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Memo to: Coneval
Memo from: Sabina Alkire and James Foster
Date: 30 May 2009
Oxford

We understand that the Committee may be contemplating a methodology for multidimensional poverty measurement based on the general approach we proposed in a recent paper (Alkire and Foster, 2007). While there is some broad agreement on the aggregation method – in particular, the adjusted headcount M_0 is a natural pick since it satisfies dimensional monotonicity and can be decomposed across dimensions (unlike the multidimensional headcount *H*) and can be applied to ordinal data[1] – there may be less agreement on how to proceed with the identification step. We have recently revisited this topic and would like to take this opportunity to share our thoughts with you. This memo presents a concrete and intuitive proposal for your consideration.

Background

The identification step has three components. First, the setting of the dimension specific cutoffs $z = (z_1,\ldots,z_d)$; second, the setting of the dimension specific weights $w = (w_1,\ldots,w_d)$, which indicate the relative importance of deprivations in determining whether a person is poor; third, the setting of a cross dimensional cutoff *k*, to separate the multidimensionally poor from those who are nonpoor. As has been customary throughout the discussions, we will take the vector *z* as given. This presumes that Coneval undertakes the appropriate studies to determine what constitutes being truly deprived in each dimension.[2] Given this assumption, we present a new approach to solving the identification problem and setting the parameters *w* and *k*.

In previous meetings we have discussed the following two methods for fixing *w* and *k*:

1. **Participatory Normative:** Through public debate and deliberation (Sen 1996) one might be able to select reasonable initial values for these parameters. The process may be iterative, with additional analysis and discussion after the first implementation to settle on final values. Additional robustness checks can ensure that results obtained are relatively stable to variations around the chosen values. **Pros:** Explicitly involves public debate to make informed value judgements. **Cons:** No theoretical structure; the process may be costly; it is difficult to ensure that the public actually consulted is representative; if repeated each year this will jeopardize comparability, but if not it imposes previous years' values.

2. **Statistically Empirical:** One might select a cutoff and perhaps weights based on data-driven techniques, of which the Bristol method is an example.

[1] If some data are cardinal the adjusted poverty gap M_1 or FGT measure M_2 can also be used.
[2] Indeed, our main axiomatic approach depends crucially on this assumption, since if the cutoffs are too high – as would be the case if the cutoffs were aspiration levels rather than truly minimum levels for functioning in the dimension – a deprivation could *not* be interpreted as a violation of human rights. It is this interpretation that justifies our 'social deprivation' axiom.

Pros: Makes use of information already present in the dataset; arrives at parameter levels from data and appears to be more ‘scientific’.
Cons: Difficult to defend from a normative point of view: one cannot derive an ‘ought’ from an ‘is’; may deliver values that are unreasonable (by generally accepted normative criteria) or politically indefensible; has difficulties with variation over time: (i) if the derived parameters vary annually, this jeopardizes comparability, (ii) if cutoffs are re-used in future years or are based partially on older data, this seems arbitrary and inconsistent; has difficulties with transparency: (i) the process by which parameters are derived is not easily understood by the nonspecialist and this impedes serious evaluation of the process and public understanding of identification of the poor; (ii) there are actually many empirically driven techniques that generate different cutoffs and yet little discussion of overarching criteria for selecting among the available techniques.

Upon reflection, we have realized that there is an additional normative approach to solving the identification step – one that has been indirectly touched upon in the meetings but has never been explicitly stated.

3. **Axiomatic:** One might propose a series of axiomatic principles that embody underlying value judgements concerning identification. This in turn might narrow down the possible range identification methods, or even select a unique one.
Pros: General principles can be much clearer and more transparent than the above alternatives; moreover, they can be easily communicated to policymakers and are explicitly normative.
Cons: It may be difficult to obtain agreement on the basic principles; a given set of axioms may not lead to a unique identification method.

It is this final approach that we explore in this memo.

