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UNIVERSITY OF  
OXFORD

# Summer School on Multidimensional Poverty Analysis

3–15 August 2015

Georgetown University, Washington, DC, USA

*Tabita, Kenya*



*Rabiya, India*



*Stephanie, Madagascar*



*Agatha, Madagascar*



*Dalma, Kenya*



*Ann-Sasha, Kenya*



*Valérie, Madagascar*



# Multidimensional Poverty Measurement Methodologies and Counting Approaches

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# Aim of this Session

Bring diverse methodologies into common framework

Clarify each methodology, data requirements, assumptions, and key choices in measurement design

Evaluate pros and cons of each approach

Enable readers to make informed choices about which approach best addresses a given problem

# Framework

**Achievement:** performance of a person in a dimension

- $x_{ij}$ : Achievement of person  $i$  ( $=1, \dots, n$ ) in dimension  $j$  ( $=1, \dots, d$ )

## Achievement matrix

- Summarizes achievements of all  $n$  persons in  $d$  dimensions

$$X = \begin{array}{c} \text{Dimensions} \\ \left[ \begin{array}{ccc} x_{11} & \dots & x_{1d} \\ x_{21} & \dots & x_{2d} \\ \vdots & \ddots & \vdots \\ x_{n1} & \dots & x_{nd} \end{array} \right] \text{Persons}$$

## Achievement vector of a Person

- May contain achievements in  $d$  different dimensions
  - Standard of living, knowledge, quality of health

# Multidimensional Measurement

A typical dataset may look like (*with 4 dimensions and 4 persons*)

<b>Income</b>	<b>Years of Education</b>	<b>Sanitation (Improved?)</b>	<b>Access to Electricity</b>	
700	14	Yes	Yes	<b>Person 1</b>
300	13	Yes	No	<b>Person 2</b>
400	10	No	No	<b>Person 3</b>
800	11	Yes	Yes	<b>Person 4</b>

# Framework

Matrix  $X$  summarizes the joint distribution of ' $d$ ' dimensions across ' $n$ ' individuals

Row vector  $x_{i\bullet} = (x_{i1}, \dots, x_{id})$  summarizes the achievements of person  $i$  in all  $d$  dimensions

Column vector  $x_{\bullet j} = (x_{1j}, \dots, x_{nj})$  summarizes the achievements in dimension  $j$  of all  $n$  persons

# Multidimensional Measurement

Measurement of multidimensional poverty generally involves three major steps

- **Choice of space:** How should we define poverty?
- **Identification:** Who is poor?
- **Aggregation:** How should the information of all the poor be aggregated to obtain an index?

# Methodologies

- Dashboard Approach
- Composite Indices



These Marginal methods use aggregate data

- Venn Diagrams
- Dominance Approach
- Statistical Approaches
- Fuzzy Sets Approach
- Axiomatic Approach



These Methodologies use micro data and reflect the joint distribution of deprivations



# Dashboard Approach

A set of dimensions (indicators), which applies “a unidimensional measure to *each* dimension”

(Alkire, Foster, Santos, 2011)

## Applications

- Basic Needs Approach
  - As a first step, it might be useful to define the best indicator for each basic need...” (Hicks & Streeten, 1979)
- Millennium Development Goals (UN, 2000)

Examples: Percent of malnourished children, Infant Mortality Rate, Illiteracy rate

# Dashboard Approach

Let

- $n_j$  denote the population size covered by dimension  $j$ 
  - Note  $n_j$ 's are not same across all  $j$
- $x^j$  denote the vector summarizing achievements of all  $n_j$  people
- $z_j$  denote the deprivation cut-off of dimension  $j$

Then  $P_j(x^j; z_j)$  is the Deprivation index for dimension  $j$

Dashboard of indicators (DI): A  $d$ -dimensional vector containing  $d$  deprivation indices.

- Hence, technically,  $DI = [P_1(x^1; z_1), \dots, P_d(x^d; z_d)]$

# Dashboard Approach: Pros and Cons

## Advantages

- Shares information on **many dimensions** of poverty
- Can draw on **different data sources**
- Can show information on **disjoint populations**

## Disadvantages

- Lack of hierarchies amongst the indicators
- Lack a single **headline figure** (such as GDP)
- Leave the questions about **tradeoffs** completely open
- Does not **identify** who is poor
- Ignores **joint distribution** even when could reflect it

# Composite Indices

While assessing quality-of-life requires a plurality of indicators, there are strong demands to develop a single summary measure (Stiglitz, Sen, and Fitoussi, 2009)

The  $d$  deprivation indices  $P_j(x^j; z_j)$  are aggregated to obtain the composite index (CI):

$$CI = f(P_1(x^1; z_1), \dots, P_d(x^d; z_d)).$$

Further discussion on CIs: Nardo et al. (2005), Bandura (2008), Alkire and Sarwar (2009), and Santos and Santos (2013)

