OPHI
OXFORD POVERTY & HUMAN DEVELOPMENT INITIATIVE
www.ophi.org.uk

UNIVERSITY OF OXFORD

Summer School on Multidimensional Poverty

8–19 July 2013

Institute for International Economic Policy (IIEP)
George Washington University
Washington, DC

Tabita, Kenya | Rabiya, India | Stephanie, Madagascar | Agatha, Madagascar | Dalma, Kenya | Ann-Sophia, Kenya | Valérie, Madagascar

Evaluating Dimensional and Distributional Contributions to Multidimensional Poverty

Sabina Alkie & James Foster

Work in progress

What makes measures practical?

Interest in the AF methodology is largely driven by three properties:

Ordinality allows the measure to be used with ordinal, binary, or ordered categorical data.

Subgroup Decomposability facilitates regional breakdown

Dimensional Breakdown permits the dimensional composition of poverty to be seen easily

Can we incorporate inequality?

- Relevant definitions of inequality:
  - *Transfer* (Kolm 1977) - satisfied if $\alpha \geq 1$ (weak)
  - *Correlation increasing switch* – weak (M_0)

How about

- *Dimensional Transfer* ? Not respected.

Can we incorporate inequality?

- To construct a measure satisfying dimensional transfer, create the censored deprivation matrix as before, and provide the censored c_i vector.
- Then square each element of the vector.
- An inequality-adjusted $\mathbf{M_0}'$ could be computed as the mean of the vector of squared deprivation scores. $M_0' = \mu(c_i(k))^2$
- More generally, $\mathbf{M_0}' = \mu(c^\gamma(k))$, where $\gamma > 1$

Can we incorporate inequality?

- **M_0'** satisfies many properties: replication invariance, symmetry, poverty and deprivation focus, dimensional monotonicity, nontriviality, and normalisation, dimensional transfer, ordinality and subgroup decomposability.
- But it does *not* satisfy *dimensional breakdown*.
- Why?

Impossibility result

- "There is no multidimensional poverty methodology M = (ρ, M) satisfying symmetry, dimensional breakdown and dimensional transfer."
- In other words, you have to choose measures that satisfy *either* one, *or* the other.
- How to proceed?

Practical paths

- Option 1: Use an inequality-sensitive measure

  + satisfies dimensional transfer, hence shows inequality

  -- does not satisfy dimensional breakdown, so changes in censored H don't add up to changes in poverty.

  -- hard(er) to interpret; lacks intuitive partial indices.

- Option 2: Use M_0 with an inequality measure

  + can show censored H, % contribution etc. as before

  + can also show inequality among the poor by group

  + inequality among the poor is of interest, but secondary

Inequality among the Poor and Disparity in Poverty among Subgroups

Sabina Alkie & Suman Seth

Concern for Inequality

Consideration of Inequality in poverty measurement has been the norm since Sen (1976)

Three I's of poverty (Jenkins and Lambert 1997)

- It is not only important to reduce the *Incidence* and *Intensity*, but also *Inequality*

Policy implications

Consideration of Inequality in Poverty Analysis

Natural for measures in cardinal approach

- **Approaches for Cardinal data** (Chakravarty, Mukherjee and Ranade 1998, Tsui 2002, Bourguignon and Chakravarty 2003, Massoumi and Lugo 2008, Alkire and Foster 2011)

Not straightforward for measures in counting approach

However, inequality can be captured across deprivation counts, if we take c_i to be cardinally meaningful

- Deprivation count vector $c = (c_1, ..., c_n)$; $0 \leq c_i \leq 1$

Consideration of Inequality in Poverty Analysis

One Approach: Fine tune a poverty measure to capture inequality

- Bossert, Chakravarty and D'Ambrosio 2009
  - Uses symmetric or generalized mean across deprivation counts
- Jayaraj and Subramanian 2009 and Rippin (2011)
  - Weights deprivation counts by themselves (like FGT)

Merely used for *ranking*. Not suitable for understanding inequality within groups and between groups

Consideration of Inequality in Poverty Analysis

Options

a. Create a poverty index that is sensitive to inequality?

b. Use a separate inequality measure to analyze inequality among the poor?

