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# Summer School on Multidimensional Poverty Analysis

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# Multidimensional Poverty and Distribution: Inequality among the poor and Disparity across regions

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# Main Sources of this Lecture

- Alkire S., J. E. Foster, S. Seth, S. Santos, J. M. Roche, P. Ballon, Multidimensional Poverty Measurement and Analysis, Oxford University Press, forthcoming, (Ch 10.1).
- Alkire, S. and Foster, J. E. (2013), Evaluating Dimensional and Distributional Contributions to Multidimensional Poverty, *Mimeo*.
- Seth S. and S. Alkire (2014), Did Poverty Reduction Reach the Poorest of the Poor? Assessment Methods in the Counting Approach, Working Paper 68, Oxford Poverty & Human Development Initiative, University of Oxford.

# Motivation

- Poverty measurement tools affect policy design and policy incentive
  - Incidence, Intensity, Inequality
    - Distributional concerns (Sen 1976)
    - Three I's (Jenkins and Lambert 1997)

- How to incorporate distributional issues in the measurement of multidimensional poverty?

# Can We Incorporate Inequality?

- Relevant properties:
  - Weak Transfer: An averaging of achievements among the poor reduces poverty
  - Deprivation Rearrangement (Substitutes): Decrease in association between dimensions decreases poverty
  - Converse Deprivation Rearrangement (Complements): Decrease in association between dimensions increases poverty

Kolm (1977), Atkinson and Bourguignon (1982)

# Can We Incorporate Inequality?

- Transformation for transfer (using bistochastic matrix)

$$Y = BX = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 3.5 & 4.5 & 3 \\ 3.5 & 4.5 & 3 \\ 8 & 6 & 3 \end{bmatrix}, z = [5 \;\; 6 \;\; 5]$$

- Transformation for rearrangement (among the poor)

$$Y = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 3 & 2 \\ 10 & 10 & 12 \end{bmatrix} \quad X = \begin{bmatrix} 3 & 4 & 8 \\ 2 & 3 & 2 \\ 10 & 10 & 12 \end{bmatrix} \quad z = [4 \;\; 5 \;\; 3]$$

# Can We Incorporate Inequality?

- Many measures in chapter 3 satisfies these properties
  - Requires *cardinal data* and not applicable for counting measures
- Dimensional Transfer: Poverty should fall whenever the total deprivations among the poor in each dimension are unchanged, but are reallocated according to an association decreasing rearrangement among the poor

*Alkire and Foster (2013)*

  - Capturing inequality by looking at the extent of joint deprivations

# Example

- Dimensional transfer

$$g^0(Y) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad g^0(X) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

- Which one has larger inequality when dimensions are equally weighted (use union approach for illustration)?

# Example: Inequality among the Poor

Initial Deprivation Count Vector ($k = 0.3$)

(0, 0.1, **0.3**, **0.7**, **1**, **1**)

Time Line 1 → (0, 0.1, 0.1, **0.3**, **0.9**, **0.9**)

Time Line 2 → (0, 0.1, 0.2, **0.5**, **0.8**, **0.8**)

Similar reductions in incidence and intensity

**Incidence :** 4/6 → 3/6

**Intensity :** 0.75 → 0.7

Inequality?

# Two Practical Properties of $M_0$

The following two properties of $M_0$ are useful in practice:

- Ordinality allows the measure to be used with ordinal, binary, or ordered categorical data

- Dimensional Breakdown permits the dimensional composition of poverty to be seen easily

*Alkire and Foster (2013)*

# An Impossibility Result

- There is no multidimensional counting poverty measure satisfying symmetry, dimensional breakdown and dimensional transfer

*Alkire and Foster (2013)*

- In other words, one has to choose measures that satisfy *either* one, *or* the other.

- How to proceed?