# Composite Indices: Example

## The Human Poverty Index (HPI-1)

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Dimension	Indicator
Survival deprivation	$P_1$ : Probability at birth of not surviving to age 40.
Education deprivation	$P_2$ : Adult illiteracy rate
Economic deprivation	$P_3$ : Equally weighted average of: <ul style="list-style-type: none"><li>• % of population without access to an improved water source</li><li>• % of children under weight for age</li></ul>

---

$$\text{HPI-1} = \left\{ \left[ (P_1)^3 + (P_2)^3 + (P_3)^3 \right] / 3 \right\}^{1/3}$$

# Composite Indices: Pros and Cons

## Advantages

- Provide a **summary** measure, useful for comparisons, ordering
- Can combine **different data sources**
- Can combine information on **disjoint populations**
- Can draw on **normalized indices**
- Offer a hierarchy and make **trade offs** explicit (see Ravallion 2011)

## Disadvantages

- Implicit or no **Identification**
- Ignore **joint distribution** even when possible to capture

# Joint Distribution

- Where  $x_{ij}$  is the achievement of person  $i$  in dimension  $j$
- $z_j$  is the deprivation cutoff of attribute or dimension  $j$

Dimensions

$$X = \begin{bmatrix} x_{11} & \dots & x_{1d} \\ x_{21} & \dots & x_{2d} \\ \vdots & \ddots & \vdots \\ x_{n1} & \dots & x_{nd} \end{bmatrix}$$

Persons

$$z = (z_1, \dots, z_d)$$

Marginal Distribution  
(without reference to  
other dimensions)

# Joint Distribution

Same dimensional distributions but are they the same?

## Distribution I

## Distribution II

	Income	Education	Shelter	Water
1.	D	ND	ND	ND
2.	ND	D	ND	ND
3.	ND	ND	D	ND
4.	ND	ND	ND	D

	Income	Education	Shelter	Water
1.	ND	ND	ND	ND
2.	ND	ND	ND	ND
3.	ND	ND	ND	ND
4.	D	D	D	D

ND: Not Deprived

D: Deprived

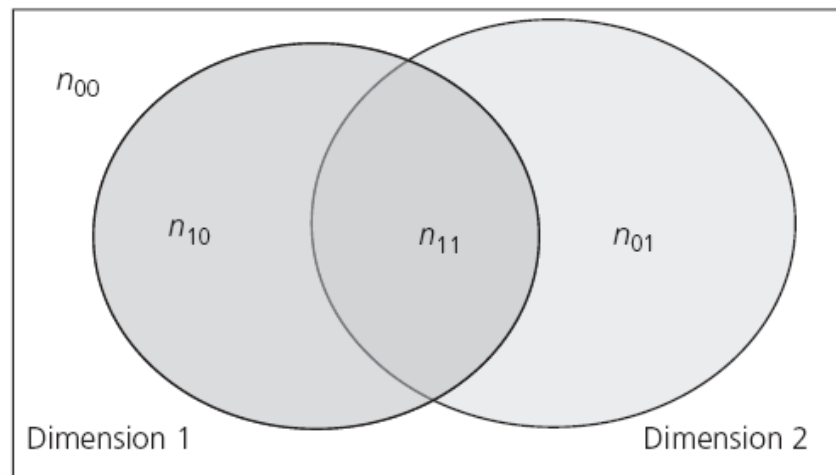


# Venn Diagrams

Diagrammatic representation showing all possible logical relations between a finite number of dimensions with binary options (*Introduced by John Venn in 1880*)

**Table 3.1 Joint distribution of deprivation in two dimensions**

		Dimension 2		Total
		Non-deprived	Deprived	
Dimension 1	Non-deprived	$n_{00}$	$n_{01}$	$n_{0+}$
	Deprived	$n_{10}$	$n_{11}$	$n_{1+}$
Total		$n_{+0}$	$n_{+1}$	$n$

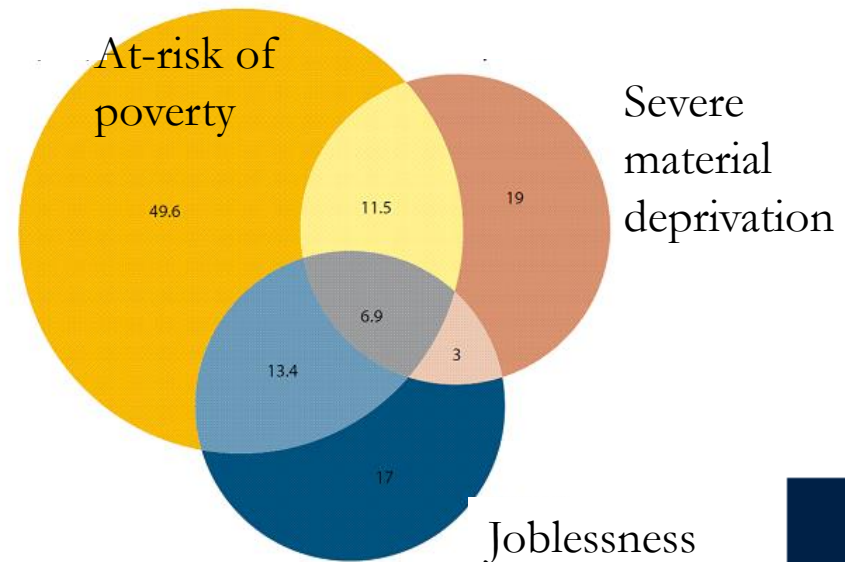


# Application of Venn Diagrams

This tool has been used to study overlaps of deprivations in different dimensions

