Counting and Multidimensional Poverty Measurement

by

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QEH
31 May 2007

Why Multidimensional Poverty?

- Appealing conceptual framework
  - Capabilities
- Data availability
- Tools
  - Poverty measures
  - Poverty orderings
    - Extended from single dimensional approach

Problems

- Poverty of what
  - Which dimensions and variables
  - How to deal with ordinal variables
  - How to make variables commensurate
- Aggregation
  - Interpretation
  - Properties
- Identification
  - Extremes: Union or intersection
    - What's in between?

Outline

- Motivation and Review
- Identification method
- Adjusted headcount
  - Needs only ordinal variables
  - Useful axiomatic justification
- Family based on FGT
  - Intuitive extension
- Empirical application
- General method and extensions

Hypothetical Challenge

- A government would like to create an official multidimensional poverty indicator
- Desiderata
  - It must understandable and easy to describe
  - It must conform to “common sense” notions of poverty
  - It must fit the purpose for which it is being developed
  - It must be technically solid
  - It must be operationally viable
  - It must be easily replicable
- What would you advise?

Not Exactly Hypothetical

- Mexican Government
  - Must alter official poverty methods
  - Include six other dimensions
- Summer 2007
  - Draft of paper
- August 2007
  - Present proposed methodology
- Question: What to advise?

Multidimensional Poverty Strategy

Twin cutoffs

Poverty line for each domain

Bourguignon and Chakravarty, *JEI*, 2003

"a multidimensional approach to poverty defines poverty as a shortfall from a threshold on each dimension of an individual's well being."

Cutoff in terms of numbers of dimensions

Ex: UNICEF, *Child Poverty Report*, 2003

-Two or more deprivations

Ex: Mack and Lansley, *Poor Britain*, 1985

-Three or more out of 26

Focus on P_α family - general case later

Note No weighting

"we have no reliable basis for doing [otherwise]" Mayer and Jencks, 1989

- will relax later

Needs cardinal variable

- relaxed for P_0

Review: Some Income Poverty Measures

Single variable, e.g., consumption, income

Sen (1976) two steps

Identification step "who is poor?"

- Typically use poverty line
  - Absolute, meaning unchanging over time
  - Cutoff is always somewhat arbitrary

Aggregation step "which overall indicator?"

- Headcount ratio P_0 = percentage poor
  - Example: Incomes = (7,3,4,8) poverty line z = 5
  - Who's poor? $g^0 = (0,1,1,0)$
  - Headcount $P_0 = \mu(g^0) = 2/4$
  - Example: (7,3,3,8) No change!

Review: Some Income Poverty Measures

Per capita poverty gap P_1

Example: incomes = (7,3,4,8) poverty line z = 5

Normalized gaps = g^1 = (0, 2/5, 1/5, 0)

Poverty gap = $P_1 = \mu(g^1)$ = 3/20

Example: (7,3,3,8) P_1 = 4/20 (sensitive to decrements)

However: (7,2,4,8) P_1 = 4/20 (insensitive to inequality)

FGT P_2

Example: incomes = (7,3,3,8) poverty line z = 5

Squared Normalized gaps = g^2 = (0, 4/25, 4/25, 0)

FGT = $P_2 = \mu(g^2)$ = 8/100

Example: (7,2,4,8)

Squared Normalized gaps = g^2 = (0, 9/25, 1/25, 0)

P_1 = 10/100 (sensitive to inequality)

Will use to construct multidimensonal poverty measures.

