Mid-Term Exam Key

1. The monotonicity principle requires that the overall poverty goes down as a poor person's achievement decreases while other achievements remaining the same.

---False. If a poor person's achievements go down, monotonicity requires that poverty go up

2. The national multidimensional poverty measures complement the Global MPI, just like national income poverty estimates complement the international $1.25/day poverty measure

---True. While the international cutoff for income-based poverty us $1.25, countries have their own poverty lines for income poverty. Similarly, while the global MPI has a cut-off of 1/3 (k=33.33%), national MPIs may have their own cut-offs

3. In unidimensional poverty if two achievement vectors are x = (7, 4, 15, 12) and y = (6, 13, 20, 8), then for any poverty line, vector y has larger poverty than vector x.

---False. Let's take k=7. X has 2 poor people as per the cut-off and Y has only 1.

4. The unidimensional poverty gap ratio is sensitive to the inequality among the poor, but the squared poverty gap is not sensitive to inequality among the poor.

---False. Both M1 and M2 are sensitive to inequality among the poor. Both satisfy monotonicity in responding to changes in the depth of deprivations in all domains with cardinal data

5. The multidimensional headcount ratio (H) satisfies the property of dimensional monotonicity.

---False. If a multi-dimensionally 'poor' individual becomes deprived in an additional dimension, H would not change. A would change. A satisfies dimensional monotonicity

6. The average deprivation share (A) among the multi-dimensionally poor cannot be lower than the poverty cutoff (K)

---True. If your k=2, i.e. 20% of indicators, for any person to be multi-dimensionally poor, the person has to be deprived in at least 20% of the indicators. Thus, A will be at least 20% if not more.

7. Everything else unchanged, if a poor person becomes non-poor and there is no change in the population size, then the average deprivation share among the poor should go down.

--- Uncertain (or cannot specify).
Let's take a simple example. Consider the simple deprivation matrix and the associated count deprivation vector:

$$g^0 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ & & \end{bmatrix} \quad ci = \begin{bmatrix} 2 \\ 2 \\ 1 \\ \\ \end{bmatrix}$$

Let's say the k=2. Therefore,

H=2/3

A = $\left(\frac{2}{3} + \frac{2}{3} + \frac{0}{2} \right)$ = 4/6 = 2/3

Now let's say that the second person becomes non-poor by improving in indicator 2. The deprivation matrix and count deprivation vector become:

$$g^0 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ & & \end{bmatrix} \quad ci = \begin{bmatrix} 2 \\ 1 \\ 1 \\ \\ \end{bmatrix}$$

Now, H=1/3

A = $\left(\frac{2}{3} + \frac{0}{} + \frac{0}{1} \right)$ = 2/3

Therefore, A did not change even though one person became non-poor and the population remained the same.

8. The MPI is only one possible application of the Alkire-Foster method but other applications are also possible

---True. The MPI currently is M0. Other applications include M1, M2 and other indices like Women in Agriculture Empowerment Index are all options

9. The focus axiom in the measurement of unidimensional poverty requires that:

---c. By definition the focus axiom argues that "If y is obtained from x by an increment to a non-poor person's income and the poverty line remains unchanged, then poverty is unchanged: P(y;z) = P(x;z)".
Simply put, the focus axiom argues that the focus should be on the deprivations of the poor. You cannot make poverty declining by improving the state of the non-poor.

10. The aim of the Multidimensional Poverty Index (MPI) published in the Human Development Report is to:

---c. Replace the Human Poverty Index and complement monetary poverty measures. The MPI is a composite index of poverty spanning many dimensions. The idea is to forward it as a measure of human development that complements monetary measure.

11. If the average deprivation share among the poor (A) goes up by ten percent over time and the adjusted headcount ratio (M_0) does not change, then the multidimensional headcount ratio (H):

---c. Must have fallen by 9.09 percent. Need to do some algebra for this.
Mo=H*A, let's call this the $M0_t$ i.e. M0 in time 't'

If A goes up by 10%, then the new A is A+A/10= 11A/10

Since Mo has remained the same, we know H has changed, but we do not know by how much. Let the % change in H be x%, i.e. x/100. Therefore, the magnitude of change in H is: (x/100)*H.
The new 'H' is then H+ *∗ 100 = 100 + 100*
The 'new' $M0_{t+1}$, i.e. Mo in time t+1 is: (*100 + 100*)* *11 10*

Now, we are told that $M0_t = M0_{t+1}$ ---- i.e.
H*A = (*100 + 100*)* *11 10*
We need to solve this equation for the value of x:
H*A= *(100+)100 *11 10*
Or, H*A= *∗ ∗11 (100+)1000*
Or, *∗ ∗ = 1100+11 1000*
Or, 1= *1100+11 1000*
Or, x= *1000−110011*
, which works out to -9.09%.