# Two Possible Way Outs

1. Use a poverty measure that satisfies dimensional breakdown and use another poverty measure that satisfies dimensional transfer and ordinality

2. Use a poverty measure that satisfies dimensional breakdown and in addition analyze inequality among the poor separately

# The First Approach

Additionally use a poverty measure that satisfies dimensional transfer and ordinality

– Jayaraj and Subramanian (2009), Bossert, Chakravarty and D'Ambrosio (2013), Alkire and Foster (2013)

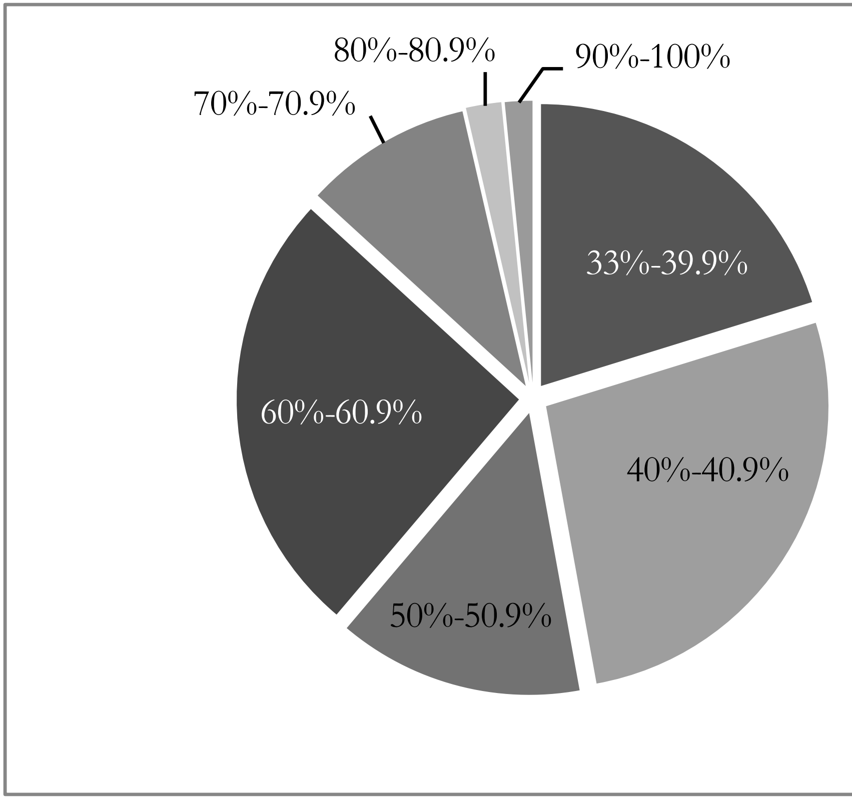
Other Practical Limitations:

1. Mainly used for ordering
2. The final figure obtained may not be intuitive and may lack usefulness for inter-temporal analysis
3. Does not pay attention to subgroup disparity in poverty

# The Second Approach

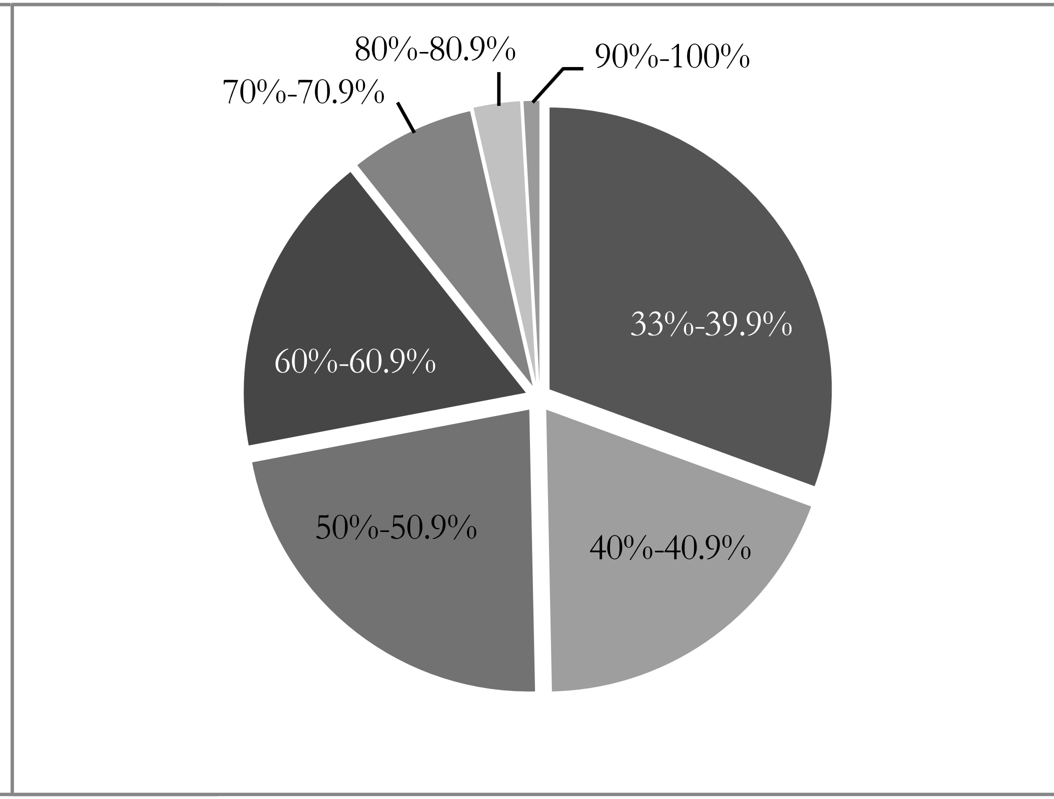
- Conduct analysis on inequality separately
- An example: Use of *standard deviation* in child poverty
  - Delamonica and Minujin (2007), Roche (2013)
- How may this approach be useful?
  - Additional information besides incidence and intensity
  - Can be used with a poverty measure that respects ordinality property and satisfies dimensional breakdown
  - If decomposable, can observe inequality decomposition within and between population subgroup

