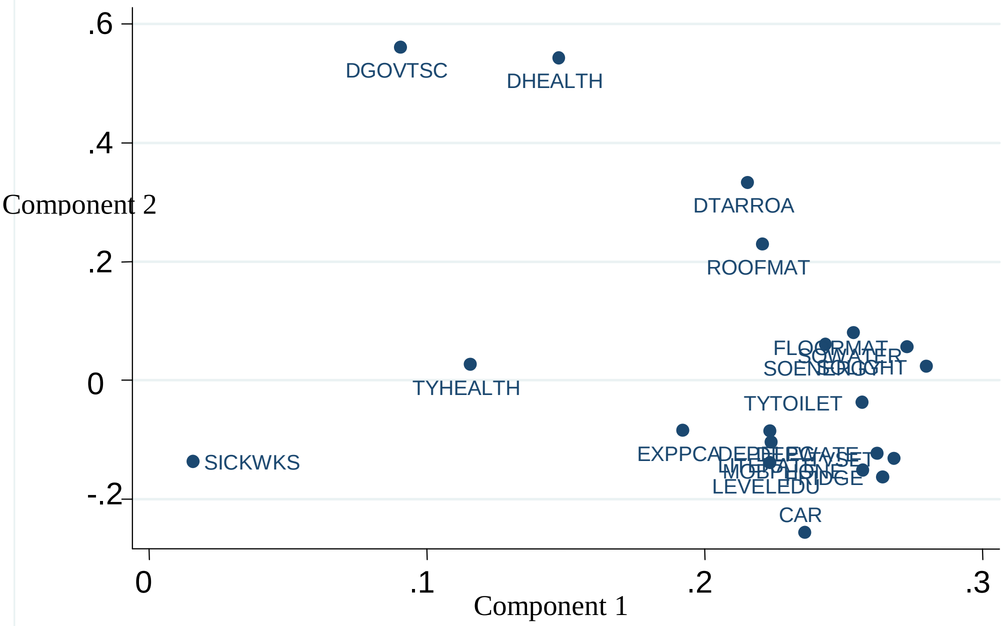
APPENDIX I

Figure 1A: Component loadings Plot




Source: Drawn by authors using STATA 9.2

**Table 1A: Principal components /Eigenvalues**

| Component | Eigenvalue | Difference | Proportion | Cumulative |
|---|---|---|---|---|
| Comp1 | 10.0319 | 8.62275 | 0.5016 | 0.5016 |
| Comp2 | 1.4091 | .07322 | 0.0705 | 0.5720 |
| Comp3 | 1.33587 | .239718 | 0.0668 | 0.6388 |
| Comp4 | 1.09615 | .133584 | 0.0548 | 0.6936 |
| Comp5 | .962571 | .158632 | 0.0481 | 0.7418 |
| Comp6 | .803939 | .087813 | 0.0402 | 0.7820 |
| Comp7 | .716126 | .093884 | 0.0358 | 0.8178 |
| Comp8 | .622241 | .05437 | 0.0311 | 0.8489 |
| Comp9 | .567862 | .037759 | 0.0284 | 0.8773 |
| Comp10 | .530103 | .039216 | 0.0265 | 0.9038 |
| Comp11 | .490887 | .163934 | 0.0245 | 0.9283 |
| Comp12 | .326953 | .033031 | 0.0163 | 0.9447 |
| Comp13 | .293922 | .046809 | 0.0147 | 0.9594 |
| Comp14 | .247112 | .012960 | 0.0124 | 0.9717 |
| Comp15 | .234152 | .053777 | 0.0117 | 0.9834 |
| Comp16 | .180374 | .029591 | 0.0090 | 0.9925 |

| | | | | |
|---|---|---|---|---|
| Comp17 | .150783 | .15078 | 0.0075 | 1.0000 |
| Comp18 | 0 | 0 | 0.0000 | 1.0000 |
| Comp19 | 0 | 0 | 0.0000 | 1.0000 |
| Comp20 | 0 | - | 0.0000 | 1.0000 |

Source: Computed by authors using STATA 9.2.