12. Why is employment excluded from the International MPI published in the Human Development Report?
---c. Because data on employment is not available for most of the surveys used for the MPI

13. How many persons are not deprived in any dimension?
----a. 1.
First, we construct the deprivation matrix:

$$X = \begin{bmatrix} \$100 & 3Years & 0 \\ \$200 & 4Years & 0 \\ \$1200 & 7Years & 1 \\ \$500 & 3Years & 1 \\ \$300 & 5Years & 0 \end{bmatrix}$$

=$5005   1

Replacing entries: 1 if deprived, 0 otherwise. Then, summing the deprivations for each individual, we obtain the deprivations count vector.

$$g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad ci = \begin{bmatrix} 3 \\ 3 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

Looking at the deprivations count vector, we see that only individual 3 has 0 deprivations. Therefore, there is only **one** person who is not deprived in any dimension. The correct answer is a.

14. If the poverty cut-off is k = 2, what is the Multidimensional Headcount Ratio (H) and the average deprivation share among the poor (A)?
----c. 3/5, 8/9
We identify the poor and censor the data of the non-poor, creating the censored deprivation matrix. We can also compute a matrix with the share deprivation of each poor.

$$ci(2) = \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \\ 2 \end{bmatrix} \qquad g^0(2) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad ci(2)/3 = \begin{bmatrix} 3/3 \\ 3/3 \\ 0 \\ 0 \\ 2/3 \end{bmatrix}$$

Three of the individuals are poor:

H = 3 / 5

The average share of deprivation of poor is given by:

A = (3/3 + 3/3 + 2/3) / 3 = 8 / 9

15. What is the M0 of matrix X when k = 2?
---a. 8/15
We can compute this in two ways:

- M0 = HA = 3 /5 * 8 / 9 = 8 / 15
- M0 = $u(g^0(2))$ = 8 / 15

16. When k = 2, what proportion of the population is poor and at the same time is deprived in the education dimension?
---a. 2/5. Here is why:

The question is simply asking for the censored headcount in education. The censored headcount in education is the mean of the second column of the censored deprivation matrix:
CH2 = 2 / 5 (where 2 refers to the column number).

17. Assuming k=2, the contribution of each dimension to the M0 are:
---b. 3/8 for income, 2/5 for educ, 3/5 for nutrition. Here is why:

We know that:
$M_0 = I_1\,CH_1 + I_2\,CH_2 + I_3\,CH_3$ (where the numbers refer to the indicators/columns.
So, the contribution of indicator I is obtained as follows:

Contrib. of dimension i = $(I / M_0)\,CH_0$
Computing the censored headcounts for our three dimensions:
CH1 = 3 / 5
CH2 = 2 / 5
CH3 = 3 / 5
Contribution of income = [(1/3) * (3/5)] / (8/15) = (1/5) / (8/15) = 3 / 8
Contribution of education = [(1/3) * (2/5)] / (8/15) == (2/15) / (8/15) = 2/8
Contribution of nutrition = [(1/3) * (3/5)] / (8/15) = (1/5) / (8/15) = 3 / 8

18. Suppose that the population represented in matrix X can be divided into two groups. The first three people belong to group A and the last two belong to group B. Assuming k = 2, what is the contribution of each group to overall poverty (M0)?
---d. 6/15 for income, 3/15 for educ and 6/15 for nutrition. Here is why:

Remember that M0 can be written as follows:

$M_0 = M_{0x} + M_{0y}$ (where x and y refer to population subgroups). In this example, the sub-groups are called A and B

Thus, the contribution of poverty in the group A to the overall poverty can be computed as follows:

Contrib. of group A = M_{0x} / M_0

Let's "break" the censored deprivation matrix into the subgroups matrices.

First group (A):   Second group (B):

$$g^0_{\,a}(2)=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 0 & 0 & 0\end{bmatrix} \qquad g^0_{\,b}(2)=\begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 1\end{bmatrix}$$

Taking the average of the sub-matrices, we obtain the M0 for each group:

M0$_a$ = 6 / 9 = 2 / 3   M0$_b$ = 2/6 = 1 / 3

Contrib. of group A = [(3/5) * (2/3)] / (8/15) = 3 / 4
Contrib. of group B = [(2/5) * (1/3)] / (8/15) = 1 / 4

19. Suppose the weight on education dimension is increased to 2 while the weights on the income and under-nutrition dimensions are reduced to 0.5 each. What is the adjusted headcount ratio (M_0), multidimensional headcount ratio (H) and the average deprivation share (A) of X?

---a. 8/15, 3/5, 8/9. Here is why.

The deprivation matrix, g^0 =

$$g0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

But now we have weights, which we need to multiply with the deprivation scores. The first column, income, has a weight of 0.5, the second column, education, is 2 and the third column, under-nutrition is 0.5. We need to weight deprivations with the weights give. Note that weights are only used for deprivations, i.e. we need to multiply elements that are 1 by the weights. The weighted deprivation matrix is as follows:

$$g0 = \begin{bmatrix} 0.5 & 2 & 0.5 \\ 0.5 & 2 & 0.5 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

The weighted deprivation count vector c is:

$$c = \begin{bmatrix} 3 \\ 3 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

Note that with weights, person 4 is now poor and person 5 is not.

H=3/5

The censored weighted deprivation count vector c(k=2) is:

$$c(k = 2) = \begin{bmatrix} 3 \\ 3 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

A is the average deprivation among the poor in the censored weight deprivation count vector above:

$$A = \frac{(3/3) + (3/3) + (2/3)}{3} = \frac{8/3}{3} = \frac{8}{9}$$

M0 = (3/5) * (8/9)= 8/15