Data

Matrix of well-being scores in D domains for N persons

Persons

$$y = \begin{bmatrix} \mathbf{13.1} & \mathbf{15.2} & \mathbf{12.5} & \mathbf{20.0} \\ \mathbf{14} & \mathbf{7} & \mathbf{10} & \mathbf{11} \\ \mathbf{4} & \mathbf{5} & \mathbf{1} & \mathbf{3} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Domains

Data

Matrix of well-being scores in J domains for N persons

$$
y = \begin{bmatrix} \mathbf{13.1} & \mathbf{15.2} & \mathbf{12.5} & \mathbf{20.0} \\ \mathbf{14} & \mathbf{7} & \mathbf{10} & \mathbf{11} \\ \mathbf{4} & \mathbf{5} & \mathbf{1} & \mathbf{3} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \quad z = \begin{matrix} \mathbf{13} \\ \mathbf{12} \\ \mathbf{3} \\ \mathbf{1} \end{matrix}
$$

Persons (columns); Domains (rows)

Domain specific cutoffs

Data

Matrix of well-being scores in D domains for N persons

	Persons				z	
y =	***13.1***	***15.2***	**12.5**	***20.0***	**13**	Domains
	14	**7**	**10**	**11**	**12**	
	4	***5***	**1**	***3***	**3**	
	1	**0**	**0**	***1***	**1**	

Domain specific cutoffs

These entries achieve target cutoffs

Data

Matrix of well-being scores in several domains for N persons

$$
\begin{array}{cc}
 & \text{Persons} \qquad\qquad \text{z} \\
y = & \begin{bmatrix} \boldsymbol{13.1} & \boldsymbol{15.2} & \mathbf{12.5} & \boldsymbol{20.0} \\ \boldsymbol{14} & \mathbf{7} & \mathbf{10} & \mathbf{11} \\ \boldsymbol{4} & \boldsymbol{5} & \mathbf{1} & \boldsymbol{3} \\ \boldsymbol{1} & \mathbf{0} & \mathbf{0} & \boldsymbol{1} \end{bmatrix} \begin{array}{c} \mathbf{13} \\ \mathbf{12} \\ \mathbf{3} \\ \mathbf{1} \end{array} \quad \text{Domains}
\end{array}
$$

Domain specific cutoffs

These entries achieve target cutoffs

These entries do not

Normalized Gaps

Persons / z / Domains

$$y = \begin{bmatrix} \mathit{13.1} & \mathit{15.2} & 12.5 & \mathit{20.0} \\ \mathit{14} & 7 & 10 & 11 \\ \mathit{4} & \mathit{5} & 1 & \mathit{3} \\ \mathit{1} & 0 & 0 & \mathit{1} \end{bmatrix} \quad z = \begin{matrix} 13 \\ 12 \\ 3 \\ 1 \end{matrix}$$

Replace these entries with 0

Replace these with normalized gap $(z_j - y_{ji})/z_j$

Normalized Gaps

$$
\begin{array}{c}
\text{Persons} \qquad\qquad\qquad z \\
g^{1} = \left[\begin{array}{cccc|c}
\mathit{0} & \mathit{0} & \mathbf{0.04} & \mathit{0} & \mathbf{13} \\
\mathit{0} & \mathbf{0.42} & \mathbf{0.17} & \mathbf{0.08} & \mathbf{12} \\
\mathit{0} & \mathit{0} & \mathbf{0.67} & \mathit{0} & \mathbf{3} \\
\mathit{0} & \mathbf{1} & \mathbf{1} & \mathit{0} & \mathbf{1}
\end{array}\right] \quad \text{Domains}
\end{array}
$$

Replace these entries with 0

Replace these with normalized gap $(z_j - y_{ji})/z_j$

Normalized Gaps

Persons

$$g^1 = \begin{bmatrix} \mathit{0} & \mathit{0} & \mathbf{0.04} & \mathit{0} \\ \mathit{0} & \mathbf{0.42} & \mathbf{0.17} & \mathbf{0.08} \\ \mathit{0} & \mathit{0} & \mathbf{0.67} & \mathit{0} \\ \mathit{0} & \mathbf{1} & \mathbf{1} & \mathit{0} \end{bmatrix}$$

Domains

Replace these entries with 0

Replace these with normalized gap $(z_j - y_{ji})/z_j$

Deprivation Counts

Persons

$$
g^0 = \begin{bmatrix} \boldsymbol{\mathit{0}} & \boldsymbol{\mathit{0}} & \mathbf{1} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \boldsymbol{\mathit{0}} & \boldsymbol{\mathit{0}} & \mathbf{1} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \mathbf{1} & \mathbf{1} & \boldsymbol{\mathit{0}} \end{bmatrix}
$$

Domains

Replace these entries with 0

Replace these entries with 1

Deprivation Counts

Persons

$$g^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Domains

Counts

$c = (0,\ 2,\ 4,\ 1)$

= number of deprivations

Identification

Q/Who is poor?