- Atkinson et al. (2010), Ferreira and Lugo (2013), Naga and Bolzani (2006), Roelen et al. (2009), Alkire and Seth (2013), Decancq, Fleurbaey, and Maniquet (2014), Decancq and Neumann (2014)

Multiple indicators from the Europe 2020 target



# Venn Diagrams: Pros and Cons

## Advantages

- A **visual tool** to explore overlapping binary deprivations
- Considers the **joint distribution** of deprivations
- **Intuitive** and easy to understand for 2-4 dimensions

## Disadvantages

- May not identify who is multidimensionally poor
- No summary measure (thus, no complete ordering)
- Regardless of the scale, every dimension is converted into the **binary** options, losing information on depth
- Difficult to read for 5 or more dimensions

# Dominance Approach

Ascertains whether poverty is unambiguously lower or higher *regardless* of parameters and poverty measures

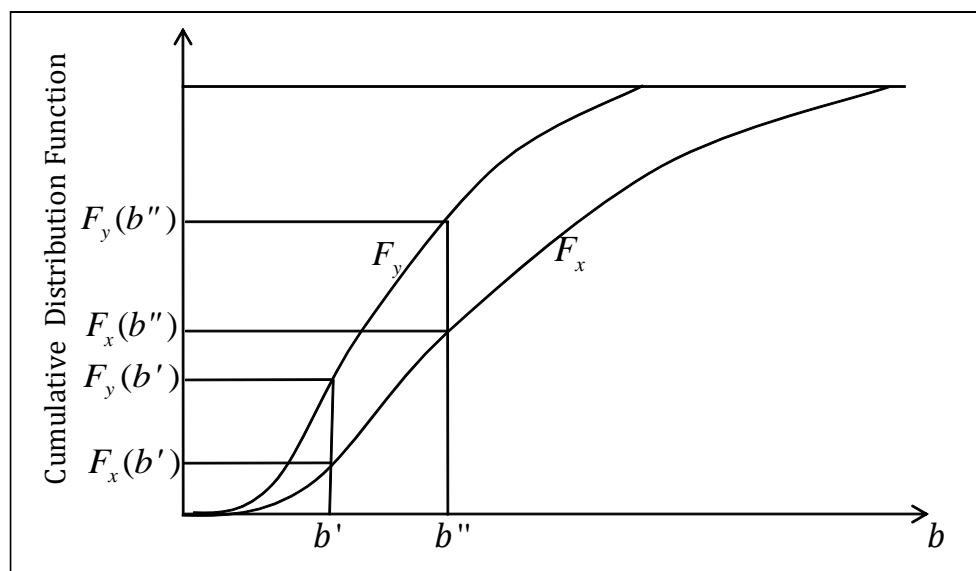
- **Unidimensional:** Atkinson (1987), Foster and Shorrocks (1988)
- **Multidimensional:** Bourguignon and Chakravarty (2002), Duclos, Sahn & Younger (2006)

Such a claim certainly has **strong political power!**

- Avoids the possibility of contradictory rankings

Key tool: *Cumulative distribution function*

# Dominance Approach



## Unidimensional Dominance

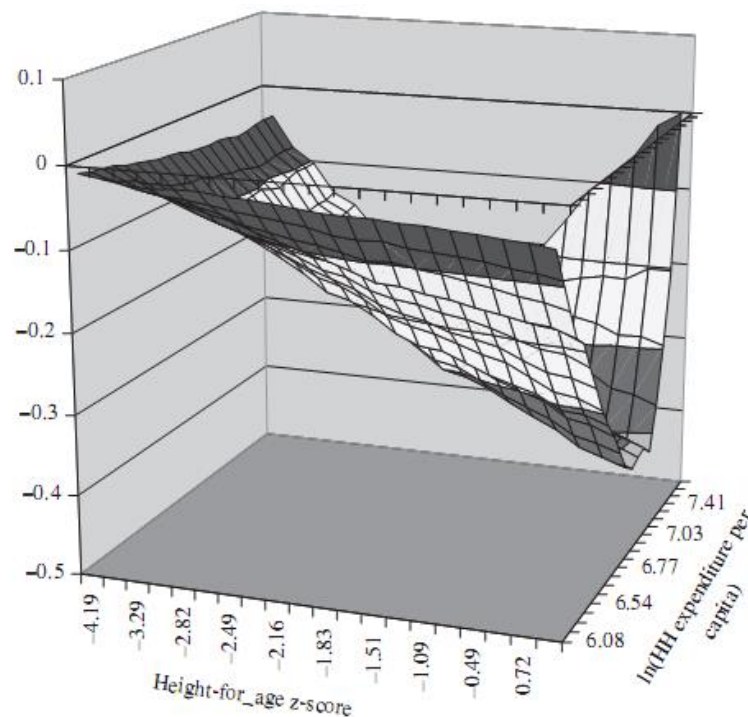


Fig. 5. Urban Minus Rural Dominance Surface for Viet Nam

## Bi-dimensional Dominance

Duclos, Sahn and Younger (2006)

