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A note on the standard errors of the members of the Alkire Foster family and its components

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Comments and corrections welcome.

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1 Basic notation

Consider a matrix X , whose N rows have information on the attainments of N individuals. Each column, therefore, hosts the distribution of each attainment across the population. The number of columns/variables is D . A typical attainment element of the matrix is: $x_{nd} (\in \mathbb{R})$, that is, the attainment of individual n in dimension/variable d .

In the identification stage, the variable-specific poverty lines are denoted by z_d ,¹ and for the second identification stage, i.e. to determine who is multidimensionally poor, variables are weighted by weights w_d such that: $w_d \in \mathbb{R}_+ \wedge \sum_d^D w_d = D$. The matrix of deprivations is formed by replacing x_{nd} with a deprivation gap, g_{nd} , such that:

$$\begin{aligned} g_{nd}(k) &= \frac{z_d - x_{nd}}{z_d} \text{ if } z_d > x_{nd} \wedge c_n \geq k \\ g_{nd}(k) &= 0 \text{ otherwise} \end{aligned} \quad (1)$$

where $k \leq D$ is the multidimensional-deprivation cut-off and c_n is the weighted number of deprivations suffered by individual n . If $c_n \geq k$ individual n is said, and identified, to be multidimensionally poor. $c_n \equiv \sum_{d=1}^D w_d I(z_d > x_{nd})$ ²

Now the multidimensional headcount can be defined:

$$H(X; k, Z) \equiv \frac{1}{N} \sum_{n=1}^N \left[\sum_{d=1}^D w_d g_{nd}(k) \right]^0 = \frac{1}{N} \sum_{n=1}^N I(c_n \geq k) \quad (2)$$

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¹From a vector of poverty lines, $Z : (z_1, \dots, z_d, \dots, z_D)$.

² $I()$ is an indicator that takes the value of 1 if the expression in parenthesis is true. Otherwise it takes the value of 0.

Also the average number of deprivations of the multidimensionally poor in the same period is defined:

$$A(X; k, Z) \equiv \frac{\sum_{n=1}^N \sum_{d=1}^D w_d [g_{nd}(k)]^0}{D \sum_{n=1}^N \left[\sum_{d=1}^D w_d g_{nd}(k) \right]^0} = \frac{\sum_{n=1}^N I(c_n \geq k) c_n}{DNH(X; k, Z)} \quad (3)$$

Finally the adjusted headcount ratio is:

$$M^0(X; k, Z) \equiv H(X; k, Z) A(X; k, Z) = \frac{\sum_{n=1}^N I(c_n \geq k) c_n}{DN} \quad (4)$$

More generally, the Alkire-Foster family can be defined as follows:

$$\mathcal{AF} : \{M^\alpha(X; k, Z) \equiv \frac{\sum_{n=1}^N \sum_{d=1}^D w_d g_{nd}^\alpha}{DN} \forall \alpha \in \mathbb{N}\}. \quad (5)$$

When $\alpha = 0$: $M^0 = \frac{\sum_{n=1}^N c_n}{DN}$, as in (4). A more useful definition of $M^\alpha(X; k, Z)$, for the purposes of deriving standard errors is the following:

$$M^\alpha(X; k, Z) \equiv \frac{1}{N} \sum_{n=1}^N I(c_n \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right) \forall \alpha \in \mathbb{N}, \quad (6)$$

where $\left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha = \left[\frac{z_d - x_{nd}}{z_d} \right]^\alpha$ if $z_d > x_{nd}$. Otherwise: $\left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha = 0$.

2 The case of a simple random sample

Since $M^\alpha(X; k, Z)$ is the average of $I(c_n \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right)$ across the population.