Proposal: a separate inequality measure may provide more information

An advantage of (b) is that – if decomposable, it can be used to analyze inequality within groups and between groups

Consideration of Inequality in Poverty Analysis

Q: Which inequality measure to use?

- It depends on which properties we want the measure to satisfy

An example: Use of *standard deviation* in child poverty

- Delamonica and Minujin (2007), Roche (2013)

What Type of Inequality Matters?

Should the consideration for inequality be based on relative or absolute distances in deprivations?

- 'Leftist' vs. 'rightist' viewpoint (Kolm 1976)

Example: $c_1 = (0,0,\mathbf{0.1},\mathbf{0.3})$ and $c_2 = (0,0,\mathbf{0.4},\mathbf{1})$

Which vector is more unequal across **the poor** (Union)?

- Relative (scaling): c_1 has more inequality (Hard to defend)
- Absolute (difference): c_2 has more inequality

Example: Two States of India (Union)

State A	
Deprivation Score	**in Millions**
Not deprived	5.4
0-0.3	24.1
0.3-0.6	3.0
0.6-0.8	0.2
0.8-0.9	-
0.9-1	-
Total Poor	**27.2**
Total Population	**32.6**

State B	
Deprivation Score	**in Millions**
Not deprived	4.8
0-0.3	21.2
0.3-0.6	24.4
0.6-0.8	9.3
0.8-0.9	1.9
0.9-1	1.0
Total Poor	**56.8**
Total Population	**62.6**

Which state has more inequality among **the poor** (Union)?

GE(2): 0.253	**Gini: 0.372**	**GE(2): 0.144**	**Gini: 0.304**

A: Kerala, B: Rajasthan, Year: 2006

from Alkire and Seth (2013)

What Type of Inequality Matters?

We argue: ‘distance’ is more appropriate than ‘scaling’ in understanding inequality in counting framework

The additional properties we want the measure to satisfy

- Symmetry
- Replication invariance (population principle)
- Zero inequality when everybody has same deprivation score
- Increase in inequality due to regressive transfer (Dalton)
- Additive Decomposability
    - Overall = Total within-group + between-group
- Within-group Mean Independence

Additive Decomposability

$c = (0.1,0.2,0.3,0.4)$, $c_1 = (0.1,0.2)$ and $c_2 = (0.3,0.4)$

$$I(c) = \underbrace{w_1 I(c_1) + w_2 I(c_2)}_{\text{Total within-group}} + Bet(c_1,c_2)$$

$\boldsymbol{c} = (0.3,0.4,0.4,0.5)$, $\boldsymbol{c}_1 = (0.3,0.4)$ and $\boldsymbol{c}_2 = (0.4,0.5)$

Q: Should the total within-group inequality be different in c and *c*?

– Within-group Mean Independence

The Inequality Measure?

The only <u>absolute</u> inequality measure that satisfies these properties is *variance* (its positive multiple, technically)

$$I(x) = \alpha\Sigma_i(x_i - \mu(x))^2/n$$

where, $I(x)$: positive multiple of variance of vector x

$\mu(x)$: mean of elements in x

n: population size of x

$\alpha > 0$

Chakravarty (2001)

Bosmans and Cowell (2011)

Bounds of Variance

Minimum possible value of variance: 0

Maximum possible value of I(x): $(b-a)^2/4$

- b is the maximum value; a is the minimum value, (b-a) is the range

Choose $\alpha = 4/(b-a)^2$, then $I(x) = V(x)$ ranges between 0 & 1

- Maximum inequality: 1
- Minimum Inequality: 0

Revisit the Example

State A	
Deprivation Score	**in Millions**
Not deprived	5.4
0-0.3	24.1
0.3-0.6	3.0
0.6-0.8	0.2
0.8-0.9	-
0.9-1	-
Total Poor	**27.2**
Total Population	**32.6**

