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Summer School on Multidimensional Poverty Analysis

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Review: Unidimensional Poverty Measurement

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Session IV, 3 August 2015

Main Sources of this Lecture

- Alkire S., J. E. Foster, S. Seth, S. Santos, J. M. Roche, P. Ballon, Multidimensional Poverty Measurement and Analysis, Oxford University Press, forthcoming, (Ch 2.1).
- Foster, J. E., S. Seth, M. Lokshin, and Z. Sajaia (2013), A Unified Approach to Measuring Poverty and Inequality: Theory and Practice, The World Bank, Free Download: https://openknowledge.worldbank.org/handle/10986/13731
- Foster J. E. and A. K. Sen (1997), Annexe of "On Economic Inequality", Oxford University Press.
- Foster (2006) "Poverty Indices"

Preliminaries

- Reference population
  - We refer as '*Society*' (e.g. country, region etc.)
- Unit of measurement
  - We refer as '*Person*' (could be households)
  - Suppose there are n persons in the society (n may vary)
- Variables or dimensions for assessing poverty
  - We refer as '*Space*' (e.g. income sources, commodities)
  - Suppose there are d such variables (fixed set)

Preliminaries

- Achievement: performance of a person in a dimension
  - x_{ij}: Achievement of person i (=1,…,n) in dimension j (=1,…, d)
- Achievement matrix
  - Summarizes achievements of all n persons in d dimensions
- Achievement vector of a Person
  - May contain incomes from d different sources or d different commodities consumed

Dimensions

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1d} \\ x_{21} & \cdots & x_{2d} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nd} \end{bmatrix}$$

Persons

Preliminaries

- Overall achievement of each person
  - x_i : Obtained by meaningfully combining d dimensions
    - Also referred as *resource variable* or *welfare indicator*
  - When the space is a set of income sources
    - Total income of person i: $x_i = \Sigma_j x_{ij} = x_{i1} + \ldots + x_{id}$
  - When the space is a set of commodities consumed
    - There is a set of d commodity prices: $(p_1,\ldots, p_d)$ and total consumption expenditure of person i:

$$x_i = \Sigma_j p_j x_{ij} = p_1 x_{i1} + \ldots + p_d x_{id}$$

Preliminaries

- Overall achievement vector: $x = (x_1, \ldots, x_n)$
  - Example: Suppose there are four persons in a society with overall incomes \$9, \$4, \$15 and \$8
  - Then $x = (9, 4, 15, 8)$ is a vector representing the incomes of the society

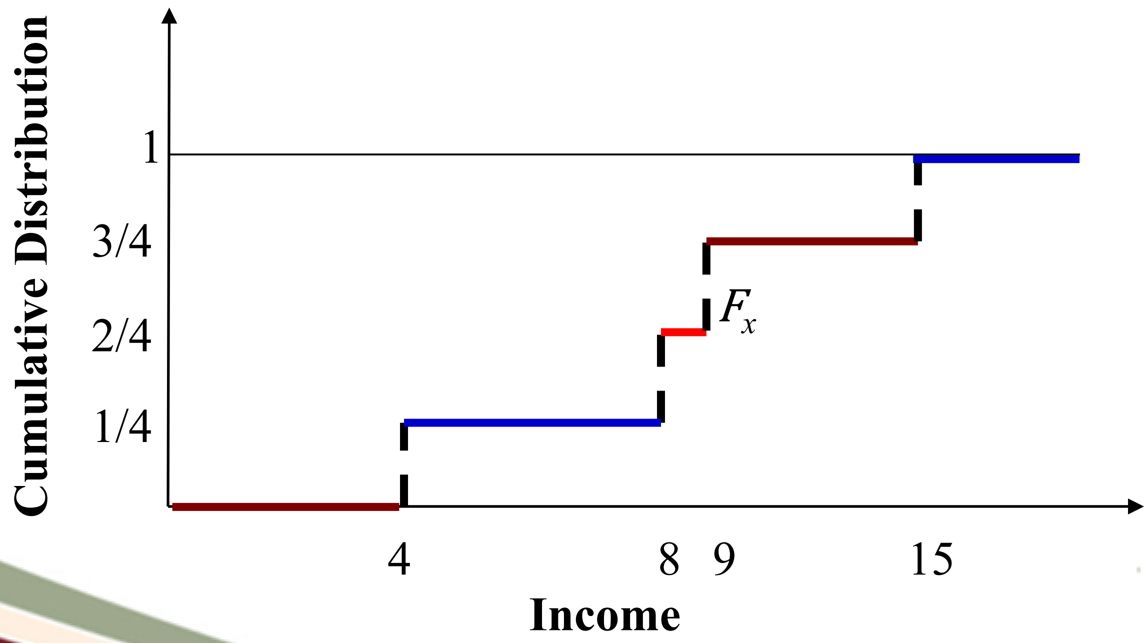
- Ordered overall achievement vector
  - An ordered vector ranks or orders individuals by their achievements
  - Example: $(9, 4, 15, 8) \rightarrow (4, 8, 9, 15)$