Persons

$$g^0 = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

Counts

$c = (0, 2, 4, 1)$

= number of deprivations

Identification: Union

Q/Who is poor?

A/ Poor if deprived in at least one dimension ($\mathbf{c_i \geq 1}$)

Persons

$$g^0 = \begin{bmatrix} \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \end{bmatrix}$$

Domains

Counts

$c = (0, \mathbf{2}, \mathbf{4}, \mathbf{1})$

= number of deprivations

Identification: Union

Q/Who is poor?

A/ Poor if deprived in at least one dimension $(c_i \geq 1)$

Persons

$$g^0 = \begin{bmatrix} \mathit{0} & \mathit{0} & \mathbf{1} & \mathit{0} \\ \mathit{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathit{0} & \mathit{0} & \mathbf{1} & \mathit{0} \\ \mathit{0} & \mathbf{1} & \mathbf{1} & \mathit{0} \end{bmatrix}$$

Domains

Counts

$$c = (0,\ \mathbf{2},\ \mathbf{4},\ \mathbf{1})$$

= number of deprivations

Difficulties

Single deprivation may be due to something other than poverty (UNICEF)

Union approach often predicts *very* high numbers - political constraints.

Identification

Q/Who is poor?

Persons

$$g^{0} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

$$c = (0,\ 2,\ 4,\ 1)$$

Identification: Intersection

Q/Who is poor?

A/ Poor if deprived in all dimensions ($c_i \geq 4$)

Persons

$$g^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Domains

$$c = (0,\ 2,\ \mathbf{4},\ 1)$$

Identification: Intersection

Q/Who is poor?

A/ Poor if deprived in all dimensions ($c_i \geq 4$)

Persons

$$g^0 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

Domains

$$c = (0, 2, \mathbf{4}, 1)$$

Difficulty: Demanding requirement (especially if J large)
Often identifies a very narrow slice of population

Identification

Q/Who is poor?

Persons

$$g^0 = \begin{bmatrix} \boldsymbol{\mathit{0}} & \boldsymbol{\mathit{0}} & \mathbf{1} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \boldsymbol{\mathit{0}} & \boldsymbol{\mathit{0}} & \mathbf{1} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \mathbf{1} & \mathbf{1} & \boldsymbol{\mathit{0}} \end{bmatrix}$$

Domains

$$c = (0,\ 2,\ 4,\ 1)$$

Counting Based Identification

Q/Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$

Persons

$$g^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Domains

$$c = (0,\ 2,\ 4,\ 1)$$

Counting Based Identification

Q/Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$

Persons

$$g^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Domains

$$c = (0, \ \mathbf{2}, \ \mathbf{4}, \ 1)$$

Example: 2 out of 4

Counting Based Identification

Q/Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$

Persons

$$g^0 = \begin{bmatrix} \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \end{bmatrix}$$

Domains

$$c = (0,\ \mathbf{2},\ \mathbf{4},\ 1)$$

Example: 2 out of 4

Note: Especially useful when number of dimensions is large

Union becomes too large, intersection too small

Counting Based Identification

Implementation method: Censor nonpoor data

Persons

$$g^{0} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

$$c = (0,\ \mathbf{2},\ \mathbf{4},\ 1)$$

Counting Based Identification

Implementation method: Censor nonpoor data

Persons

$$g^{0}(k) = \begin{bmatrix} \mathit{0} & \mathit{0} & \mathbf{1} & \mathit{0} \\ \mathit{0} & \mathbf{1} & \mathbf{1} & \mathit{0} \\ \mathit{0} & \mathit{0} & \mathbf{1} & \mathit{0} \\ \mathit{0} & \mathbf{1} & \mathbf{1} & \mathit{0} \end{bmatrix}$$