APPENDIX II

TECHNICAL NOTES

Multivariate methods deal with the simultaneous treatment of several variables. In a strict statistical sense, they concern the collective study of a group of variables that take into consideration the correlation structure within the group. This section reviews the multivariate approaches of Principal Components Analysis (PCA) and Multiple Correspondence Analysis (MCA). Unlike the Fuzzy set approach, PCA and MCA may not be transparent to readers with little knowledge of statistics, because they require some knowledge of advanced Statistics and matrix algebra. In what follows we strive to highlight some technical issues inherent in the methods.

**1. ANALYTICAL FRAMEWORK OF PCA**

PCA is essentially a data reduction technique. It is a technique often used to reduce the dimensionality of the data while retaining most of the variability in the original dataset. The basic idea behind this method is to determine orthogonal linear combinations (the principal components - PCs) by rotating the original system of variables such that the first PC explains most of the variance of the variables (Kamanou, 2000). If x is a random variable of dimension p with finite p x p variance-covariance matrix V[x] = Σ, principal components analysis solves the problem of finding directions of the greatest variance of the linear combinations of x's. In other words, the principal components of the variables $x_1,\ldots,x_p$ are linear combinations $a_1^{'}x,......,a_p^{'}x$ such that

$y_j = a_j^{'}x$ j= 1, …, k ……………………………………………………….. (1)

The motivation behind this problem is that the first PC will have the greatest variance and extract the largest amount of information from the data, the second component will be orthogonal to the first one, and extract the greatest information in that sub-space; and so on.

The solution to equation (1) is found by solving the eigenproblem for the correlation matrix Σ. To understand the concept of the eigenproblem, we follow Krishnakumar and Nagar (2007). Let us further denote by $\theta_1,\ldots,\theta_k$ the k eigenvalues of

$\Sigma$ and by $a_1,\ldots\ldots,a_k$ the corresponding eigenvectors. Then the principal components ($y_j$) equation (1) can be expressed alternatively in matrix form as:

$y = A'x$ ……………………………………………………………………… (2)

where $A = [a_1 \ldots a_k]$ is the matrix of eigenvectors of $\Sigma$. We have $A'A = AA' = I_k$ and $\Sigma = A\Theta A'$ or $A'\Sigma A = \Theta$ where $\Theta = diag(\theta_j)$, $j = 1,\ldots, k$ with the $\theta_j$'s arranged in descending order of magnitude. We also have $\Sigma^{-1} = A\,\Theta^{-1}A'$. The variances of the PCs are equal to corresponding eigenvalues, i.e. $V(y_j) = \theta_j,\ \forall_j$

Now, how do we derive the composite indicator ( i.e the deprivation index) from PCs? The two most commonly used approaches are:

1) The first principal component i.e. the one corresponding to the greatest eigenvalue $\theta_j$

If we take the first PC; $y_1 = a_1' x$ as an aggregate index then we have $V(y_1) = \theta_1$.

2) A weighted average of all the principal components $p_j$'s, j = 1,...,k with the weights $w_j$ being given by the proportion of the total variance explained by each PC. As for the weighted average its variance can be calculated as follows. Let us write it as:

$$\hat{H} = \sum_{j=1}^{k} w_j p_j \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where $$w_j = \frac{\theta_j}{\sum_{j=1}^{k} \theta_j} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

The model implicitly assumed by treating the first PC as a suitable index of welfare based on ordinal variables can be formulated following Kolenikov and Angeles (2004). Let us assume that two ordinal variables $x_x$ and $x_2$ are obtained by categorizing $x_1^*$ and $x_2^*$ that come from a bivariate normal distribution with standard normal marginals.

Denote corr($x_1^*, x_2^*$) = ρ and $\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix} \approx N\left[0, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right]$, $-1 \leq \rho \leq 1$ ………………… (5)

The categorizing thresholds for the two variables are given by $\alpha_{1,1}$ and $\alpha_{2,1}$ ($\alpha_{i,0} = -\infty$ , $\alpha_{i,2} = +\infty$, i=1,2), then the proportion in each cell(i,j) is given by:

$$\pi_{i,j} = \pi(i,j, \rho,\alpha) = Prob[x_1 = i,\ x_2 = j] =$$
$$= \Phi_2(\alpha_{1,i}, \alpha_{2,j}\,;\rho) - \Phi_2(\alpha_{1,i-1}, \alpha_{2,j}\,;\rho) - \quad (6)$$
$$- \Phi_2(\alpha_{1,i}, \alpha_{2,j-1}\,;\rho) + \Phi_2(\alpha_{1,i-1}, \alpha_{2,j-1}\,;\rho)$$

where $\Phi_2(.)$ represents the cdf of the bivariate standard normal distribution.