# Example: A Descriptive Tool




Madagascar (2009)

**MPI = 0.357, H = 67%, A = 53%**




Rwanda (2010)

**MPI = 0.350, H = 69% A = 50.8%**

# The Second Approach

- Use a separate inequality measure to capture inequality among the poor

- Which inequality measure?
  - Depends on value judgments
  - Consider an example

# Example (Use Union Approach)

- Suppose vector $\underline{c}$ is obtained from vector $c$ over time
  - $c = (0.4, 0.4, 0.9, 0.9)$ and $\underline{c} = (0.1, 0.1, 0.3, 0.3)$

- Has poverty gone down?
  - Indeed according to any poverty measure (integrated as well) that satisfies *dimensional monotonicity*
    - The property requires that if a poor person remains poor but becomes non-deprived in a dimension in which the person was deprived earlier, poverty should fall

# Example (Use Union Approach)

- Suppose vector $\underline{c}$ is obtained from vector $c$ over time
  - $c$ = (0.4, 0.4, 0.9, 0.9) and $\underline{c}$ = (0.1, 0.1, 0.3, 0.3)

- How has poverty gone down?
  - Incidence? No
  - Intensity? Yes
  - Inequality among the poor?
    - Inequality of what?
    - Depends on value judgments!
    - Absolute (translation invariance) or relative (scale invariance)?
    - Across deprivations scores or across attainment scores?

# Example (Use Union Approach)

- Suppose vector $\underline{c}$ is obtained from vector $c$ over time
  - $c = (0.4, 0.4, 0.9, 0.9)$ and $\underline{c} = (0.1, 0.1, 0.3, 0.3)$

- Inequality of what?
  - Inequality (relative) across <u>deprivation scores</u> of the poor?
  - Then inequality has gone up
  - $GE(c,2) = 0.074$ and $GE(\underline{c},2) = 0.125$
    - Approach followed by Rippin (2011)
  - Is measuring inequality across the <u>deprivation scores</u> right?

# Example (Use Union Approach)

- Suppose vector $\underline{c}$ is obtained from vector $c$ over time
  - $c$ = (0.4, 0.4, 0.9, 0.9) and $\underline{c}$ = (0.1, 0.1, 0.3, 0.3)

- Inequality of what?
  - What if we capture inequality across <u>attainment scores</u>?
    - $c_a$ = (0.6,0.6,0.1,0.1) and $\underline{c}_a$ = (0.9,0.9,0.7,0.7)
  - Then inequality (relative) among the poor has gone down
  - GE($c_a$,2) = 0.255 and GE($\underline{c}_a$,2) = 0.008

# Inequality across Attainment Scores

- Is this then the right way to reflect inequality?