Domains

$$c(k) = (0, \mathbf{2}, \mathbf{4}, 0)$$

Counting Based Identification

Implementation method: Censor nonpoor data

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Domains

$$g^{\mathbf{0}}(k) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

$$c(k) = (0,\ \mathbf{2},\ \mathbf{4},\ 0)$$

Similarly for $y(k)$, $g^1(k)$, etc

Counting Based Identification

Implementation method: Censor nonpoor data

Persons

$$g^{0}(k) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Domains

$$c(k) = (0,\ \mathbf{2},\ \mathbf{4},\ 0)$$

Similarly for $y(k)$, $g^{1}(k)$, etc
Note: Includes both union and intersection

Counting Based Identification

Implementation method: Censor nonpoor data

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Domains

$$c(k) = (0, \mathbf{2}, \mathbf{4}, 0)$$

Similarly for $y(k)$, $g^1(k)$, etc

Note: Includes both union and intersection

Next: Turn to aggregation

Headcount Ratio

Persons

$$g^{0}(k)=\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

$$c(k) = (0, \mathbf{2}, \mathbf{4}, 0)$$

Dimension cutoff $k = 2$

Headcount ratio $H = 1/2$

Critique

Suppose the number of deprivations rises for person 2

Persons

$$g^{0}(k) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

$$c(k) = (0, \mathbf{2}, \mathbf{4}, 0)$$

Dimension cutoff $k = 2$

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$$g^0(k) = \begin{bmatrix} \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \end{bmatrix}$$

Domains

$$c(k) = (0, \mathbf{3}, \mathbf{4}, 0)$$

Dimension cutoff $k = 2$

Headcount ratio **H = 1/2** No change!

Violates dim. monotonicity

Adjusted Headcount Ratio

Return to original matrix

Persons

$$g^{0}(k) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

$$c(k) = (0, \mathbf{2}, \mathbf{4}, 0)$$

Adjusted Headcount Ratio

Need to augment information of H

Persons

$$g^{\mathbf{0}}(k)=\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

$$c(k) = (0,\ \mathbf{2},\ \mathbf{4},\ 0)$$

Adjusted Headcount Ratio

Need to augment information of H

Persons

$$g^{0}(k)=\begin{bmatrix} \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \end{bmatrix}$$

Domains

$$c(k) = (0, \mathbf{2}, \mathbf{4}, 0)$$

shares of deprivations (0, 1/2, 1, 0)

Adjusted Headcount Ratio

Need to augment information of H

Persons

$$g^{0}(k) = \begin{bmatrix} \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \end{bmatrix}$$

Domains

$c(k) = (0, \mathbf{2}, \mathbf{4}, 0)$

shares of deprivations (0, 1/2, 1, 0)

Average deprivation share among poor

A = 3/4

Adjusted Headcount Ratio

Adjusted headcount ratio = D_0 = HA

Persons

$$g^0(k) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Domains

c(k) = (0, **2**, **4**, 0)

shares of deprivations (0, 1/2, 1, 0)

Average deprivation share among poor

A = 3/4

Adjusted Headcount Ratio

Adjusted headcount ratio = D_0 = HA = $\mu(g^0(k))$

Persons

$$g^0(k) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Domains

c(k) = (0, **2**, **4**, 0)

shares of deprivations (0, 1/2, 1, 0)

Average deprivation share among poor

A = 3/4

Adjusted Headcount Ratio

Adjusted headcount ratio = D_0 = HA = $\boldsymbol{\mu(g^0(k))}$ **= 6/16 = .375**

Persons

$$g^0(k) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

c(k) = (0, **2**, **4**, 0)

shares of deprivations (0, **1/2**, **1**, 0)

Average deprivation share among poor

A = 3/4

Adjusted Headcount Ratio

Adjusted headcount ratio = D_0 = HA = $\boldsymbol{\mu(g^0(k))}$ **= 6/16 = .375**