# Dominance Approach: Pros and Cons

## Advantages

- Offers tool for strong empirical assertions about poverty comparisons
- Considers the joint distribution of achievements/deprivations
- Avoids ‘controversial’ decisions on parameter values

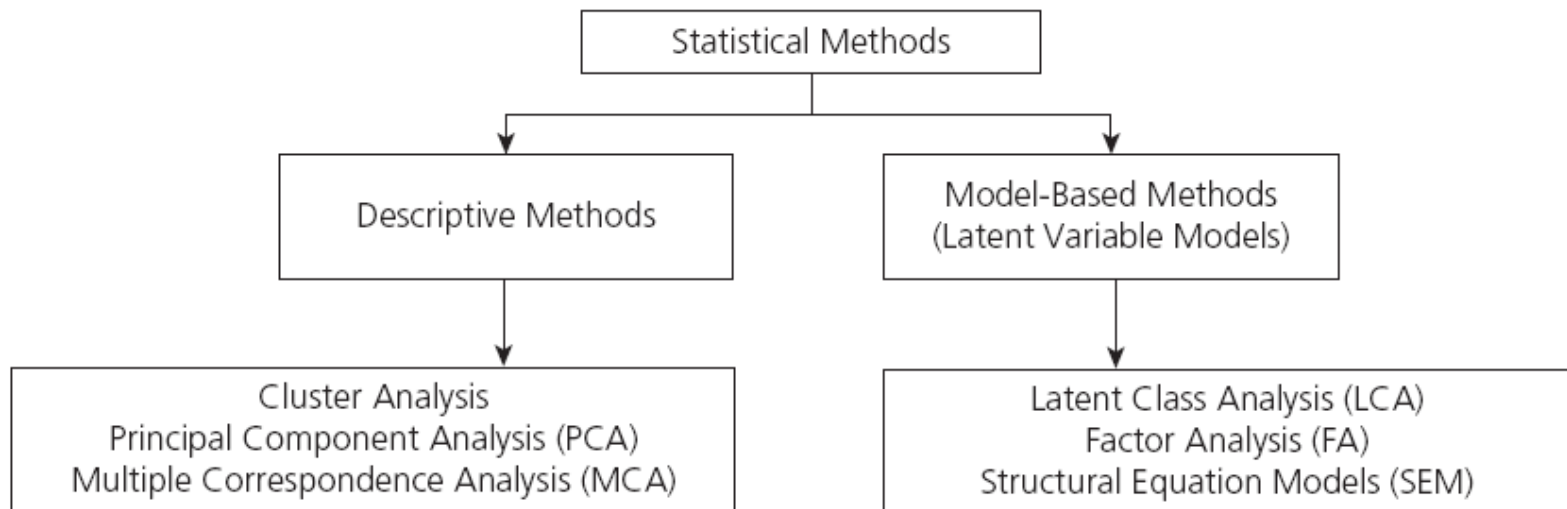
## Disadvantages

- No summary measure, No complete ordering
- Allows pair-wise dominance, but not cardinally meaningful difference
- Dominance conditions depend on relationship between dimensions
- For 2+ dimensions, limited applicability for smaller datasets
- Stringent less intuitive conditions for dominance beyond first order