Then under the assumption that each value, $I(c_n \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right)$, is identically and independently distributed across the population, then the result follows:

$$\sqrt{N} (M^\alpha(X; k, Z) - \mu^\alpha(X; k, Z)) \xrightarrow{d} N(0, \sigma_\alpha^2), \quad (7)$$

where $\mu^\alpha(X; k, Z) \equiv E \left[I(c_n \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right) \right]$

and $\sigma_\alpha^2 \equiv E \left[I(c_n \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right) - \mu^\alpha(X; k, Z) \right]^2$. The empirical counterparts are the following:

$$\widehat{\sigma}_\alpha^2 = \frac{1}{N} \sum_{n=1}^N \left[I(c_n \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right) - M^\alpha(X; k, Z) \right]^2 \quad (8)$$

Therefore the standard error of $M^\alpha(X; k, Z)$ is:

$$SE(M^\alpha(X; k, Z)) = \sqrt{\frac{\widehat{\sigma_\alpha^2}}{N}} \quad (9)$$

In the case of $H(X; k, Z)$ a similar reasoning applies,

i.e. $\sqrt{N}(H(X; k, Z) - \eta(X; k, Z)) \xrightarrow{d} N(0, \sigma_H^2)$, where $\eta(X; k, Z) \equiv E[I(c_n \geq k)]$. So its standard error is:

$$SE(H(X; k, Z)) = \sqrt{\frac{\widehat{\sigma_H^2}}{N}}, \quad (10)$$

where:

$$\widehat{\sigma_H^2} = \frac{1}{N} \sum_{n=1}^N [I(c_n \geq k) - H(X; k, Z)]^2 = H(X; k, Z) [1 - H(X; k, Z)] \quad (11)$$

The case of $A(X; k, Z)$ is less straightforward because: $A(X; k, Z) = \frac{M^0(X; k, Z)}{H(X; k, Z)}$. Hence A does not have an exact standard error, but an asymptotic one. Deriving it require a first-order Taylor expansion of $A(X; k, Z)$ around $\frac{\mu^0(X; k, Z)}{\eta(X; k, Z)}$:

$$A(X; k, Z) - \frac{\mu^0(X; k, Z)}{\eta(X; k, Z)} \cong \frac{1}{H(X; k, Z)} (M^0(X; k, Z) - \mu^0(X; k, Z)) - \frac{M^0(X; k, Z)}{[H(X; k, Z)]^2} (H(X; k, Z) - \eta(X; k, Z)) \quad (12)$$

From (7) and its equivalent for $H(X; k, Z)$, and from (12), it can be deduced that:

$$\sqrt{N} \left(A(X; k, Z) - \frac{\mu^0(X; k, Z)}{\eta(X; k, Z)} \right) \xrightarrow{d} N(0, \sigma_A^2), \quad (13)$$

where σ_A^2 is the asymptotic variance of the left-hand side of (13):

$$\sigma_A^2 = \frac{1}{[H(X; k, Z)]^2} \sigma_\alpha^2 + \left(\frac{M^0(X; k, Z)}{[H(X; k, Z)]^2} \right)^2 \sigma_H^2 - 2 \frac{M^0(X; k, Z)}{[H(X; k, Z)]^3} \sigma_{\alpha, H}, \quad (14)$$

where $\sigma_{\alpha, H}$ is the covariance of $I(c_n \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right)$ and $I(c_n \geq k)$. To estimate the empirical counterpart of σ_A^2 , $\widehat{\sigma_A^2}$, the empirical counterparts for σ_α^2 and σ_H^2 , and the covariance, $\sigma_{\alpha, H}$, are needed. The first two are in (8) and (11), respectively. The covariance's it is:

$$\begin{aligned} \widehat{\sigma_{\alpha, H}} &= \frac{1}{N} \sum_{n=1}^N [I(c_n \geq k) - H(X; k, Z)] \left[I(c_n \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right) - M^\alpha(X; k, Z) \right] \\ &= \frac{1}{N} \sum_{n=1}^N \left[I(c_n \geq k)^2 \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right) \right] - H(X; k, Z) M^\alpha(X; k, Z) \end{aligned} \quad (15)$$