Notation

Before proceeding, let us recall the notation pertinent to our methodology for measuring multidimensional poverty. (A more extensive guide is given below in Appendix 1).

d	number of dimensions
n	number of persons
i	a typical individual
j	a typical dimension
$j = 1$	the income dimension
$j \neq 1$	a social dimension j = 2,…,d
y_i	the d-dimensional achievement vector for person i
y	the n by d matrix of achievements across all persons
g_i^0	the d-dimensional vector of person i’s deprivations obtained from the achievement vector and the dimension specific cutoffs; $g_i^0 = (1,1,0,\ldots)$ means that i is deprived in the first two dimensions, not deprived in the third, etc.
w	a d-dimensional vector of weights summing to d; for example $w = (1,\ldots,1)$ represents equal weights
w_j	the weight on dimension j

$\overline{w}$	the nested weighting vector which gives half of the weight to income and half of the weight to social dimensions, and then splits the weight on social dimensions equally among the individual dimensions, i.e., $\overline{w} = (\frac{d}{2}, \frac{d}{2(d-1)}, \ldots, \frac{d}{2(d-1)})$
c_i	person i's weighted deprivation count, found by taking the inner product $c_i = wg_i^0$; for example, if the person is only deprived in the first two dimensions, then $c_i = w_1 + w_2$
c	the n-dimensional vector of weighted deprivation counts, one for each person
k	the cross-dimensional cutoff, satisfying $0 < k \leq d$
$\overline{k}$	the specific cross-dimensional cutoff that is halfway between 0 and d, i.e., $\overline{k} = \frac{d}{2}$
$\rho_{wk}(y_i)$	the identification function associated with w and k: if i's weighted deprivation count c_i exceeds k, then $\rho_{wk}(y_i) = 1$ and person i is identified as being poor; otherwise, $\rho_{wk}(y_i) = 0$ and i is not poor.[3]
$H(y)$	the multidimensional headcount ratio
$M_\alpha(y)$	the dimension adjusted FGT family: for example M_0 is the dimension adjusted headcount, M_1 is the dimension adjusted per capita poverty gap, and M_2 is the dimension adjusted FGT measure
$\mathcal{M}$	the overall measurement methodology: $\mathcal{M} = (\rho_{wk}, M_\alpha)$

Our Proposal

The goal is to apply the axiomatic method to guide the selection of a specific identification method from the set of all dual cutoff identification methods (Alkire and Foster, 2007). This reduces to the selection of the dimensional weights w and the cross-dimensional cutoff k used in constructing the 'identification function' ρ_{wk}, which is a function of the individual achievement vector y_i and maps to value 1 when the person is poor and to value 0 when the person is not poor.[4] It proves more convenient to identify a person as poor when ci > k (rather than ci $\geq$ k). This 'strict' definition allows us to select more intuitive values for parameters $\overline{w}$ and $\overline{k}$, but is otherwise equivalent in generality to the 'weak' definition.

Our axiomatic approach to identification draws heavily upon discussions from the meeting of October 2008 in CIDE, and other related conversations. Following the lead of these discussions, we are considering multidimensional poverty as having two components: economic deprivation and social deprivation. We differentiate between the two types of variables and adopt the convention that dimension 1 is 'income' or 'economic attainment' while dimensions 2 through d are 'social attainments'.

Our first principles define when a person is economically deprived and socially deprived.

> ***Economic Deprivation* (*ED*)**: A person is economically deprived if the person's income falls below the income cutoff.

[3] Note that here we are taking k to be a strict cutoff whereas in Alkire and Foster it was defined weakly, such that a person who had k or more dimensions was identified as deprived.
[4] Note that we are explicitly considering the vector of weights and so the identification function is denoted as ρ_{wk} rather than ρ_k given in Alkire and Foster (2007).

> ***Social Deprivation* (*SD*)*:*** A person is socially deprived if *any* social achievement falls below its respective cutoff.

The first principle adopts the standard interpretation of economic deprivation as a shortfall in the space of income (consumption, wealth), where income acts as a fungible, general means to other valuable ends. The second principle focuses on the social dimensions such as those pertaining to health, education, and social protection, and is based on the assumption that each social dimension is intrinsically important, and that an attainment below the respective cutoff represents a denial of a basic human right. Consequently, socially deprivation is identified using a union approach: A person is socially deprived if the person falls below the minimum cutoff in *any* of the d - 1 social dimensions.[5]

The next principle links the intuitive notions of economic deprivation and social deprivation to multidimensional poverty.