State B	
Deprivation Score	**in Millions**
Not deprived	4.8
0-0.3	21.2
0.3-0.6	24.4
0.6-0.8	9.3
0.8-0.9	1.9
0.9-1	1.0
Total Poor	**56.8**
Total Population	**62.6**

V: 0.052 $\alpha = 4$ **V: 0.188**

Range of Deprivation Scores = 1

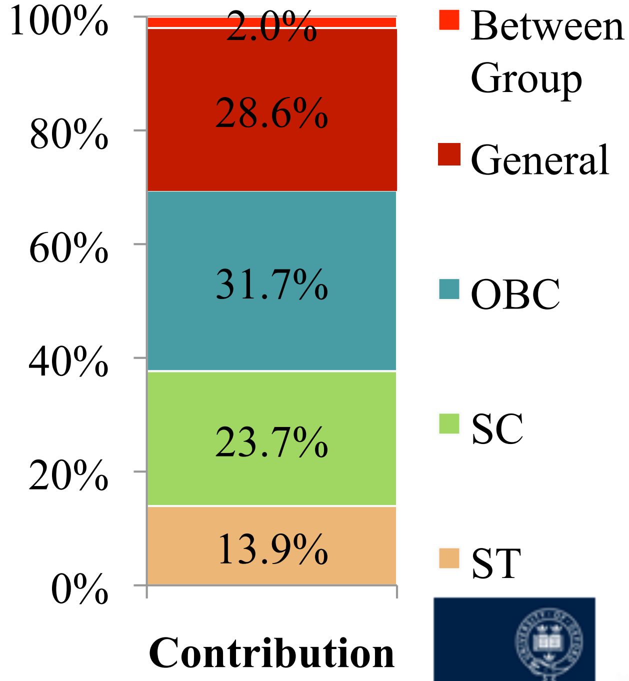
The Natural Decomposition

Total inequality across the poor into between-group and within group components

Inequality Decomposition across Castes and Tribes in India (1998)

	Intensity of Pov	Share of Poor	Inequality (Poor)	Total Within group	Between Group
ST	57.0%	12.6%	0.110		
SC	55.0%	22.1%	0.107		
OBC	52.1%	33.3%	0.095		
General	50.6%	32.0%	0.089		
India	**52.9%**	**100%**	0.100	0.098	0.002

Alkire and Seth (2013)



What Happened Over Time?

1999	Intensity (MPI)	Share of Poor	Inequality (Poor)	Total Within group	Between Group
ST	57.0%	12.6%	0.110		
SC	55.0%	22.1%	0.107		
OBC	52.1%	33.3%	0.095		
General	50.6%	32.0%	0.089		
India	**52.9%**	**100%**	**0.100**	**0.098**	**0.002**
2006					
ST	56.3%	12.9%	0.115		
SC	52.6%	22.9%	0.098		
OBC	50.8%	42.1%	0.090		
General	49.7%	22.0%	0.092		
India	**51.7%**	**100%**	**0.097**	**0.096**	**0.0017**

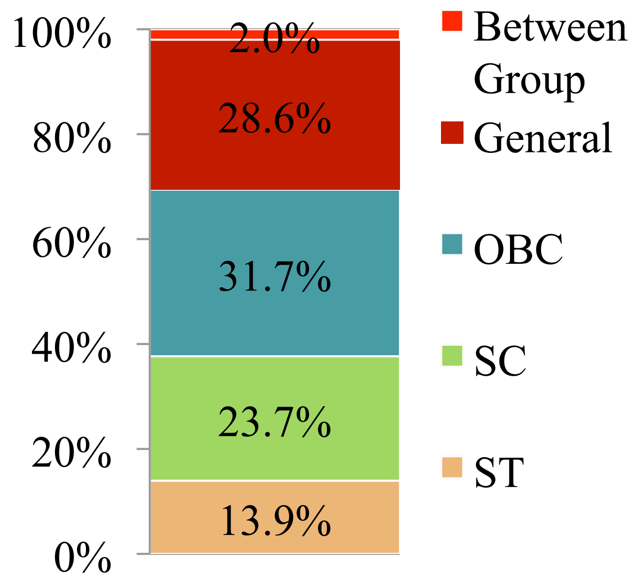
What Happened Over Time?