And finally the asymptotic standard error of A is:

$$SE(A(X; k, Z)) = \sqrt{\frac{\widehat{\sigma_A^2}}{N}}. \quad (16)$$

2.1 Calculating percentage changes using cross-sectional data

Defining the percentage change of any of the above statistics requires indexing the matrix of attainments by a time period, e.g. t or $t - a$. In the case of cross-sectional data, the population sizes in each different period also need to be indexed accordingly. Then, for instance, the percentage change of any member of the Alkire-Foster family is:

$$\Delta\%M_a^\alpha(X^t, X^{t-a}; k, Z) \equiv \frac{M^\alpha(X^t; k, Z) - M^\alpha(X^{t-a}; k, Z)}{M^\alpha(X^{t-a}; k, Z)} = \frac{M^\alpha(X^t; k, Z)}{M^\alpha(X^{t-a}; k, Z)} - 1 \quad (17)$$

Since $\Delta\%M_a^\alpha$ is also made of the ratio of two averages, its standard error is also asymptotic and is derived like A 's. although in the case of cross-sectional data, $M^\alpha(X^t; k, Z)$ and $M^\alpha(X^{t-a}; k, Z)$ are independent.

In the case of $\Delta\%M_a^\alpha(X^t, X^{t-a}; k, Z)$, the required first-order Taylor expansion is:

$$\begin{aligned} \frac{M^\alpha(X^t; k, Z)}{M^\alpha(X^{t-a}; k, Z)} - \frac{\mu^\alpha(X^t; k, Z)}{\mu^\alpha(X^{t-a}; k, Z)} &\cong \frac{1}{M^\alpha(X^{t-a}; k, Z)} (M^\alpha(X^t; k, Z) - \mu^\alpha(X^t; k, Z)) \\ &\quad - \frac{M^\alpha(X^t; k, Z)}{[M^\alpha(X^{t-a}; k, Z)]^2} (M^\alpha(X^{t-a}; k, Z) - \mu^\alpha(X^{t-a}; k, Z)) \end{aligned} \quad (18)$$

Resorting to the same reasoning used to derive the asymptotic standard error of A , i.e. considering the equivalent of expressions (12) through (16) applied to (18) yield the following results:

$$\sqrt{\frac{N^t N^{t-a}}{N^t + N^{t-a}}} \left(\frac{M^\alpha(X^t; k, Z)}{M^\alpha(X^{t-a}; k, Z)} - \frac{\mu^\alpha(X^t; k, Z)}{\mu^\alpha(X^{t-a}; k, Z)} \right) \xrightarrow{d} N(0, \sigma_{\Delta\%M}^2), \quad (19)$$

where $\sigma_{\Delta\%M}^2$ is the asymptotic variance of the left-hand side of (19):

$$\sigma_{\Delta\%M}^2 = \frac{1}{[M^\alpha(X^{t-a}; k, Z)]^2} \sigma_{\alpha(t)}^2 \lambda + \left(\frac{M^\alpha(X^t; k, Z)}{[M^\alpha(X^{t-a}; k, Z)]^2} \right)^2 \sigma_{\alpha(t-a)}^2 [1 - \lambda] \quad (20)$$

where $\sigma_{\alpha(t)}^2$ is the variance of $I(c_n \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}}{z_d} \right]_+^\alpha \right)$ in period t , $\sigma_{\alpha(t-a)}^2$ is the respective variance for period $t-a$; and $\lambda = \frac{N^{t-a}}{N^t + N^{t-a}}$. Notice that the covariance element is absent from the right-side of (20). That is the case because $M^\alpha(X^t; k, Z)$ and $M^\alpha(X^{t-a}; k, Z)$ are independent.