> ***Identification* (*I*)*:*** A person is multidimensionally poor if and only if the person is both economically deprived and socially deprived.

This axiom follows Committee discussions in taking an intersection approach to identification with respect to economic deprivation and social deprivation. In other words, the identification method defines as poor all persons in the intersection of the economically deprived group and the socially deprived group. Any person who has sufficient economic resources is not considered to be multidimensionally poor, even if the person happens to be socially deprived. Alternatively, a person who is only economically deprived, but has no evidence of being socially deprived, is not considered to be multidimensionally poor.

Our first observation is that these three basic axioms, by themselves, lead to a unique functional form for the identification function, namely, the form $\rho_{\overline{w}\overline{k}}(y_i)$ associated with $\overline{w} = (\frac{d}{2}, \frac{d}{2(d-1)}, \ldots, \frac{d}{2(d-1)})$ and $\overline{k} = \frac{d}{2}$. (The proof is given in Appendix 2).

> ***Theorem* 1** Suppose that the identification function $\rho_{wk}(y_i)$ satisfies axioms *ED*, *SD*, and *I*. Then $\rho_{wk}(y_i) = \rho_{\overline{w}\overline{k}}(y_i)$ for all y_i.

By Theorem 1, every identification function satisfying the three axioms must take the same values as the identification function with parameters $\overline{w}$ and $\overline{k}$, and hence the underlying identification method is unique. This is a remarkable result linking basic, comprehensible principles to a specific functional form for indentifying the multidimensionally poor.

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[5] Of course, the relevance of this principle rests on whether the variables in question are in fact important social dimensions, and whether the dimension specific cutoffs identify true deprivations. The final section of this note explores modifications of *SD* if the data are less reliable and focused.

Notice that while $\overline{w}$ and $\overline{k}$ are not the only parameter values consistent with the three axioms, all acceptable values lead to an *identical* identification function (which identifies who is poor in society). And with a fixed identification function, the number of the poor people associated with a given attainment matrix y must also be determinate. Consequently, the headcount ratio H takes on the same value across all acceptable values of w and k. So long as the number of poor people stay the same, the relative importance of the different dimensions (as indicated by w) has no effect on it evaluation of poverty. In contrast, the adjusted FGT indices are all sensitive to w and their values will (appropriately) vary as w emphasizes one or the other dimensions. We offer the following two axioms which effectively fix the weights used in identifying the poor.

> ***Balance* (*B*)***:* The weight on economic deprivation should be no greater than the aggregate weight on social deprivations; the aggregate weight on social deprivations should not exceed the weight on economic deprivation.
> ***Equal Rights* (*ER*)***:* No social dimension should receive greater weight than any other social dimension.

We have the following result. (The proof is given in Appendix 2.)

> ***Theorem* 2** Suppose that the identification function $\rho_{wk}(y_i)$ satisfies axioms *ED*, *SD*, *I*, *B*, and *ER*. Then $w = \overline{w}$ and $\overline{k} \leq k < \overline{k} + \overline{w}_2$.

In words, the additional axioms imply that the weights must be the ones given in $\overline{w}$, which first split evenly between economic and social variables, and then split evenly between social variables. This, in turn, places a restriction on how large k can be and remain consistent with the three original axioms: the cutoff k must be at least $\frac{d}{2}$ and fall below $\frac{d}{2} + \frac{d}{2(d-1)}$. Note, though, that this variation in k does *not* affect the level of multidimensional poverty as measured by any adjusted FGT measure. In particular, once the five axioms are agreed to, the adjusted headcount ratio can be unambiguously obtained, even allowing for the allowable variation in cutoff k. Therefore, there is no loss at all in selecting $k = \overline{k}$. In this sense, then, the axioms deliver a unique identification function that, along with the choice of the adjusted headcount measure as the aggregation function, yields a well-defined and justifiable methodology ($\rho_{\overline{w}\overline{k}}, M_0$) for measuring multidimensional poverty. (See Appendix 1 for a Guide to this methodology.)