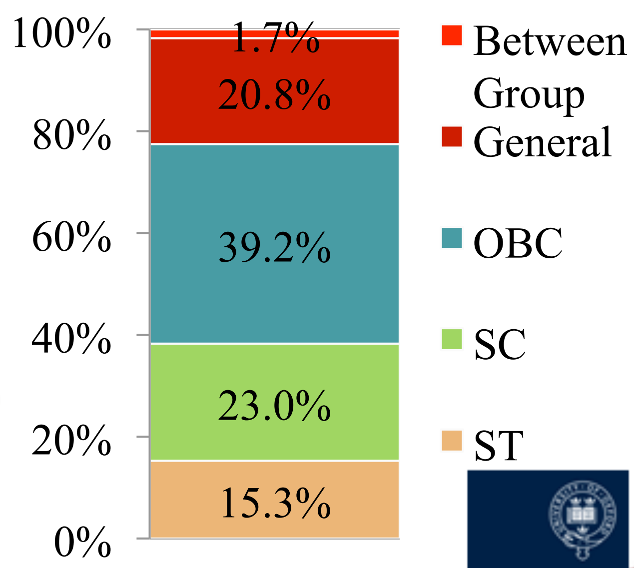
1999	Intensity (MPI)	Share of Poor	Inequality (Poor)	Total Within group	Between Group
ST	57.0%	12.6%	0.110		
SC	55.0%	22.1%	0.107		
OBC	52.1%	33.3%	0.095		
General	50.6%	32.0%	0.089		
India	**52.9%**	**100%**	**0.100**	**0.098**	**0.002**
2006					
ST	56.3%	12.9%	0.115		
SC	52.6%	22.9%	0.098		
OBC	50.8%	42.1%	0.090		
General	49.7%	22.0%	0.092		
India	**51.7%**	**100%**	**0.097**	**0.096**	**0.0017**

Inequality among the poor fell for SC and OBC, but not for ST

What Happened Over Time?

1999	Intensity (MPI)	Share of Poor	Inequality (Poor)	Total Within group	Between Group
ST	57.0%	12.6%	0.110		
SC	55.0%	22.1%	0.107		
OBC	52.1%	33.3%	0.095		
General	50.6%	32.0%	0.089		
India	**52.9%**	**100%**	**0.100**	**0.098**	**0.002**
2006					
ST	56.3%	12.9%	0.115		
SC	52.6%	22.9%	0.098		
OBC	50.8%	42.1%	0.090		
General	49.7%	22.0%	0.092		
India	**51.7%**	**100%**	**0.097**	**0.096**	**0.0017**





Cross Country Comparisons

Two countries with similar MPI but similarly unequal

Country	Year	Headcount Ratio	MPI	Average Deprivation Count (Poor)	Inequality (Poor)
Colombia	2010	5.4%	0.022	40.9%	0.041
Lesotho	2009	35.3%	0.156	44.1%	0.042

Between Group Term

What does the between group term capture?

It captures disparity in *intensity* across population subgroups

It, however, does not capture disparity in poverty across subgroups

Disparity in Intensity vs. Disparity in Poverty

Between group inequality among poor is not sufficient for disparity between poverty across groups

- Horizontal Inequality (Stewart 2000)
- Sub-national Disparity (Alkire, Roche, Seth 2011)

Example:

$c = (0,0,0,6,6,6,6,6,7,7)$, $c_A = (0,0,6,6,7)$ and $c_B = (0,6,6,6,7)$

$\mathit{c} = (0,0,0,6,6,6,6,6,6,6)$, $\mathit{c}_A = (0,0,0,6,6)$ and $\mathit{c}_B = (6,6,6,6,6)$

Overall inequality, within group inequalities, between group inequalities among the poor – all lower in *c*'s than in c's

Disparity in poverty between subgroups?

