Poverty Measurement and the Distribution of Deprivations among the Poor

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Introduction

Two forms of technologies for evaluating poverty

- for identification and measurement of poverty

1 Unidimensional methods apply when:

- Single welfare variable – eg, calories
- Variables can be combined into an aggregate variable – eg, expenditure

2 Multidimensional methods apply when:

- Variables cannot be meaningfully aggregated – eg, sanitation conditions and years of education
- Desirable to leave variables disaggregated because sub-aggregates are policy relevant – eg food and nonfood consumption

Introduction

- Recently, strong demand for tools for measuring poverty multidimensionally
  - Governments, international organizations, NGOs
- Literature has responded with new measures
  - Anand and Sen (1997)
  - Tsui (2002)
  - Atkinson (2003)
  - Bourguignon and Chakravarty (2003)
  - Deutsch and Silber (2005)
  - Chakravarty and Silber (2008)
  - Maasoumi and Lugo (2008)

Introduction

- Problems
  - Most are not applicable to **ordinal** variables
    - Encountered in poverty measurement
  - Or otherwise yield methods that are far too **crude**
    - Violate Dimensional Monotonicity
    - Non-discerning identification: Very few poor or very few nonpoor

Introduction

- Methodology introduced in Alkire-Foster (2011)
  - Identification: Dual cutoff z and k
  - Measure: Adjusted headcount ratio M_0
- Addressed these problems
  - Applies to **ordinal**
    - And even categorical variables
  - Not so **crude**
    - Satisfies Dimensional Monotonicity
    - Discerning identification: not all poor or all nonpoor
- Satisfies key properties for policy and analysis
  - **Decomposable** by population
  - **Breakdown** by dimension after identification

Introduction

- Specific implementations include:
  - **M**ultidimensional **P**overty **I**ndex (UNDP)
    - Cross country implementation of M_0 by OPHI and HDRO
  - Official poverty index of **Colombia**
    - Country implementation of M_0 by Government of Colombia
  - **G**ross **N**ational **H**appiness index (Bhutan)
    - Country implementation of $(1-M_0)$ by Center for Bhutan Studies
  - **W**omen's **E**mpowerment in **A**griculture **I**ndex (USAID)
    - Cross country implementation of $(1-M_0)$ by USAID, IFPRI, OPHI

Introduction

- One possible critique
  - M_0 is **not sensitive to distribution among the poor**
- Two forms of distribution sensitivity
  - To inequality **within dimensions**
    - Kolm (1976)
  - To positive association **across dimensions**
    - Atkinson and Bourguignon (1982)
- Many measures satisfy one or both
  - For example adjusted FGT of Alkire-Foster (2011)
  - However, adjusted FGT not applicable to **ordinal** variables

This Paper

Asks

Can M_0 be altered to obtain a method that is both

- **sensitive to distribution** among the poor
- and applicable to **ordinal data**?

Answer

Yes. In fact, as easy as constructing unidimensional measures satisfying the transfer principle

Key

Intuitive transformation from **unidimensional** to **multidimensional** measures

Offers insight on the structure of M_0 and related measures

This Paper

- However we lose
  - **Breakdown** by dimension after identification
- Question
  - Is there any multidimensional measure that is sensitive to the distribution of deprivations and also can be broken down by dimension?
- Answer
  - Classical **impossibility** result
  - Can have one **or** the other but **not both**!
- Bottom line
  - Recommend using M_0 with an associated inequality measure

Outline

- Poverty Measurement
  - Unidimensional
  - Multidimensional
- Transformations
  - Measures
  - Axioms
- Impossibilities and Tradeoffs
- Conclusions

Poverty Measurement

- Traditional framework of Sen (1976)
- Two steps
  - Identification: "Who is poor?"
    - Targeting
  - Aggregation "How much poverty?"
    - Evaluation and monitoring

Unidimensional Poverty Measurement

- Typically uses **poverty line** for identification
  - Poor if income weakly below the cutoff (alternatively, strictly)
    - Example: Income distribution x = (7,3,4,8) poverty line $\pi = 5$
    - Who is poor?