Obviously if person 2 has an additional deprivation, D_0 rises

Persons

$$g^0(k) = \begin{bmatrix} \mathit{0} & \mathit{0} & \mathbf{1} & \mathit{0} \\ \mathit{0} & \mathbf{1} & \mathbf{1} & \mathit{0} \\ \mathit{0} & \mathit{0} & \mathbf{1} & \mathit{0} \\ \mathit{0} & \mathbf{1} & \mathbf{1} & \mathit{0} \end{bmatrix}$$

Domains

c(k) = (0, **2**, **4**, 0)

shares of deprivations (0, **1/2**, **1**, 0)

Average deprivation share among poor

A = 3/4

Adjusted Headcount Ratio

Adjusted headcount ratio = D_0 = HA = $\boldsymbol{\mu(g^0(\tau))}$ **= 6/16 = .375**

Obviously if person 2 has an additional deprivation, D_0 rises Dim. Mon.

Persons

$$g^0(k) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

c(k) = (0, **2**, **4**, 0)

shares of deprivations (0, **1/2**, **1**, 0)

Average deprivation share among poor

A = 3/4

Adjusted Headcount Ratio

Observations

Uses ordinal data

Transform variable and poverty line: D_0 unchanged

D_0 is "meaningful" in the sense of Roberts, *Measurement Theory*, 1979

Works with: Self reported health, years of schooling and income

Similar to traditional gap $P_1 = HI$

HI = per capita poverty gap = total income gap of poor/total population

HA = per capita deprivation = total deprivations of poor/total population

Adjusted Headcount Ratio

Observations

For k = 1 (union approach)

$$D_0 = (\sum_j j\ H_j)/J$$

Also used by Brandolini and D'Alessio (1998)

Link with Human Poverty Index

$$D_0 = \mu(g^0(k)) \leq HPI \leq \mu_3(g^0(k))$$

But similar values!

Satisfies several typical properties of multidimensional poverty

Symmetry, Replication invariance, Weak monotonicity, Scale invariance

Normalization, Decomposability,

And the new one: Dimension monotonicity

Adjusted Headcount Ratio

Axiomatic Treatment

Note $D_0 = \sum_i p(v_i)/N$

where v_i is i's deprivation vector, and i's individual deprivation function is $p(v_i) = 0$ for $|v_i| < k$ and $p(v_i) = k$ for $|v_i| \geq k$

Q/ Why this functional form for p?

A/ Suppose f satisfies

1) **Weak monotonicity**: $f(v') \geq f(v)$ if $v' = v + e_m$

   Individual deprivation does not fall if increase dimensions of deprivation

2) **Semi-consistency**: $f(v) \geq f(v')$ implies $f(w) \geq f(w')$

   whenever $v - w = v' - w' = e_m$

   Ordering preserved if remove same deprivation from both vectors.

3) **Simple anonymity**: $f(v) = f(v')$ for all v, v' with exactly J-1 deprivations   All deprivation vectors with one achievement ranked equally.

Th: If f satisfies (1) - (3), then f is some increasing function of p.

Prf: Analogous to Pattanaik and Xu (1990)

Adjusted Headcount Ratio

Adjusted headcount = D_0 = HA = $\mu(g^0(k))$

Persons

$$g^{\mathbf{0}}(k) = \begin{bmatrix} \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \end{bmatrix}$$

Domains

Assume cardinal variables

Adjusted Headcount Ratio

Adjusted headcount = D_0 = HA = $\mu(g^0(k))$

Persons

$$g^{\mathbf{0}}(k) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

Assume cardinal variables

Q/ What happens when a poor person who is deprived in dimension j becomes even more deprived?

Adjusted Headcount Ratio

Adjusted headcount = D_0 = HA = $\mu(g^0(k))$

Persons

$$g^0(k) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Domains

Assume cardinal variables

Q/ What happens when a poor person who is deprived in dimension j becomes even more deprived?