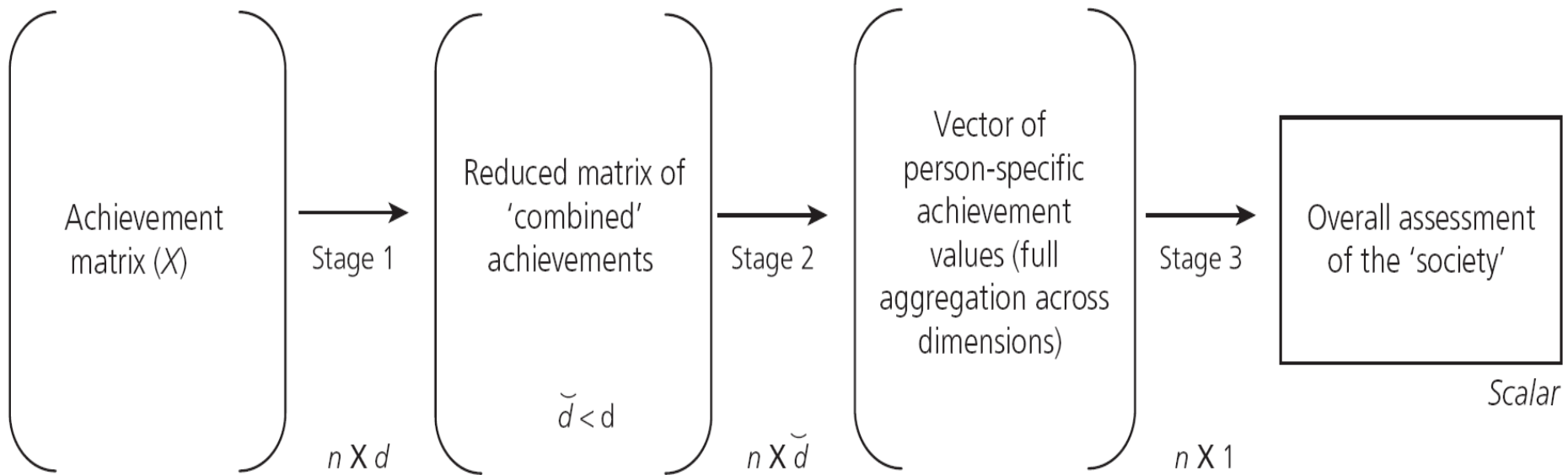
# Statistical Approaches

The main aim is to reduce dimensionality

- May be used for *poverty identification*, *poverty aggregation*, or both
- Are used during measurement design for
  - Exploring **relationships** across variables
  - Setting **weights**



# Sub-Steps in Aggregation within Multivariate Statistical Methods





# Statistical Approaches: Pros and Cons

## Advantages

- Addresses **multidimensionality**
- Considers **joint** distribution
- MCA can be used with **ordinal** data
- Helps **clarify relations** among indicators: strengthen indicator design

## Disadvantages

- Poverty identification and measurement are often **not transparent**
  - Not straightforward for communicating, not checked for robustness
- Identification is mostly **relative** (based on percentiles of the score)
- **Comparisons** across space and time may be difficult
- No automatic **normative** or **theoretical** justification

# Fuzzy Sets Approach

In poverty measurement, thresholds/cutoffs dichotomize people into sets of the *deprived and non-deprived* or *poor and non-poor*

Yet there may be *uncertainty* about where to set cutoffs

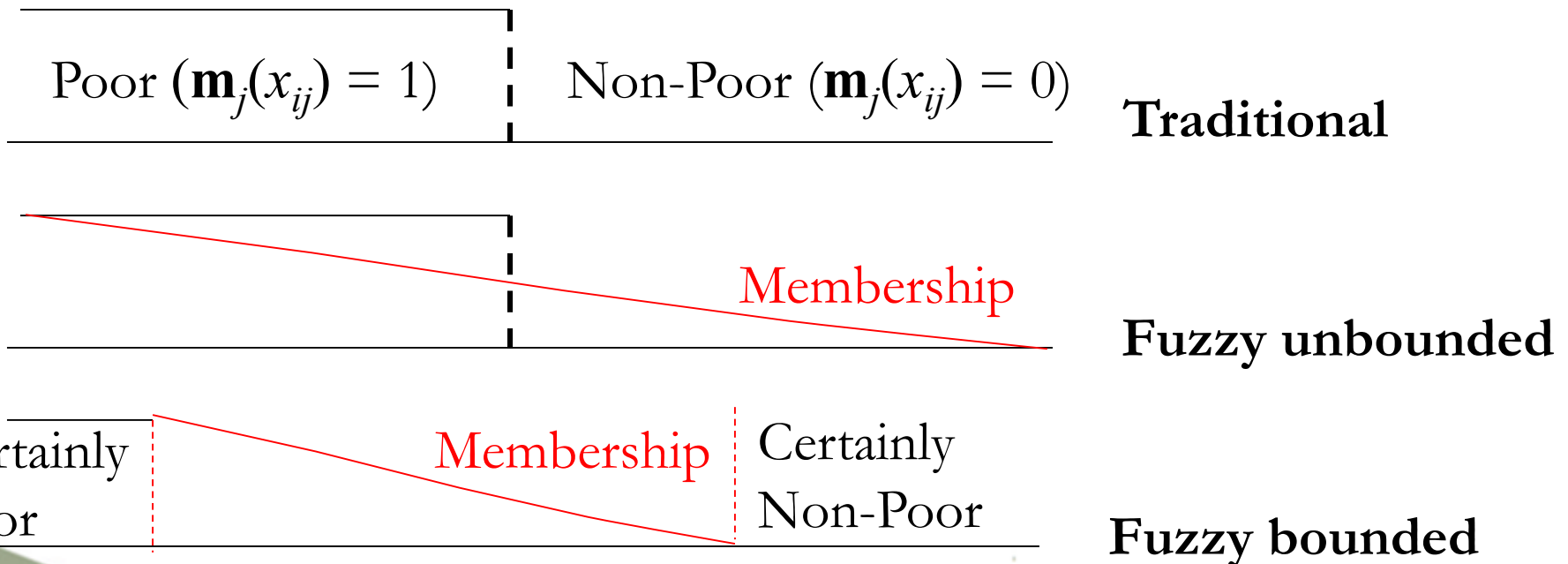
- “... it is undoubtedly more important to be **vaguely right** than to be **precisely wrong**.” (Sen 1992: 48-9)