Unidimensional Poverty Measurement

- Typically uses **poverty line** for identification
  - Poor if income weakly below the cutoff (alternatively, strictly)
    - Example: Income distribution x = (7,3,4,8) poverty line $\pi = 5$
    - Who is poor?
- Typically uses **poverty measure** for aggregation
  - Formula aggregates data to poverty level
    - Examples: Watts, Sen
    - Example: FGT $P_\alpha(x;\pi) = \mu(g_1^\alpha, \ldots, g_n^\alpha) = \mu(g^\alpha)$
      - Where: g_i^α is $[(\pi - x_i)/\pi]^\alpha$ if *i* is poor and 0 if not, and $\alpha \geq 0$ so that
        - $\alpha = 0$ headcount ratio
        - $\alpha = 1$ per capita poverty gap
        - $\alpha = 2$ squared gap, often called FGT measure

Unidimensional Poverty Measurement

Example

Incomes $x = (7,\underline{1},\underline{4},8)$

Poverty line $\pi = 5$

Deprivation vector $g^0 = (0,1,1,0)$

Headcount ratio $P_0(x;\pi) = \mu(g^0) = 2/4$

Normalized gap vector $g^1 = (0, 4/5, 1/5, 0)$

Poverty gap $= HI = P_1(x;\pi) = \mu(g^1) = 5/20$

Squared gap vector $g^2 = (0, 16/25, 1/25, 0)$

FGT Measure $= P_2(x;\pi) = \mu(g^2) = 17/100$

Unidimensional Poverty Measurement

FGT Properties

- For $\alpha = 0$ (headcount ratio)
  - Invariance Properties: Symmetry, Replication Invariance, Focus
  - Composition Properties: Subgroup Consistency, Decomposability
- For $\alpha = 1$ (poverty gap)
  - +Dominance Property: Monotonicity
- For $\alpha = 2$ (FGT)
  - +Dominance Property: Transfer

Unidimensional Poverty Measurement

- Poverty line actually has **two** roles
  - In identification step, as the separating **cutoff** between the target group and the remaining population.
  - In aggregation step, as the **standard** against which shortfalls are measured
- In some applications, it may make sense to **separate** roles
  - A **poverty standard** π for constructing gap and aggregating
  - A **poverty cutoff** $\pi_t < \pi$ for targeted identification
    - Example 1: Measuring ultra-poverty Foster-Smith (2011)
      - Forcing standard π down to cutoff π_t distorts the evaluation of ultrapoverty
    - Example 2: Measuring hybrid poverty Foster (1998)
- Targeted poverty measure $P(x;z,z_t)$

Unidimensional Poverty Measurement

Example: Targeted FGT $P_\alpha(x; \pi, \pi_t)$

- **Incomes** $x = (7,\underline{1},4,8)$
- **Poverty standard** $\pi = 5$
- **Targeted poverty cutoff** $\pi_t = 3$

Deprivation vector $g^0 = (0,1,0,0)$ (*use* π_t *for identification*)

- **Targeted headcount ratio** $P_0(x; \pi, \pi_t) = \mu(g^0) = 1/4$

Normalized gap vector $g^1 = (0, 4/5, 0, 0)$ (*use* π *for gap*)

- **Targeted poverty gap** = $HI = P_1(x; \pi, \pi_t) = \mu(g^1) = 4/20$

Squared gap vector $g^2 = (0, 16/25, 0, 0)$

- **Targeted FGT Measure** = $P_2(x; \pi, \pi_t) = \mu(g^2) = 16/100$

Unidimensional Poverty Measurement

Targeted FGT Properties

- For $\alpha = 0$ (targeted headcount ratio)
  - Invariance Properties: Symmetry, Replication Invariance, and Targeted Focus
  - Composition Properties: Subgroup Consistency, Decomposability,
- For $\alpha = 1$ (targeted poverty gap)
  - +Dominance Property: Targeted Monotonicity
- For $\alpha = 2$ (targeted FGT)
  - +Dominance Property: Targeted Transfer

Unidimensional Poverty Measurement

- Idea of targeted poverty measure $P(x;\pi,\pi_t)$
  - Allows flexibility of targeting group below poverty cutoff π_t while maintaining the poverty standard at π
  - Particularly helpful when different groups of poor have different characteristics and hence need different policies
- Note
  - Other more nuanced forms of targeting are possible
  - This is a key topic for research

Multidimensional Poverty Measurement

- How to evaluate poverty with many dimensions?
- Previous work mainly focused on **aggregation**
- While for the **identification** step it:
  - First set cutoffs to identify deprivations
  - Then identified poor in one of three ways
    - Poor if have *any* deprivation
    - Poor if have *all* deprivations
    - Poor according to some function left unspecified
- Problem
  - First two are **impractical** when there are many dimensions
    - Need intermediate approach
  - Last is **indeterminate**, and likely **inapplicable** to ordinal data

AF Methodology

Alkire and Foster (2011) methodology addresses these problems

It specifies an **intermediate** identification method that is consistent with **ordinal** data

Dual cutoff identification

- Deprivation cutoffs $z_1 \ldots z_j$ one per each of *j* deprivations
- Poverty cutoff k across aggregate weighted deprivations

Idea

- A person is poor if multiply deprived enough

Example

AF Methodology

Achievement Matrix

Dimensions

$$Y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix}$$

Persons

$$\mathbf{z = (\,13 \quad 12 \quad 3 \quad 1\,)}$$

Cutoffs

AF Methodology

Deprivation Matrix

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 4 \\ 1 \end{bmatrix}$$

$\Longrightarrow$

Censored Deprivation Matrix, k=2

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 4 \\ 0 \end{bmatrix}$$

Identification Who is poor?