A/ Nothing. D_0 is unchanged. **Violates monotonicity**.

Adjusted Headcount Ratio

Need to augment the information of D_0

Persons

$$g^{\mathbf{0}}(k) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{1} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

Adjusted Poverty Gap

Return to normalized gaps

Persons

$$g^1(k) = \begin{bmatrix} \boldsymbol{\mathit{0}} & \boldsymbol{\mathit{0}} & \mathbf{0.04} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \mathbf{0.42} & \mathbf{0.17} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \boldsymbol{\mathit{0}} & \mathbf{0.67} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \mathbf{1} & \mathbf{1} & \boldsymbol{\mathit{0}} \end{bmatrix}$$

Domains

Adjusted Poverty Gap

Return to normalized gaps

Persons

$$g^1(k) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \mathbf{0.04} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{0.42} & \mathbf{0.17} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \mathbf{0.67} & \boldsymbol{0} \\ \boldsymbol{0} & \mathbf{1} & \mathbf{1} & \boldsymbol{0} \end{bmatrix}$$

Domains

Average **gap** across all deprived dimensions of the poor:

$G(k) = (0.04+0.42+0.17+0.67+1+1)/6$

Adjusted Poverty Gap

Adjusted Poverty Gap = D_1 = $\mathbf{D_0 G}$ = HAG

Persons

$$g^1(k) = \begin{bmatrix} \boldsymbol{\mathit{0}} & \boldsymbol{\mathit{0}} & \mathbf{0.04} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \mathbf{0.42} & \mathbf{0.17} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \boldsymbol{\mathit{0}} & \mathbf{0.67} & \boldsymbol{\mathit{0}} \\ \boldsymbol{\mathit{0}} & \mathbf{1} & \mathbf{1} & \boldsymbol{\mathit{0}} \end{bmatrix}$$

Domains

Average **gap** across all deprived dimensions of the poor:

$G(k) = (0.04+0.42+0.17+0.67+1+1)/6$

Adjusted Poverty Gap

Adjusted Poverty Gap = D_1 = $\mathbf{D_0 G}$ = HAG = $\boldsymbol{\mu(g^1(k))}$

Persons

$$g^1(k) = \begin{bmatrix} \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{0.04} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{0.42} & \mathbf{0.17} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{\mathit{0}} & \mathbf{0.67} & \mathbf{\mathit{0}} \\ \mathbf{\mathit{0}} & \mathbf{1} & \mathbf{1} & \mathbf{\mathit{0}} \end{bmatrix}$$

Domains

Average **gap** across all deprived dimensions of the poor:

$$G(k) = (0.04+0.42+0.17+0.67+1+1)/6$$

Adjusted Poverty Gap

Adjusted Poverty Gap = D_1 = $\mathbf{D_0 G}$ = HAG = $\boldsymbol{\mu(g^1(k))}$

Persons

$$g^1(k) = \begin{bmatrix} \mathit{0} & \mathit{0} & \mathbf{0.04} & \mathit{0} \\ \mathit{0} & \mathbf{0.42} & \mathbf{0.17} & \mathit{0} \\ \mathit{0} & \mathit{0} & \mathbf{0.67} & \mathit{0} \\ \mathit{0} & \mathbf{1} & \mathbf{1} & \mathit{0} \end{bmatrix}$$

Domains

Obviously, if in a deprived dimension, a poor person becomes even more deprived, then D_1 will rise.

Adjusted Poverty Gap

Adjusted Poverty Gap = D_1 = $\mathbf{D_0G}$ = HAG = **$\mu(g^1(k))$**

Persons

$$g^1(k) = \begin{bmatrix} \mathit{0} & \mathit{0} & \mathbf{0.04} & \mathit{0} \\ \mathit{0} & \mathbf{0.42} & \mathbf{0.17} & \mathit{0} \\ \mathit{0} & \mathit{0} & \mathbf{0.67} & \mathit{0} \\ \mathit{0} & \mathbf{1} & \mathbf{1} & \mathit{0} \end{bmatrix}$$

Domains

Obviously, if in a deprived dimension, a poor person becomes even more deprived, then D_1 will rise.