And finally the asymptotic standard error of $\Delta\%M_a^\alpha$ is:

$$SE(\Delta\%M_a^\alpha(X^t, X^{t-a}; k, Z)) = \sqrt{\widehat{\sigma_{\Delta\%M}^2} \left(\frac{1}{N^t} + \frac{1}{N^{t-a}} \right)}. \quad (21)$$

2.2 Calculating percentage changes using panel data

With panel data the matrices of attainments for the two periods stem from the same observations. Therefore, unlike the case of cross-sectional data, $M^\alpha(X^t; k, Z)$ and $M^\alpha(X^{t-a}; k, Z)$ are no longer independent. The same first-order Taylor expansion as in (18) is useful in this context. But now the results are the following:

$$\sqrt{N} \left(\frac{M^\alpha(X^t; k, Z)}{M^\alpha(X^{t-a}; k, Z)} - \frac{\mu^\alpha(X^t; k, Z)}{\mu^\alpha(X^{t-a}; k, Z)} \right) \xrightarrow{d} N(0, \sigma_{\Delta\%M}^2), \quad (22)$$

where now the asymptotic variance, $\sigma_{\Delta\%M}^2$, is similar to that of expression (14):

$$\sigma_{\Delta\%M}^2 = \frac{1}{[M^\alpha(X^{t-a}; k, Z)]^2} \sigma_{\alpha(t)}^2 + \left(\frac{M^\alpha(X^t; k, Z)}{[M^\alpha(X^{t-a}; k, Z)]^2} \right)^2 \sigma_{\alpha(t-a)}^2 - 2 \frac{M^\alpha(X^t; k, Z)}{[M^\alpha(X^{t-a}; k, Z)]^2} \sigma_{\alpha(t), \alpha(t-a)}. \quad (23)$$

The covariance between the two measures in different periods, $\sigma_{\alpha(t), \alpha(t-a)}$, is:

$$\begin{aligned} \widehat{\sigma_{\alpha(t), \alpha(t-a)}} &= \frac{1}{N} \sum_{n=1}^N \left[I(c_n^t \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}^t}{z_d} \right]_+^\alpha \right) - M^\alpha(X^t; k, Z) \right] \\ &\quad \left[I(c_n^{t-a} \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}^{t-a}}{z_d} \right]_+^\alpha \right) - M^\alpha(X^{t-a}; k, Z) \right] \\ &= \frac{1}{N} \sum_{n=1}^N \left[I(c_n^t \geq k) I(c_n^{t-a} \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}^t}{z_d} \right]_+^\alpha \right) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{nd}^{t-a}}{z_d} \right]_+^\alpha \right) \right] \\ &\quad - M^\alpha(X^t; k, Z) M^\alpha(X^{t-a}; k, Z) \end{aligned} \quad (24)$$

Finally the asymptotic standard error of $\Delta\%M_a^\alpha$ for the panel-data case is:

$$SE(\Delta\%M_a^\alpha(X^t, X^{t-a}; k, Z)) = \sqrt{\frac{\widehat{\sigma_{\Delta\%M}^2}}{N}}. \quad (25)$$

3 The case of a two-stage, stratified household survey (following Deaton (1997))

This section derives the standard errors for the main statistics of the Alkire-Foster family, combining the above results with the formulas provided by Deaton (1997) in order to estimate means and variances of means from two-stage, stratified samples.

3.1 Basic notation

The data are assumed to come from a stratified sample. Following the notation by Deaton (1997), there are S strata each subindexed by s . Within each stratum households are drawn

in two stages. In the first stage n_s clusters are drawn in each stratum separately. Then in the second stage m_c households are drawn in every cluster, each indexed by i . The respective Alkire-Foster statistics are indexed accordingly. For instance, M_{sc}^0 is the adjusted headcount ratio of cluster c from stratum s . The data also come accompanied by weights, w , each subindexed as corresponds (not to be confused with dimension weights, w_d). These weights are inverse to the probability of being sampled into the dataset.