Preliminaries

Cumulative Distribution Function (CDF)

The ordered vector of x = (9, 4, 15, 8) can be represented by a cdf.

The cdf of distribution x is denoted by F_x

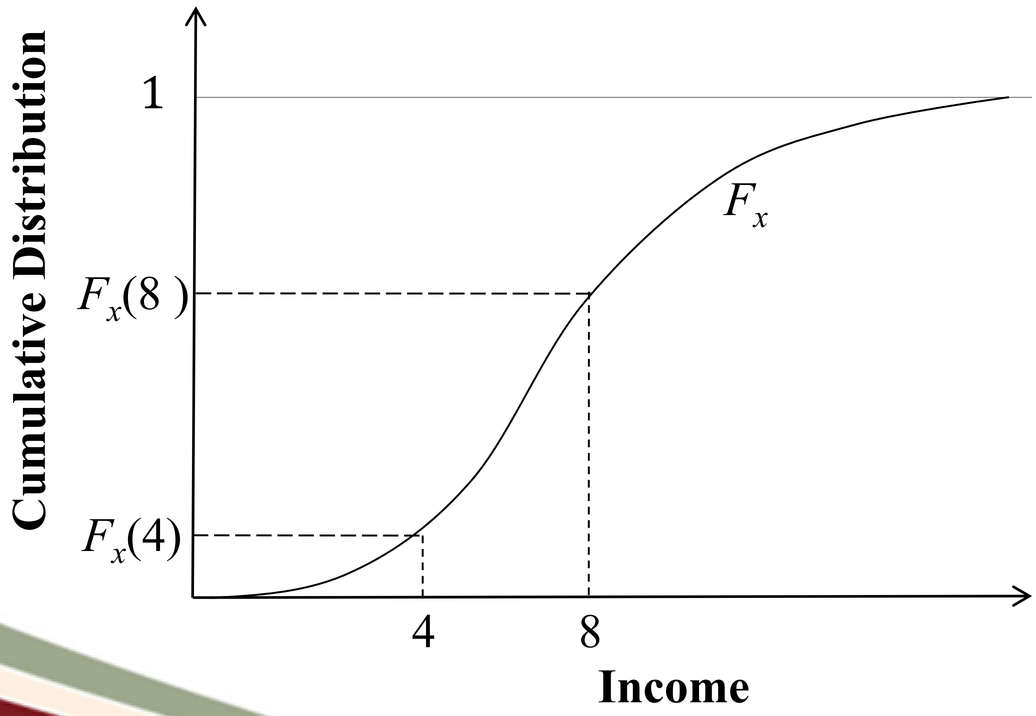


$$F_x = \begin{cases} 0 & \text{for } b<4 \\ 1/4 & \text{for } 4 \le b < 8 \\ 2/4 & \text{for } 8 \le b < 9 \\ 3/4 & \text{for } 9 \le b < 15 \\ 1 & \text{For } b \ge 15 \end{cases}$$

Preliminaries

Cumulative Distribution Function (CDF)

For a society with large population size, a typical cdf looks like



What does a CDF tell us?

It tells us the share of the population having income less than a particular income level

e.g., $F_x(4)$ is the share of the population having income less than \$4

Preliminaries

- A policy maker is generally interested in the following three aspects of a distribution or a vector
  - Size (Welfare), e.g. per-capita income
  - Spread (Inequality), e.g. Gini coefficient
  - Base (Poverty)
    - Welfare of the population below a certain level of income

In this summer school, we focus on the third aspect

Poverty Measurement

Unidimensional poverty measurement involves two steps (Sen 1976): Identification and Aggregation

Identification: Who is poor?