Possible Modifications of the Social Deprivation Axiom

Our previous discussion was predicated on assumptions that the data on deprivations in individual social dimensions a) were proxies for *human rights* and b) identified truly *deprived* persons and c) were reasonably *accurate*, without high measurement errors.

If the data do not meet those three criteria, then the union approach used within *SD* is not advisable. The discussions noted that a union approach (where if a person is deprived in *any* single social dimension they are socially deprived) may well identify too many people as socially deprived. To address this, our proposal might be altered in three ways. One relates to the *dimension-specific thresholds* which identify deprivation in each dimension. The second relates to the proposal of equal *weights* between deprivations and the final, to the *number* of social deprivations required in *SD*. The potential alterations are as follows:

1. Use more discriminating dimension-specific thresholds on social dimensions. There are three cutoffs in income space: extreme poverty, capability poverty, and patrimony. It may be useful to adopt a lower vector of thresholds for social deprivations that would sharply reduce the probability that a person having one social deprivation is actually *not* socially deprived. This alternative is preferred in that the methods proposed in this paper might then be retained.

2. Apply dimension-specific weights that represent the probability that someone deprived in that social attainment is actually deprived. If the quality of data varies by dimension, one could select weights such that those dimensions for which data are not a reliable predictor of deprivation receive a lighter weight, and other dimensions receive greater weight. If k were set across a weighted sum, it might be set such that identification involved deprivations in one *or more* social dimensions, depending upon their weights. For example, a household that was deprived in health insurance alone would not be considered socially deprived, whereas if it were deprived in health insurance and any other dimension, it would be. However if food had a higher weight, then a household were deprived in food alone might be considered socially deprived. This option may be challenging to communicate.

3. Alter the social deprivation principle to require two or more social deprivations rather than one. The *SD* principle could be changed to read: a person is socially deprived if *at least x* social achievements fall below their respective cutoffs. By requiring a person to be deprived in at least two social achievements, for example, we allow the possibility that any single deprivation may be due to a measurement error. Concretely, identification would be inaccurate if all households that enjoy private health care are considered to be health deprived, or if households having a very elderly member without an 8th grade education were considered education deprived. Requiring two dimensions maintains simplicity while decreasing over-coverage by introducing more rigorous identification conditions. It is also easier to communicate than option 2 above. However it may increase errors of omission because a household deprived in only one dimension would never be identified as poor.

Appendix 1: Updated Guide to Measuring Multidimensional Poverty

The accompanying is brief synthesis of the multidimensional poverty methodology suggested in Foster (2008) and Alkire and Foster (2007), updated to reflect subsequent discussions.

Introduction

Poverty measurement can be broken down conceptually into two distinct steps: first, the identification step, which defines the criteria for distinguishing poor persons from the non-poor, and second, the aggregation step, by which data on poor persons are brought together into an overall indicator of poverty (Sen, 1976).

I. Identification: The Dual Cutoff Approach

Consider a matrix y of achievements in d dimensions for n persons. The vector z gives the deprivation cutoffs in each dimension; a person is deprived in a given dimension if the achievement is less than or equal to the respective cutoff. Whether the data are cardinal or ordinal, we can construct the matrix g^0 by replacing all non-deprived entries with zero, and all deprived levels of achievement with a one.

If dimensions are weighted differentially, we consider w to be a d dimensional row vector of positive numbers summing to d, whose j^{th} coordinate w_j is the weight associated with dimension j such that $w_1 + \ldots + w_d = d$.

In the case of 7 dimensions used in our proposal, when j =1 the dimension is income and obtains 50% of the overall weight; and the other social deprivations are equally weighted. Thus the specific weighting vector is $\overline{w} = (\frac{d}{2}, \frac{d}{2(d-1)} \cdots \frac{d}{2(d-1)})$. In numbers, the weights are 3.5 on income, and $\frac{3.5}{6}$ for the others, or $\overline{w}$ = (3.50, 0.58, … 0.58).