Fuzzy set approach explore how to be **vaguely right**

- Zadeh (1965), Cerioli & Zani (1990), Cheli & Lemmi (1995), Chiapero-Martineti (1994, 1996, 2000)

# Fuzzy Sets – Identification

Extend venn diagrams: rather than identifying the deprived/  
non-deprived or poor/non-poor, allow varying degrees of membership  $\mathbf{m}_j(x_{ij})$  to each set



The selection of the membership function  $\mathbf{m}_j(x_{ij})$  is key

# Fuzzy Sets Approach: Pros and Cons

## Advantages

- Offers **summary measure**, **hierarchy** among dimensions, explicit **tradeoffs** and a complete **ranking**
- Can consider **joint distribution of deprivations**
- Compatible with many **aggregation** methodologies (Chakravarty 2006)

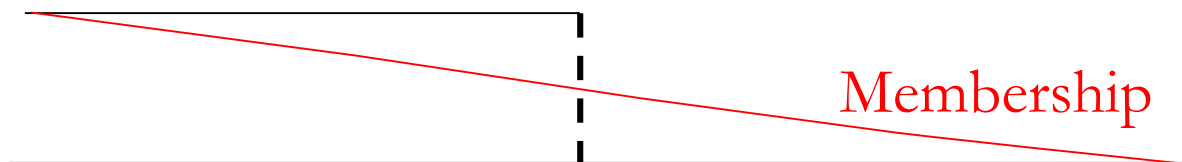
## Disadvantages

- **Justification** of membership function is not straightforward
- Robustness tests are not mostly provided
- Some membership functions may **misuse** ordinal data
- Fuzzy sets results may **conflict** with Dominance results

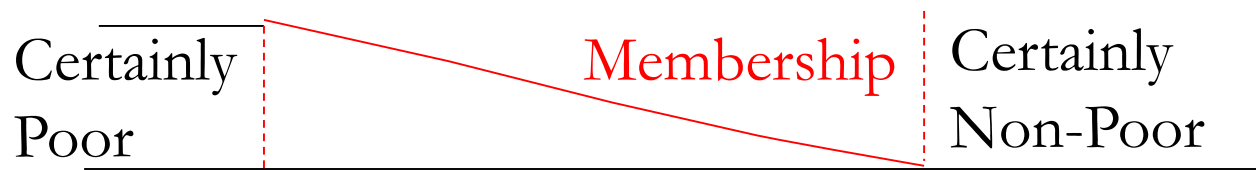
# Fuzzy Sets Approach - Disadvantages

Its contribution is not clear in poverty measurement

- Unbounded membership (Why need a poverty measure!)



- Bounded membership (may provide wrong policy incentive)



- What about the Fuzzyness of Fuzzy bounds?

# Axiomatic Approach

Develops poverty measures that comply with a number of desirable properties

*Unidimensional:* Sen (1976), Watts (1969), Foster, Greer and Thorbecke (1984), Chakravarty (1983), Clark, Hemming and Ulph (1981), Atkinson (1987), among others.

*Multidimensional:* Chakravarty, Mukherjee and Ranade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Chakravarty and D'Ambrosio (2006), Alkire and Foster (2007, 2011), Bossert, Chakravarty and D'Ambrosio (2009), Maasoumi & Lugo (2008), Decancq, Fleurbaey, and Maniquet (2014) among others

[Most extend FGT (1984), then Watts (1969)

or Chakravarty (1983)

# Axiomatic Approaches: Pros and Cons

## Advantages

- Allows looking at **joint distribution** of deprivations
- Offers **summary measure** of poverty
- Provides **clearer understanding** on how measures behave due to different transformations (biggest advantage)

## Disadvantages

- Relies on **normative** judgments (require various robustness tests)
- No single measure can satisfy all desirable properties (properties themselves often need strong justifications)
- Final poverty measures can be **difficult to interpret** intuitively when they are made to satisfy many properties simultaneously

Tabita, Kenya

Rabiya, India

Stephanie, Madagascar

Agatha, Madagascar

Dalima, Kenya

Ann-Sophia, Kenya

Valérie, Madagascar



# Counting Approach: History



# Counting Approach

Refers to a **particular method for identifying the poor**

- Entails ‘counting the number of dimensions in which people suffer deprivation, (...) the number of dimensions in which they fall below the threshold’. Atkinson (2003: 51)

Associated to the ‘direct method’ to measure poverty  
(Sen, 1981)

Widely used in practice and policy since mid 1970s

# Counting Approach

## Steps for **identifying** a poor person

1. Define a **list** of relevant indicators
2. Assign a **weight** to each considered indicator
3. Define a threshold (***deprivation cutoff***) for each indicator
4. Create **binary deprivation scores** for each person in each indicator: “1” = deprived, “0” = non-deprived
5. Produce a **deprivation score** by taking a **weighted sum** of deprivations
6. Set a threshold (or ***poverty cutoff***) such that if a person has a deprivation score at or above the threshold, the person is considered **poor**