- If poverty cutoff is $k = 2$
  - Then the two middle persons are poor

Now censor the deprivation matrix

Ignore deprivations of nonpoor

AF Methodology

If data cardinal, construct two additional censored matrices

Censored Gap Matrix

$$g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Censored Squared Gap Matrix

$$g^2(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Aggregation

$$M_\alpha = \mu(g^\alpha(k)) \text{ for } \alpha \geq 0$$

Adjusted FGT M_α is the mean of the respective censored matrix

AF Methodology

Properties

- For $\alpha = 0$ (Adjusted headcount ratio)
  - Invariance Properties: Symmetry, Replication Invariance, Deprivation Focus, Poverty Focus
  - Dominance Properties: Weak Monotonicity, Dimensional Monotonicity, Weak Rearrangement
  - Composition Properties: Subgroup Consistency, Decomposability, Dimensional Breakdown
- For $\alpha = 1$ (Adjusted poverty gap)
  - +Dominance Property: Monotonicity, Weak Transfer
- For $\alpha = 2$ (Adjusted FGT)
  - +Dominance Property: Transfer

AF Methodology

Note

- The poverty measures with $\alpha > 0$ use gaps, hence require **cardinal** data
- Impractical given data quality
- Focus here on measure with $\alpha = 0$ that handles **ordinal** data

Adjusted Headcount Ratio M_0

- Practical and applicable

Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\mu(g^0(k))$

	Domains				c(k)	c(k)/d	
$g^0(k) =$	0	0	0	0	0		
	0	1	0	1	2	2 / 4	Persons
	1	1	1	1	4	4 / 4	
	0	0	0	0	0		

H = multidimensional headcount ratio = 1/2

A = average deprivation share among poor = 3/4

Adjusted Headcount Ratio

Properties

- Invariance Properties: Symmetry, Replication Invariance, Deprivation Focus, Poverty Focus
- Dominance Properties: Weak Monotonicity, Dimensional Monotonicity, Weak Rearrangement, a form of Weak Transfer
- Composition Properties: Subgroup Consistency, Decomposability, Dimensional Breakdown

Note

- No transfer property **within dimensions**
  - Requires cardinal variables!
- No transfer property **across dimensions**
  - Here there is some scope

New Property

Recall: **Dimensional Monotonicity** Multidimensional poverty should rise whenever a poor person becomes deprived in an additional dimension (*cet par*) (AF, 2011)

New: **Dimensional Transfer** Multidimensional poverty should fall whenever the total deprivations among the poor in each dimension are unchanged, but are reallocated according to an association decreasing rearrangement among the poor

Adjusted Headcount Satisfies Dimensional Monotonicity, but just violates Dimensional Transfer.

Q/ Are there other related measures satisfying **DT**?

New Measures

- Idea
  - Construct attainment matrix
    - For poor, replace deprivations with attainments
    - For non-poor, replace 0's with 1's
      - In measuring poverty, nonpoor are seen as having full attainments
  - Count attainments and create vector or distribution
  - Apply a unidimensional poverty measure P to obtain a multidimensional poverty measure B
- Hypothesis
  - The properties of P are directly linked to the properties of B
  - Perhaps B satisfying dimensional transfer can be found

Attainments

Recall **Achievement** matrix

Dimensions

$$Y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} \quad \textbf{Persons}$$

$$\boldsymbol{z} = (\mathbf{13} \quad \mathbf{12} \quad \mathbf{3} \quad \mathbf{1}) \quad \textbf{Cutoffs}$$

Attainments

Construct **attainment** matrix

1 if person attains cutoff in a given domain

0 if not

Domains

$$a^0 = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \underline{\mathbf{0}} & \mathbf{1} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} \\ \mathbf{1} & \underline{\mathbf{0}} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