We apply w_d to the g^0 matrix. From the rows g_i^0 of the weighted deprivation matrix g^0, we construct the vector c of (weighted) deprivation counts, whose i^{th} entry $c_i = w g_i^0$ is the (weighted) sum of the dimensions in which i is deprived.

Next, we fix a dimensional cutoff k such that k takes a value between 0 and d. If the weighted deprivation count $c_i > k$, then person i is identified as being poor; otherwise, i is not poor.[6] In our proposal, we have a cutoff halfway between the limits, and so $\overline{k}$ = 3.5.

What is the justification for the identification strategy $\rho_{\overline{w}\overline{k}}(y_i)$? Consider the following axioms:

> *Economic Deprivation* (*ED*)*:* A person is economically deprived if the person's income falls below the income cutoff.
> *Social Deprivation* (*SD*)*:* A person is socially deprived if *any* social achievement falls below its respective cutoff.

[6] Note that here we are taking k to be a strict cutoff whereas in Alkire and Foster it was defined weakly, such that a person who had k or more dimensions was identified as deprived.

Identification (*I*): A person is multidimensionally poor if and only if that person is both economically and socially deprived.

It can be shown that satisfying the three axioms leads to a unique identification function $\rho_{\bar{w}\bar{k}}(y_i)$. There are indeed a remaining range of possible values for weights and k that respect these axioms, but we note that all would identify the same set of poor people. Out of these choices we chose the most intuitive and simple value, which was 50% of the total possible value for c_i. Applying this cutoff allows us to identify whether each person is poor or not. This completes identification.

II. Measurement

Having fixed the method of identification, we now censor the data of the non-poor, or replace their entries with zeros, to obtain the censored matrix $g^0(k)$ and a censored vector $c(k)$.

We define the **headcount ratio** H to be the percentage of persons who are multidimensionally poor. In other words, $H = q/n$ where q is the number of poor. The headcount is a useful and intuitive measure, and can be used to compare the overall multidimensional poverty levels for different regions, ethnic groups, rural/urban locations, kinds of households, and so on. However the headcount has several weaknesses:

1) Headcount cannot be broken down by dimension, to reveal *how* the components of poverty differ for different groups – yet this information is very relevant to policy
2) Headcount does not increase if the average number of social deprivations increases. For example, if an area that used to have two deprivations on average now suffers five deprivations on average, headcount would remain unchanged.

To construct a measure that fulfils other desirable properties we propose using a further measure that is suited to ordinal data, which we call the adjusted headcount ratio.

The Adjusted Headcount Ratio

Let $A = \Sigma_i (c_i(k)/d)/q$ be the average deprivation share of the poor. The (dimension) **adjusted headcount ratio** M_0 can be defined as $\boldsymbol{M_0 = HA}$, or the headcount ratio times the average deprivation share. The measure can also be defined as $M_0 = \mu(g^0(k))$, or the mean of the matrix $g^0(k)$. In words, M_0 can be viewed as the total number of deprivations experienced by poor persons divided by the highest possible number of deprivations (or dn).

This measure satisfies symmetry, scale invariance, normalization, replication invariance, focus, weak monotonicity, and subgroup decomposability. It can be applied to ordinal data.[7]

[7] We are not at this time presenting the other two measures of this class, namely the Adjusted Poverty Gap $\boldsymbol{M_1}$, or the adjusted *FGT* measure $\boldsymbol{M_\alpha}$. These both require some cardinal data, but then reflect the depth of deprivation in each (cardinal) dimension. M_2 additionally signals the inequality among the poor.

Using the $M_{0,}$ one can graphically compare the different compositions of multidimensional poverty for different sub-groups of the population. Further, one can chart how the dimensional composition of poverty changes over time.

Appendix 2: Proofs

***Theorem* 1** Suppose that the identification function $\rho_{wk}(y_i)$ satisfies axioms *ED*, *SD*, and *I*. Then $\rho_{wk}(y_i) = \rho_{\bar{w}\bar{k}}(y_i)$ for all y_i.