- Example (Attainment scores only among the poor)
  – $c_a$ = (0.5,0.5,0.1,0.1) and $\underline{c}_a$ = (0.9,0.9,0.2,0.2)
  – Least improvement among the poorest (in red)
  – What should happen to inequality among the poor?
    - GE($c_a$,2) = 0.222 and GE($\underline{c}_a$,2) = 0.202
  – Value judgment?
    - Hard to argue that poverty has fallen by improving the situation of the poorest

# The Second Approach

- Which inequality measure to use?
  - Depends on value judgments or properties

Properties

  - Same level of inequality should be reflected whether across deprivation scores or across attainment scores

# The Second Approach

- Which inequality measure to use?
  - Depends on value judgments or properties

Properties

- We also want to capture disparity across population subgroups
  - Between-group inequality
- Additive Decomposability: Total inequality should be presented as a sum of two components: a within-group component and a between group component

# A Policy Relevant Property

- Within-group Mean Independence: Total within-group component does not change if there is no change in inequality within any subgroup
  - Analogous to path independence (Foster and Shneyerov, 2000)
  - The weight attached to each within-group inequality component is the population share of the group

# Within-group Mean Independence

Consider $c$ and two subgroups $c^1$ and $c^2$

Additive Decomp.: $I(c) = \overbrace{\omega_1 I(c^1) + \omega_2 I(c^2)}^{\text{Total within-group}} + \text{Bet}(c^1,c^2)$

Suppose for $\underline{c} \neq c$, $I(\underline{c}^1) = I(c_1)$ and $I(\underline{c}_2) = I(c_2)$

- Unchanged population size in two subgroups

Q: Should the total <u>within-group inequality</u> be different in $c$ and $\underline{c}$?

*Path independence* (Foster and Shneyerov, 1999)

# The Inequality Measure

The inequality measure that we use:

$$I(y) = \frac{\tilde{\beta}}{t}\sum_{i=1}^{t}[y_i - \mu(y)]^2$$

$I(y)$: positive multiple of variance of distribution $y$

$\mu(y)$: mean of distribution $y$

$t$: Number of elements in $y$

$\tilde{\beta} > 0$

Chakravarty (2001)

# Inequality Decomposition

- $I(y)$ can be decomposed across subgroups as follows

$$I(y) = \underbrace{\sum_{\ell=1}^{m} \frac{t^{\ell}}{t} I(y^{\ell})}_{\text{Total within-group}} + \underbrace{\tilde{\beta} \sum_{\ell=1}^{m} \frac{t^{\ell}}{t} \left(\mu(y^{\ell}) - \mu(y)\right)^{2}}_{\text{Between-group}}$$

  - Number of subgroups: $m$
  - Score vector of subgroup □: $y^{□}$
  - The population share of subgroup □: $t^{□}/t$

# Inequality among the Poor

- Two applications:
  - Inequality among the poor

$$I^q = \frac{\tilde{\beta}}{q} \sum_{i=1}^{q} [c_i(k) - A]^2$$

  - Inequality across population subgroups

$$I^n = \tilde{\beta} \sum_{\ell=1}^{m} \frac{n^\ell}{n} \left(M_0(X^\ell) - M_0\right)^2$$

# Disparity in Poverty across Subgroup

- A valid question: Has the national reduction in MPI been uniform across subgroups?
  - Analogous to horizontal inequality (Stewart 2010)
  - Sub-national disparity (Alkire, Roche and Seth 2011)

- It may be possible that the overall inequality and within group inequalities in all subgroups fall but still disparity in poverty increases

# Example ( $\tilde{\beta} = 4$ )

| Country | Year | $M_0$ | A | H | Inequality Among the Poor ($I^q$) | Disparity Between MPIs ($I^n$) | Number of Regions |
|---|---|---|---|---|---|---|---|
| Yemen | 2006 | 0.283 | 53.90% | 52.50% | 0.122 | 0.052 | 21 |
| India | 2005 | 0.283 | 52.70% | 53.70% | 0.104 | 0.05 | 29 |
| Togo | 2010 | 0.25 | 50.30% | 49.80% | 0.086 | 0.042 | 6 |
| Bangladesh | 2011 | 0.253 | 49.50% | 51.20% | 0.084 | 0.005 | 7 |

**Source**: Seth and Alkire (2014)

- Yemen and India: Same MPI, different inequality
- Bangladesh and Togo: Similar inequality but different sub-national disparity (with similar number of regions)

# Concluding Remarks

- Integrated approaches to poverty measurement in counting approach are appropriate for ordering, but may not be intuitive with strong policy implications
- Various important properties conflicts with each other
- An alternative approach is to study inequality among the poor using a separate inequality measure

# Concluding Remarks

- Added advantage: The inequality measure reflects same level of inequality whether deprivations are counted or achievement counterparts

- The tool can also be used to assess and monitor disparity in poverty across population subgroups