Persons

Note

Opposite of the deprivation matrix

Attainments

Counting Attainments

1 if person attains cutoff in a given domain

0 if not

Domains $\qquad\qquad$ a

$$a^0 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \underline{0} & 1 & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ 1 & \underline{0} & 1 & 1 \end{bmatrix} \qquad \begin{matrix} \mathbf{4} \\ \mathbf{2} \\ \mathbf{0} \\ \mathbf{3} \end{matrix}$$

Persons

Attainment vector

$$a = (4, 2, 0, 3)$$

Now apply unidimensional poverty measure

Transformations

Define $B(x;k) = P(a; \pi)$

where a is the distribution of attainments associated with x

P is a unidimensional poverty measure

k is a cutoff below d and $\pi = d + 1 - k$

Result

If P is the poverty gap with weak (≤) identificaton, then $B = M_0' = HA'$ where H is multidimensional headcount, A' is intensity measured as number of deprivations beyond cutoff (where multidimensional poverty identification is strict (>)).

Note: Mexico's version M_0' of adjusted headcount ratio

Transformations

Example

Let k = 2 be the multidimensional poverty cutoff where d = 4

Set $\pi = 3$ (or d +1 – k)

Recall $a = (4, 2, 0, 3)$

Strict identification deprivation vector $g^0 = (0,1,1,0)$

Gap vector $g^1 = (0, 1/3, 3/3, 0)$

Then

With strict identification $P_1 = \int (g^1) = 1/3$

Note

Mexican version $M_0' = HA' = (1/2)(2/3) = 1/3$

Transformations

- Note: Properties of B depend on properties of P
- For example: Poverty gap satisfies monotonicity, hence B satisifes dimensional monotonicity.
- Same follows for array of properties.
- Alternatively
  - Could base B on a targeted poverty measure $P(y; \pi, \pi_t)$
  - With separate identification cutoff π_t and aggregation standard π.

Transformations

Define $B(x;k) = P(a; \pi, \pi_t)$

where a is the distribution of attainments associated with x

P is a unidimensional targeted poverty measure

k is a cutoff below d; $\pi_t = d + 1 - k$; and $\pi = d$

Result

If P is the poverty gap with strict ($<$) identification, then $B = M_0 = HA$ where H is multidimensional headcount, A is intensity measured as number of deprivations (where multidimensional poverty identification is weak ($\leq$)).

Note: Exactly adjusted headcount ratio M_0

Example

$a = (4, 2, 0, 3)$ $k = 2$ and $z = 3$

Transformations

Example

Let k = 2 be the multidimensional poverty cutoff where d = 4

Set $\pi_\tau = 3$ (or d +1 – k) and $\pi = 4$ (or d)

Recall $a = (4, 2, 0, 3)$

Weak identification deprivation vector $g^0 = (0,1,1,0)$

Targeted gap vector $g^1 = (0, 2/4, 4/4, 0)$

Then

With weak identification $P_1 = \int (g^1) = 3/8$

Note

Adjusted headcount ratio $M_0 = HA = (1/2)(3/4) = 3/8$

Transformations

Note

Properties of B depend on properties of $P(y; \pi_t, \pi)$

Example

Targeted poverty gap satisfies targeted monotonicity, hence B satisifes dimensional monotonicity.

Result

If P satisfies the targeted transfer principle, then B satisfies dimensional transfer

Example

Let P be targeted FGT or Watts

Transformations

Lesson

- Trivial to construct multidimensional measures sensitive to inequality across deprivations – just use distribution sensitive unidimensional measure and transform

Question

- But at what cost?

Impossibility

- Note
  - The measure associated with targeted P_2 does not satisfy breakdown by dimension; same for ordinary P_2
- Theorem (almost proved)
  - There is no measure B satisfying both dimensional breakdown and dimensional transfer
- Proof
  - Pattanaik impossibility result

Impossibility

- Importance of Dimensional Breakdown
  - Policy
    - Composition of poverty
    - Changes over time by indicator
  - Analysis
    - Composition of poverty across groups, time
    - Interconnections across deprivations
    - Efficient allocations
- Conclusion
  - Easy to construct measure satisfying dimensional transfer
  - But at a cost: lose this key element of the toolkit

Concluding Remarks

Alternative way forward:

- Apply M_0 class of measures for ordinal data
  - Satisfies dimensional breakdown
- Construct associated measure of inequality among the poor

Note

- P_0 headcount ratio, P_1 poverty gap and FGT P_2 have long been used in concert to analyze the incidence, depth, and distribution of (income) deprivations
- Analogously, can use H headcount ratio, adjusted headcount ratio M_0 and inequality measure to analyze the incidence, breadth and distributions of deprivations

With a focus on the measure M_0 and its useful breakdown

Thank you