The multidimensional headcount is now the following:

$$H(X; k, Z) \equiv \frac{1}{\widehat{N}} \sum_{s=1}^S \sum_{c=1}^{n_s} \left(\sum_{i=1}^{m_c} w_{ics} \left[\sum_{d=1}^D w_d g_{id}(k) \right]^0 \right) = \frac{1}{\widehat{N}} \sum_{s=1}^S \sum_{c=1}^{n_s} \left(\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) \right) \quad (26)$$

where:

$$\widehat{N} = \sum_{s=1}^S \sum_{c=1}^{n_s} \sum_{i=1}^{m_c} w_{ics}$$

The average number of deprivations of the multidimensionally poor is:

$$A(X; k, Z) \equiv \frac{1}{D\widehat{N}H} \sum_{s=1}^S \sum_{c=1}^{n_s} \left(\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) c_i \right) \quad (27)$$

The adjusted headcount ratio is:

$$M^0(X; k, Z) \equiv H(X; k, Z) A(X; k, Z) = \frac{1}{D\widehat{N}} \sum_{s=1}^S \sum_{c=1}^{n_s} \left(\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) c_i \right) \quad (28)$$

And the Alkire-Foster family is now defined as follows:

$$\mathcal{AF} : \{M^\alpha(X; k, Z) \equiv \frac{\sum_{s=1}^S \sum_{c=1}^{n_s} \sum_{i=1}^{m_c} w_{ics} \left[\sum_{d=1}^D w_d g_{id}^\alpha \right]}{D\widehat{N}} \forall \alpha \in \mathbb{N}\}. \quad (29)$$

An alternative definition of $M^\alpha(X; k, Z)$, for the purposes of deriving standard errors is the following:

$$M^\alpha(X; k, Z) \equiv \frac{1}{\widehat{N}} \sum_{s=1}^S \sum_{c=1}^{n_s} \left[\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) \left(\sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{id}}{z_d} \right]_+^\alpha \right) \right] \forall \alpha \in \mathbb{N}, \quad (30)$$

3.2 Standard errors

The variance of $M^\alpha(X; k, Z)$ can be computed using Deaton's formula (1.63) (Deaton, 1997, p. 56). The formula is:

$$\widehat{var}_\alpha = \frac{1}{\widehat{N}^2} \sum_{s=1}^S \sum_{c=1}^{n_s} \left[\left(\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) \sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{id}}{z_d} \right]_+^\alpha - M_s^\alpha \right) - M^\alpha (w_{cs} - \bar{w}_s) \right]^2, \quad (31)$$

$$\text{where } M_s^\alpha = \frac{\sum_{c=1}^{n_s} \sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) \sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{id}}{z_d} \right]_+^\alpha}{\sum_{c=1}^{n_s} \sum_{i=1}^{m_c} w_{ics}}, w_{cs} = \sum_{i=1}^{m_c} w_{ics}, \text{ and } \bar{w}_s = \frac{1}{n_s} \sum_{c=1}^{n_s} w_{cs}.$$

Then the standard error of $M^\alpha(X; k, Z)$ is: $SE(M^\alpha(X; k, Z)) = \sqrt{\widehat{var}_\alpha}$.

In the case of $H(X; k, Z)$, its standard error is: $SE(H(X; k, Z)) = \sqrt{\widehat{var}_H}$ where:

$$var_H = \frac{1}{\widehat{N}^2} \sum_{s=1}^S \sum_{c=1}^{n_s} \left[\left(\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) - H_s^\alpha \right) - H(w_{cs} - \bar{w}_s) \right]^2, \quad (32)$$

and $H_s^\alpha = \frac{\sum_{c=1}^{n_s} \sum_{i=1}^{m_c} w_{ics} I(c_i \geq k)}{\sum_{c=1}^{n_s} \sum_{i=1}^{m_c} w_{ics}}$. In the case of $A(X; k, Z)$, the asymptotic standard error is based on the approximation (12). The asymptotic variance is:

$$\widehat{var}_A = \frac{1}{[H(X; k, Z)]^2} \widehat{var}_\alpha + \left(\frac{M^0(X; k, Z)}{[H(X; k, Z)]^2} \right)^2 \widehat{var}_H - 2 \frac{M^0(X; k, Z)}{[H(X; k, Z)]^3} \widehat{covar}_{\alpha, H}, \quad (33)$$

where $\widehat{covar}_{\alpha, H}$ is the empirical counterpart of the covariance of $\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) \sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{id}}{z_d} \right]_+^\alpha$ and $\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k)$:

$$\widehat{\sigma}_{\alpha, H} = \frac{1}{\widehat{N}^2} \sum_{s=1}^S \sum_{c=1}^{n_s} \left[\left(\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) \sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{id}}{z_d} \right]_+^\alpha - M_s^\alpha \right) - M^\alpha (w_{cs} - \bar{w}_s) \right] \left[\left(\sum_{i=1}^{m_c} w_{ics} I(c_i \geq k) - H_s^\alpha \right) - H^\alpha (w_{cs} - \bar{w}_s) \right] \quad (34)$$