Disparity in Intensity vs. Disparity in Poverty

In fact, when the poverty cut-off is one-fifth (Alkire and Seth 2013):

	Between Group Inequality (Poor)	**Disparity in Poverty (Castes)**
1999	0.040	0.192
2006	0.036	0.204

Contradicting changes

Cross Country Comparisons

Similar inequality among the poor but very different sub-national disparity

Country	Year	MPI	Inequality (Poor)	Total Within-Group	Between Group	Between MPI
Bolivia	2008	0.089	0.044	0.042	0.002	0.006
Zimbabwe	2011	0.172	0.045	0.044	0.001	0.021

Further Decomposition? How?

The poverty measures are based on the deprivation (*censored*) count vector $c = (c_1,\ldots,c_n)$

- Alkire and Foster (2011): $P(c) = (c_1 + \ldots + c_n)/n$ (Adj. HCR)
- Bossert *et al.* (2009): $P(c) = [(c_1^{\alpha} + \ldots + c_n^{\alpha})/n]^{1/\alpha}$
- Jayaraj and Subramanian: $P(c) = (c_1^{\alpha} + \ldots + c_n^{\alpha})/n$
- Rippin (2011): $P(c) = (c_1^{2} + \ldots + c_n^{2})/n$

Similar to Thon (1979), Clark, Hemming, and Ulph (1981), Chakravarty (1983), Shorrocks (1995), Xu and Osberg (2001) in single-dimensional context

Further Decomposition? How?

Notation:

H: Multidimensional Headcount Ratio

c^{ℓ} : Deprivation (censored) score vector of any subgroup ℓ

a^{ℓ}: Deprivation score vector of the poor in any subgroup ℓ

ν^{ℓ} : The population share of any subgroup ℓ

θ^{ℓ}: Share of poor in any subgroup ℓ

μ: The average all elements in x

$\mu(c^{\ell})$: M0 of any subgroup ℓ

$\mu(a^{\ell})$: Intensity of any subgroup ℓ

Further Decomposition? How?

Steps:

Step 1: Divide the entire population in to m subgroups

Step 2: Compute the within and between group inequality: the between group inequality is the <u>disparity in M_0</u>

Step 3: Divide further each subgroup into the group of poor and the group of non-poor

Step 4: Compute the total within group inequality and between group inequality: <u>the within group inequality among the non-poor is zero</u>

OPHI Oxford Poverty & Human Development Initiative

UNIVERSITY OF OXFORD

Further Decomposition

Decomposition:

$$\overbrace{\mu(a^{\ell})^2 H^{\ell}(1-H^{\ell})}$$

$$V(c) = \underbrace{V[\mu(c^1),\ldots,\mu(c^m)]}_{1} + H[\underbrace{\Sigma_{\ell}\, \theta^{\ell} V(a^{\ell})}_{2}] + \underbrace{\Sigma_{\ell}\, \nu^{\ell}\, V[\mu(a^{\ell}),0]}_{3}$$

1. Disparity in M_0's
2. Headcount *times* the overall within group inequality among the poor
3. Overall inequality between poor and the non-poor (less interesting for policy)

A Proposal

Use the measure:

$$\mathbb{V}(c) = V[\mu(c^1),\ldots,\mu(c^m)] + H[\Sigma_\ell\, \theta^\ell V(a^\ell)]$$

Disparity in subgroup M0

Total within-group inequality among the poor

OPHI Oxford Poverty & Human Development Initiative

UNIVERSITY OF OXFORD

Conclusion

We discuss the appropriate way of capturing inequality across the poor and proposed variance

Variance is invariant to whether we count deprivations or count achievements

Emphasize that consideration of between-group inequality is not enough to understand group disparity in poverty

Computing in STATA

- Use the censored deprivation score vector $c_i(k)$
- Inequality among the poor: Use vector $c_i(k)$ and the intensity of each subgroup to compute the inequality among the poor: $4\Sigma_i w_i(c_i^{\ell} - A^{\ell})^2/(\Sigma_i w_i)$
  - Only among the poor for each subgroup
- Disparity in M_0: Compute the subgroup M_0 and then use the overall M_0 to compute the variance