# Counting Approach

The approach has been applied within different conceptual frameworks:

- Social Exclusion in Europe (Lenoir, 1974)
- Basic Needs in Latin America (Cocoyoc Declaration, 1974)
- More recently: capability approach, human rights approach

Applications in different parts of the world

- Europe, US, Latin America, South Asia
- UNICEF, NGOs (BRAC)

# Salient Implementations: Europe

## Townsend (1979), “**Poverty in the UK**”

- Listed 60 indicators covering 12 dimensions
- Then focused on a shorter **list of 12 items**
- **Equal weights** to all indicators
- Used a minimum score of **five** ‘as suggestive of deprivation’
- **But**, used this approach only to ‘validate’ the income poverty line to be for poverty measurement

Townsend’s work inspired a prominent body of subsequent work within and outside Europe

# Salient Implementations: Europe

Mack and Lansley (1985), “**Poor Britain**” (2 innovations)

- Socially perceived necessities (Breadline Britain survey)
  - Retained 26 items considered necessary by more than 50% of population (majority rule)
- Enforced lack
  - Distinguished people not having an item as they could not afford it, from those it was a voluntary choice
- How were the poor identified?
  - Who could not afford **three or more items**
  - Indicators were equally weighted

# Salient Implementations: Europe

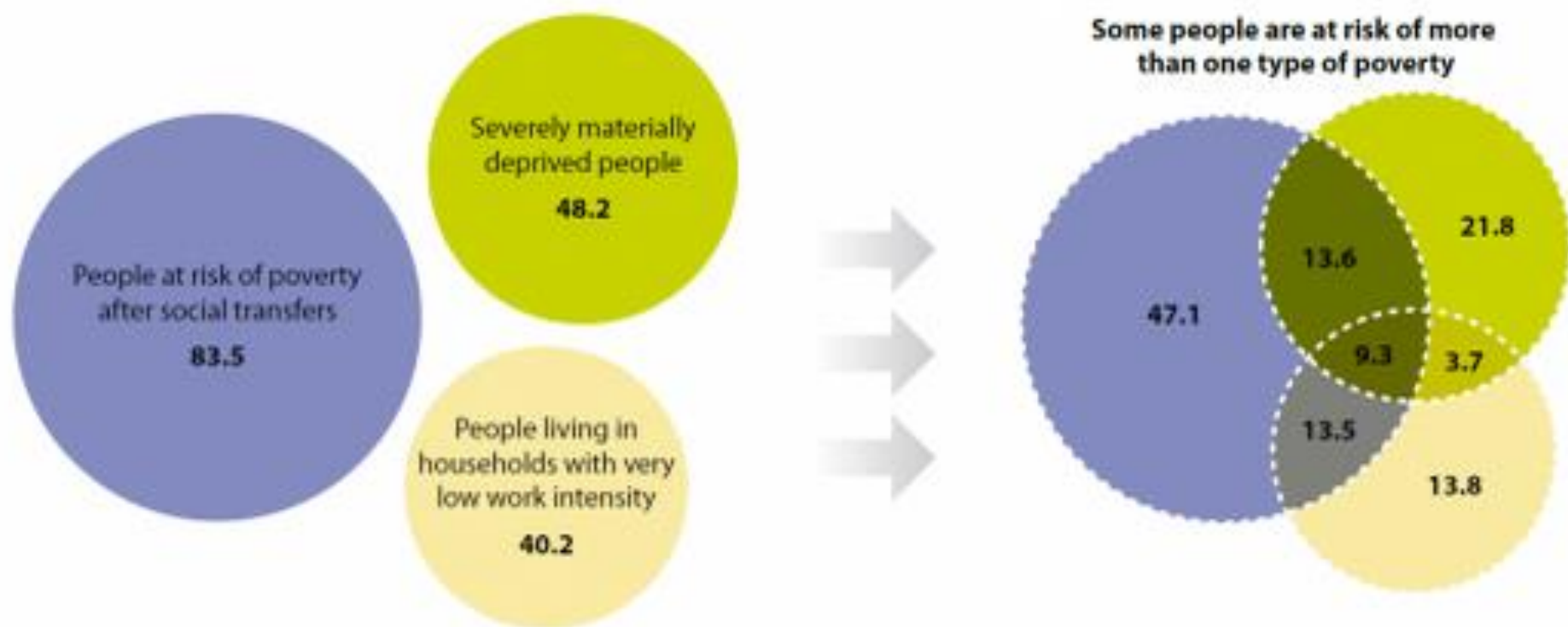
Callan, Nolan and Whelan (1993)

- Listed 8 items of the basic lifestyle dimension.
- Who were the poor?
  - Those deprived in **one or more** of the eight items **and** below the relative income poverty line (60% of average income)
  - Consistent poverty approach (intersection of two approaches)