The maximum likelihood estimate of ρ, given threshold values, can be obtained by maximizing the following equation:

$$\log L = i = \sum_{i=1}^{N} \log \pi\left(x_{i,1}, x_{i,2}; \rho, \alpha\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ldots(7)$$

The resulting ρ is what is referred to as polychoric correlation. That is, polychoric correlations are those correlations between ordinal variables and the latent continuous variables underlying each of the ordinal variables. In practice, the estimation is performed in three stages:

- First, the thresholds are estimated as $\alpha_j = \Phi^{-1}\left(\sum_{i=1}^{j} \frac{N_i}{N}\right)$, j = 1, …. , k

  where $\Phi^{-1}(.)$ is the inverse of the standard normal distribution function and $N_i$ is the number of observations in category i.
- Second, estimation of the latent (polychoric) correlations given the estimated thresholds
- Third, the bivariate information maximum likelihood procedure is used to estimate the polychoric correlation matrix

Having established the correlations, one can proceed to PCA in the regular manner to solve the eigenproblem, that is, diagonalise the polychoric matrix[10] (as in equation 2 above).

## 2. ANALYTICAL FRAMEWORK OF MCA

MCA is used to analyze the pattern of relationships among observations described by a set of nominal variables. Each nominal variable comprises several levels, and each of these levels is coded as a binary variable. For example gender, (F *vs.* M) is one nominal variable with two levels. The pattern for a male respondent will be 0 1 and 1 0 for a female. The complete data table is composed of binary columns with one and only one column taking the value "1" per nominal variable. MCA can also accommodate quantitative variables by recoding them as binary variables. We do not lose any information by proceeding like this, rather we have an advantage to make appear the

[10] Multivariate factorial analyses motly differ according to the nature of the matrix to be diagonalized

specificities of the modalities considered individually. The principle of the MCA is to extract a first factor which retains maximum information contained in this matrix.

To understand the MCA technique we make the following notations. There are $K$ nominal variables, each nominal variable has $J_k$ levels and the sum of the $J_k$ is equal to $J$. There are $I$ observations. The $I \times J$ indicator matrix is denoted **X**. Performing correspondence analysis on the indicator matrix will provide two sets of factor scores: one for the rows and the other for the columns. These factor scores are, in general scaled such that their variance is equal to their corresponding eigenvalue.

The inertia matrix which is finally diagonalized in the MCA is Burt matrix deduced from the binary matrix by $B=X^T X$. The principle of the MCA is to extract a first factor which retains maximum information contained in this matrix. And then to extract the first eigenvalue ($\lambda 1$) and the associated eigen vectors. The Burt matrix is important in MCA because using correspondence analysis on the Burt matrix gives the same factors as the analysis of **X** but is often computationally easier. But the Burt matrix also plays an important theoretical role because the eigenvalues obtained from its analysis give a better approximation of the inertia explained by the factors than the eigenvalues of **X**.

We may make two interpretations of MCA results:

a) Each variable modality has a coordinate on each of the extracted axes. It is the factorial score which represents its weight in that axes. If we consider α as a factorial axis, $\Phi_{\alpha p}$ is the factorial score of a modality $p_j$ on this axis. This score is obtained by:

$$\Phi_{\alpha p} = \sqrt{\frac{np}{p_j}}\, \lambda_\alpha\, w_{p_j}^{\alpha}$$

where n is the total number of individuals, p the total number of variables, $p_j$ is the number of individuals possessing the modality p of the variable j. $\lambda\alpha$ is the eigenvalue of factor α. $w_{p_j}^{\alpha}$ is the proportion of the eigenvalue associated to $\lambda\alpha$ corresponding to the modality $p_j$.

b) For each of the ordinal variables, the MCA calculates a discrimination measure on each of the factorial axes. It is the variance of the factorial scores of all the modalities of the variable on the axis and measures the intensity with which the variable explains the axis. These quantities are important so as to interpret a factorial axis.