And finally the asymptotic standard error of A is:

$$SE(A(X; k, Z)) = \sqrt{\widehat{\sigma}_A^2}. \quad (35)$$

3.3 Calculating percentage changes using cross-sectional data

The asymptotic variance of $\Delta\%M_a^\alpha(X^t, X^{t-a}; k, Z)$ is also based on (18):

$$\widehat{var}_{\Delta\%M^\alpha} = \frac{1}{[M^\alpha(X^{t-a}; k, Z)]^2} \widehat{var}_{\alpha(t)} + \left(\frac{M^\alpha(X^t; k, Z)}{[M^\alpha(X^{t-a}; k, Z)]^2} \right)^2 \widehat{var}_{\alpha(t-a)} \quad (36)$$

where $\widehat{var}_{\alpha(t)}$ is the variance \widehat{var}_α in period t , $\sigma_{\alpha(t-a)}^2$, $\widehat{var}_{\alpha(t-a)}$ is the respective variance for period $t - a$. The asymptotic standard error of $\Delta\%M_a^\alpha$ is:

$$SE(\Delta\%M_a^\alpha(X^t, X^{t-a}; k, Z)) = \sqrt{\widehat{var}_{\Delta\%M^\alpha}}. \quad (37)$$

3.4 Calculating percentage changes using panel data

The asymptotic variance, $\widehat{var}_{\Delta\%M^\alpha}$, is now:

$$\widehat{var}_{\Delta\%M^\alpha} = \frac{1}{[M^\alpha(X^{t-a}; k, Z)]^2} \widehat{var}_{\alpha(t)} + \left(\frac{M^\alpha(X^t; k, Z)}{[M^\alpha(X^{t-a}; k, Z)]^2} \right)^2 \widehat{var}_{\alpha(t-a)} - 2 \frac{M^\alpha(X^t; k, Z)}{[M^\alpha(X^{t-a}; k, Z)]^2} \sigma_{\alpha(t), \alpha(t-a)}. \quad (38)$$

The covariance between the two measures in different periods, $\sigma_{\alpha(t), \alpha(t-a)}$, is:

$$\sigma_{\alpha(t), \alpha(t-a)} = \frac{1}{\widehat{N}^2} \sum_{s=1}^S \sum_{c=1}^{n_s} \left[\begin{aligned} & \left(\sum_{i=1}^{m_c} w_{ics}^t I(c_i^t \geq k) \sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{id}^t}{z_d} \right]_+^\alpha - M_s^\alpha(X^t; k, Z) \right) \\ & - M^\alpha(X^t; k, Z) (w_{cs}^t - \overline{w_s^t}) \end{aligned} \right] \quad (39)$$

$$\left[\begin{aligned} & \left(\sum_{i=1}^{m_c} w_{ics}^{t-a} I(c_i^{t-a} \geq k) \sum_{d=1}^D \frac{w_d}{D} \left[\frac{z_d - x_{id}^{t-a}}{z_d} \right]_+^\alpha - M_s^\alpha(X^{t-a}; k, Z) \right) \\ & - M^\alpha(X^{t-a}; k, Z) (w_{cs}^{t-a} - \overline{w_s^{t-a}}) \end{aligned} \right]$$

Finally the asymptotic standard error of $\Delta\%M_a^\alpha$ for the panel-data case is:

$$SE(\Delta\%M_a^\alpha(X^t, X^{t-a}; k, Z)) = \sqrt{\widehat{var}_{\Delta\%M^\alpha}}. \quad (40)$$

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