# EU-2020 Poverty measure

1. **Monetary poverty** is measured by the indicator ‘people at risk of poverty after social transfers’. The indicator measures the share of people with an equivalised disposable income below the risk-of-poverty threshold. This is set at **60 % of the national median** equivalised disposable income after monetary social transfers. Social transfers are benefits provided by national or local governments, including benefits relating to education, housing, pensions or unemployment.
2. **Material deprivation** covers issues relating to economic strain, durables and housing and dwelling environment. Severely materially deprived people are living in conditions greatly constrained by a lack of resources and cannot afford **at least four of the following**: to pay their rent or utility bills or hire purchase instalments or other loan payments; to keep their home warm; to pay unexpected expenses; to eat meat, fish or other protein-rich nutrition every second day; a week-long holiday away from home; to own a car, a washing machine, a colour TV or a telephone.
3. **Very low work intensity** describes the number of people aged 0 to 59 living in households where the adults worked **less than 20% of their work potential** during the past year.

Because there are intersections between these three dimensions, they cannot simply be added together to give the total number of people at risk of poverty or social exclusion. Some people are affected by two, or even all three, types of poverty. Taking the sum of each would lead to cases being double-counted. This will become clearer when looking at the current numbers of people at risk of poverty or social exclusion (see Figure 9).



(\*) Estimated data.

**Figure 9:** Aggregation of sub-indicators of 'People at risk of poverty or social exclusion', EU-28, 2013 (\*) (Million people)

Source: Eurostat online data code [\(ilc\\_pees01\)](#)



# Other Implementations

- **UK:** Callan *et al.* (1999), Whelan *et al.* (2001), Layte *et al.* (2000), Gordon *et al.* 2000, Whelan *et al.* (2001), Whelan *et al.* (2006)
- **Netherlands:** Muffels *et al.* (1992)
- **Sweden:** Halleröd (1995) and Halleröd *et al.* (2006)
- **Europe:** Layte *et al.* (2001), Guio 2005, Guio and Maquet (2006), Guio 2009, Decanq *et al.* (2013)
- **US:** Mayer and Jencks (1989) and Bauman (1998, 1999)

# Salient Implementations: Latin America

- **Unsatisfied Basic Needs (UBN) Approach**
  - Chile (Kast and Molina, 1975)
  - Argentina (INDEC 1984, together with CEPAL)
- Used five **census** indicators
  - (i) Overcrowding, (ii) Housing, (iii) Sanitation, (iv) Children attending primary school, (v) Household head has 2 or less years of education, or high dependency rate
  - Reported headcount ratio with 1+, 2+ and 3+ deprivations
  - Used for policy mapping at very disaggregated levels
  - **Extensions:** Bolivia, Colombia, Ecuador, Guatemala, Honduras, Nicaragua, Paraguay, Peru, Uruguay, Venezuela

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# Salient Implementations: Latin America

‘Integrated method’ identified household that were income poor AND had Unsatisfied basic needs

	UBN poor	UBN non-poor
Income Poor	Chronically Poor	Recently Poor
Income Non-Poor	With Structural Deprivations	Socially Integrated

Non-counting proposal to cardinalize ordinal data and combine it with income poverty (Boltvinik)

# Salient Implementations: Arab Regions

Lebanon (1997): *Mapping Living Conditions in Lebanon*

- Lebanese Ministry of Social Affairs (MoSA) and UNDP
- Eleven indicators in four dimensions

Iraq (2006): *Mapping of Deprivation and Living Conditions in Iraq*

- seventeen indicators grouped into five dimensions: education, health, housing, home necessities, and economic conditions

# Salient Implementations: Child Poverty

## Child Poverty Measures

- Internationally comparable child poverty measures in developing countries (Gordon *et al.* 2001, 2003; UNICEF 2004)
- Indicators and cutoffs reflect the *Convention of the Rights of the Child*
- Applications using the Alkire-Foster method (including MODA)
  - Alkire and Roche (2012), Apablaza and Yalonetzky (2011), Roche (2013), Trani *et al.* (2013), de Neubourg *et al.* (2012), and Dickerson and Popli (2013)

# Salient Implementations: Targeting

Targeting 'Below the Poverty Line' households in India

- Exclusion and inclusion criteria and then counting

Kerala government's strategy for targeting the poor

- Developed originally by an NGO Kudumbashree (Thomas et al 2009)
- Nine equally weighted indicators
- Poor: deprivation in 4 or more, Destitute: Deprivation in 8 or 9

Other applications: Grameen Bank and BRAC in Bangladesh

# Counting Approach: Pros and Cons

## Advantages

- Clarity, Simplicity, Transparency, Intuition for identifying the multiply deprived
- Allows looking at joint deprivations
- Allows for both cardinal and ordinal variables



# Counting Approach: Pros and Cons

## Disadvantages

- Relies on the particular selection of indicators (appropriateness for the particular purpose)
- Relies on the weights assigned to the dimensions/indicators.
- Relies on dichotomies (deprived/non-deprived) so not sensitive to the depth for identification.
- Sometimes a counting approach is combined with aggregation methodologies that are not intuitive, in certain cases incorrectly assigning cardinal meaning to ordinal values (ex. poverty scorecards)