Proof Let y_i be any given achievement vector for person *i*.
Case i: Suppose that *i* is not economically deprived in y_i. From *ED* we know that $y_{i1} \geq z_1$, and so $g^0_{i1} = 0$. Consequently, *i*'s weighted count given $\bar{w}$ is $\bar{c}_i = \bar{w}g^0_i \leq \bar{w}_2 + ... + \bar{w}_d = \frac{d}{2}$ and so $\rho_{\bar{w}\bar{k}}(y_i) = 0$. As ρ_{wk} satisfies *I*, and *i* is not economically deprived in y_i, it follows that $\rho_{wk}(y_i) = \rho_{\bar{w}\bar{k}}(y_i)$.
Case ii: Suppose that *i* is not socially deprived in y_i. From *SD* we know that each $y_{ij} \geq z_j$ and hence $g^0_{ij} = 0$ for $j = 2,\ldots, d$. Consequently, *i*'s weighted count given $\bar{w}$ is $\bar{c}_i = \bar{w}g^0_i \leq \bar{w}_1 = \frac{d}{2}$ and so $\rho_{\bar{w}\bar{k}}(y_i) = 0$. As ρ_{wk} satisfies *I*, and *i* is not socially deprived in y_i, it follows that $\rho_{wk}(y_i) = \rho_{\bar{w}\bar{k}}(y_i)$.
Case iii: Suppose that *i* is economically deprived and socially deprived in y_i. From *ED* we know that $y_{i1} < z_1$, and so $g^0_{i1} = 1$; from *SD* we know that some $y_{ij} < z_j$ and hence $g^0_{ij} = 1$ for some $j = 2,\ldots, d$. Consequently, *i*'s weighted count given $\bar{w}$ is $\bar{c}_i = \bar{w}g^0_i \geq \bar{w}_1 + \bar{w}_j > \frac{d}{2}$ and so $\rho_{\bar{w}\bar{k}}(y_i) = 1$. As ρ_{wk} satisfies *I*, and *i* is economically and socially deprived in y_i, it follows that $\rho_{wk}(y_i) = \rho_{\bar{w}\bar{k}}(y_i)$.
Clearly, in all three cases we obtain the desired result that $\rho_{wk}(y_i) = \rho_{\bar{w}\bar{k}}(y_i)$, which completes the proof.

***Theorem* 2** Suppose that the identification function $\rho_{wk}(y_i)$ satisfies axioms *ED*, *SD*, *I*, *B*, and *ER*. Then $w = \bar{w}$ and $\bar{k} \leq k < \bar{k} + \bar{w}_2$.

Proof It is immediate from *B* and *ER* that $w = \bar{w}$. Let y_i be any given achievement vector for person *i*. If $y_{i1} < z_1$ and $y_{ij} < z_j$ for all $j = 2,\ldots, d$, then $g^0_{i1} = 1$ and $g^0_{ij} = 0$ for all $j = 2,\ldots, d$. Consequently, *i*'s weighted count given $\bar{w}$ is $\bar{c}_i = \bar{w}g^0_i = \bar{w}_1 = \frac{d}{2} = \bar{k}$. From *ED*, *SD*, and *I* we know that $\rho_{\bar{w}k}(y_i) = 0$, and hence we must have $\bar{c}_i \leq k$ and hence $\bar{k} \leq k$. Alternatively, if $y_{i1} < z_1$, $y_{i2} < z_2$, and $y_{ij} < z_j$ for all $j = 3,\ldots, d$, then $g^0_{i1} = g^0_{i2} = 1$ and $g^0_{ij} = 0$ for all $j = 3,\ldots, d$. Consequently, *i*'s weighted count given $\bar{w}$ is $\bar{c}_i = \bar{w}g^0_i = \bar{w}_1 + \bar{w}_2 = \bar{k} + \bar{w}_2$. From *ED, SD,* and *I* we know that $\rho_{\bar{w}k}(y_i) = 1$, and hence we must have $\bar{c}_i > k$ and hence $\bar{k} + \bar{w}_2 > k$.