Satisfies monotonicity

Adjusted FGT

Adjusted Poverty Gap = D_1 = $\mathbf{D_0G}$ = HAG = $\mu(g^1(k))$

Persons

$$g^1(k) = \begin{bmatrix} 0 & 0 & 0.04 & 0 \\ 0 & 0.42 & 0.17 & 0 \\ 0 & 0 & 0.67 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Domains

An increase in deprivation has the same impact no matter the size of the initial deprivation

Adjusted FGT

Consider the matrix of alpha powers of normalized shortfalls

Persons

$$
g^{1}(k)=\begin{bmatrix} 0 & 0 & 0.04 & 0 \\ 0 & 0.42 & 0.17 & 0 \\ 0 & 0 & 0.67 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}
$$

Domains

Adjusted FGT

Consider the matrix of alpha powers of normalized shortfalls

Persons

$$g^{\alpha}(k) = \begin{bmatrix} \boldsymbol{0}^{\alpha} & \boldsymbol{0}^{\alpha} & \mathbf{0.04}^{\alpha} & \boldsymbol{0}^{\alpha} \\ \boldsymbol{0}^{\alpha} & \mathbf{0.42}^{\alpha} & \mathbf{0.17}^{\alpha} & \boldsymbol{0}^{\alpha} \\ \boldsymbol{0}^{\alpha} & \boldsymbol{0}^{\alpha} & \mathbf{0.67}^{\alpha} & \boldsymbol{0}^{\alpha} \\ \boldsymbol{0}^{\alpha} & \mathbf{1}^{\alpha} & \mathbf{1}^{\alpha} & \boldsymbol{0}^{\alpha} \end{bmatrix}$$

Domains

Adjusted FGT

Adjusted FGT is $D_\alpha = \boldsymbol{\mu(g^\alpha(\tau))}$ for $\alpha \geq 0$

Persons

$$g^\alpha(k) = \begin{bmatrix} \mathbf{0}^\alpha & \mathbf{0}^\alpha & \mathbf{0.04}^\alpha & \mathbf{0}^\alpha \\ \mathbf{0}^\alpha & \mathbf{0.42}^\alpha & \mathbf{0.17}^\alpha & \mathbf{0}^\alpha \\ \mathbf{0}^\alpha & \mathbf{0}^\alpha & \mathbf{0.67}^\alpha & \mathbf{0}^\alpha \\ \mathbf{0}^\alpha & \mathbf{1}^\alpha & \mathbf{1}^\alpha & \mathbf{0}^\alpha \end{bmatrix}$$

Domains

Adjusted FGT

Adjusted FGT is $D_\alpha = \mu(g^\alpha(\tau))$ for $\alpha \geq 0$

Persons

$$g^\alpha(k) = \begin{bmatrix} 0^\alpha & 0^\alpha & 0.04^\alpha & 0^\alpha \\ 0^\alpha & 0.42^\alpha & 0.17^\alpha & 0^\alpha \\ 0^\alpha & 0^\alpha & 0.67^\alpha & 0^\alpha \\ 0^\alpha & 1^\alpha & 1^\alpha & 0^\alpha \end{bmatrix}$$

Domains

Satisfies numerous properties including decomposability, and dimension monotonicity, monotonicity (for $\alpha > 0$), transfer (for $\alpha > 1$).

Illustration: USA

- **Data Source:** National Health Interview Survey, 2004, *United States Department of Health and Human Services. National Center for Health Statistics* - ICPSR 4349.
- **Tables Generated By:** Suman Seth.
- **Unit of Analysis:** Individual.
- **Number of Observations:** 46009.
- **Variables Used:**

  Income - Ratio of family income to poverty threshold

  Education – Highest level of school completed

  Health – Reported health status
- **Poverty Threshold:**

  Income Poor: 12.1% if *Income* < *1* (below threshold),

  Education Poor: 18.6% if *Education* < *GED/High School*

  Education Poor: 12.8% if *Health = Fair or Poor*

Example

- Headcount Ratio

% of individuals poor in 1 or more dimensions **(Union Approach)**	% of households poor in 2 or more dimensions **(Intermediate App.)**	% of households poor in 3 or more dimensions **(Intersection App.)**
31.55%	10.07%	1.86%

Example

- D_0

Of those who are poor 1 or more dimensions	Of those who are poor 2 or more dimensions	Of those who are poor 3 or more dimensions
0.1449	0.0733	0.0186

- D_1

Poverty Gap of…		
Those who are poor one or more dimensions	Those who are poor two or more dimensions	Those who are poor three or more dimensions
0.0561	0.0292	0.0076

- D_2

Squared Poverty Gap of…		
Those who are poor one or more dimensions	Those who are poor two or more dimensions	Those who are poor three or more dimensions
0.0287	0.0152	0.0041

- HPI Equivalent

Of those who are poor 1 or more dimensions	Of those who are poor 2 or more dimensions	Of those who are poor 3 or more dimensions
0.1507	0.0743	0.0186

Example

Head Counts

Number of deprivations	USA	Hispanic	Non-Hispanic
0	69	44	75
1	21	34	18
2	8	18	6
3	2	4	1
Total	100	100	100

Example

Crude Head Count Measure Decomposition

Ethnicity	Freq.	Poor in one or more dimensions	Poor in two or more dimensions	Poor in three or more dimensions
Hispanic	9140	.560 (35%)	.217 (43%)	.038 (40%)
Non-Hispanic	36869	.255 (65%)	.072 (57%)	.014 (60%)
Overall Poverty Rate	46009	.316 (100%)	.101 (100%)	.019 (100%)

D_0 Measure Decomposition

Ethnicity	Freq.	Poor in one or more dimensions	Poor in two or more dimensions	Poor in three or more dimensions
Hispanic	9140	.272 (37%)	.157 (43%)	.038 (40%)
Non-Hispanic	36869	.114 (63%)	.052 (57%)	.014 (60%)
Overall Poverty Rate	46009	.145 (100)	.073 (100)	.019 (100)

Example

D_1 Measure Decomposition

Ethnicity	Freq.	Poor in one or more dimensions	Poor in two or more dimensions	Poor in three or more dimensions
Hispanic	9140	.113 (40%)	.067 (45%)	.017 (46%)
Non-Hispanic	36869	.042 (60%)	.020 (55%)	.005 (54%)
Overall Poverty Rate	46009	.056 (100%)	.029 (100%)	.008 (100%)

D_2 Measure Decomposition

Ethnicity	Freq.	Poor in one or more dimensions	Poor in two or more dimensions	Poor in three or more dimensions
Hispanic	9140	.061 (42%)	.037 (48%)	.010 (52%)
Non-Hispanic	36869	.021 (58%)	.010 (52%)	.003 (48%)
Overall Poverty Rate	46009	.029 (100%)	.015 (100%)	.004 (100%)

Extension

- General application of identification strategy
  - Derive censored matrix y*(k)
    - Replace all nonpoor entries with poverty cutoffs
  - Apply any multidimensional measure
    - P(y*(k);z)
  - Straightforward transformation of existing technology
  - Preserves key axioms, slightly redefined

Extension

- Modifying for weights
  - Weighted identification
    - Weight on income: 50%
    - Weight on education, health: 25%
  - Cutoff = 0.50
    - Poor if income poor, or suffer two or more deprivations
  - Cutoff = 0.60
    - Poor if income poor and suffer one or more other deprivations
      - Nolan, Brian and Christopher T. Whelan, Resources, Deprivation and Poverty, 1996
  - Weighted aggregation

Review Challenge

- Desiderata
  - It must understandable and easy to describe
  - It must conform to “common sense” notions of poverty
  - It must fit the purpose for which it is being developed
  - It must be technically solid
  - It must be operationally viable
  - It must be easily replicable

Thanks for your attention