This step dichotomises the population into a group of *poor* and a group of *non-poor* persons

Main tool: The Poverty Line (z)

Person i is poor if $x_i < z$ and is non-poor if $x_i \geq z$

x_i is the i^{th} element of vector x

Significance of Poverty Line (*z*)

- Poverty line enables policy makers to identify a group of people, who are subject to different social assistance programs or '*targeting*'

- Poverty line as '*a benchmark*': the objective of a policy maker should be to bring the poor at *z*
  - For poverty analysis, additional achievement of the non-poor above the poverty line is ignored

Censoring at Poverty Line

Having z as benchmark, allows us to create a censored distribution of x, referred as x^*, where

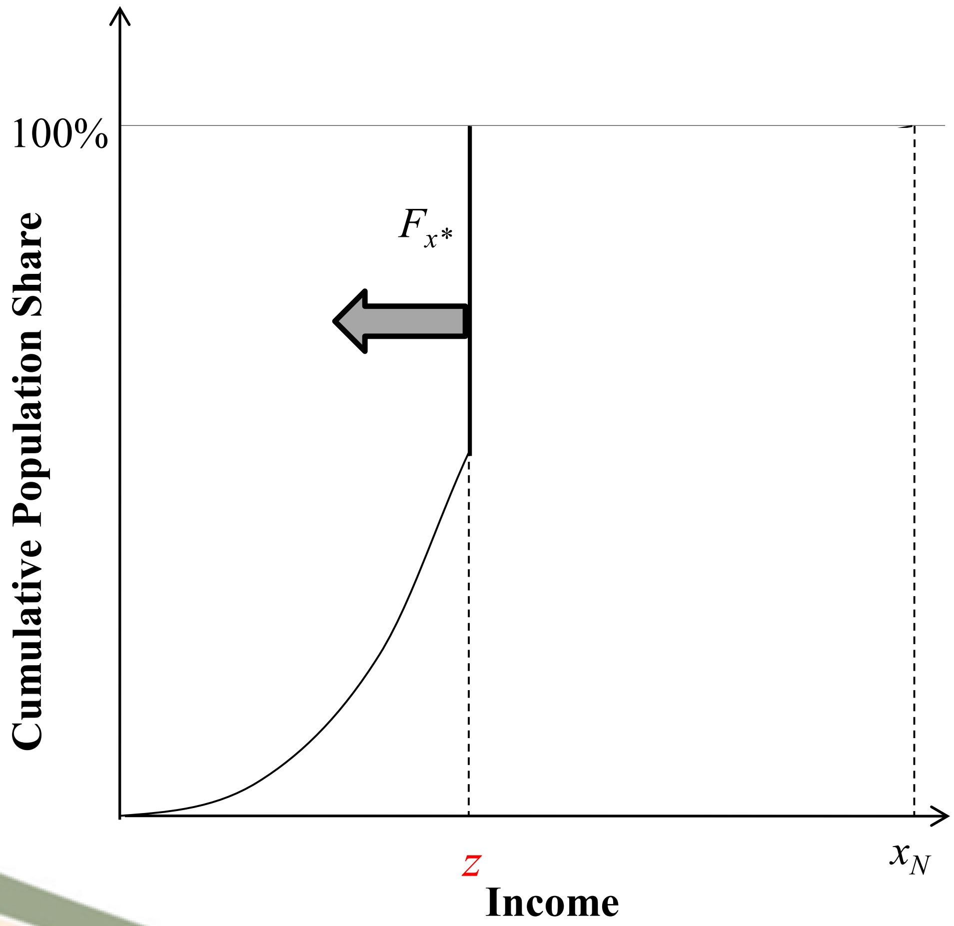
$$x_i^* = x_i \text{ if } x_i < z$$

and $$x_i^* = z \text{ if } x_i \geq z$$

Example: If $z = 10$ and $x = (9, 4, 15, 8)$, then

$$x^* = (9, 4, 10, 8)$$

Censoring at Poverty Line



Obtain the censored distribution x^*

$$x_i^* = x_i \text{ if } x_i < z$$
$$x_i^* = z \text{ if } x_i \geq z$$

Aggregation

- How poor is the society?
  - This step constructs an index of poverty summarizing the information in the censored achievement vector x^*
  - For each distribution x and poverty line z, $P(x;z)$ or $P(x^*)$ measures the level of poverty in the distribution

Behaviour of Poverty Measures

- How should a poverty measure change due to different data transformations?

- Consider certain examples

Example 1

- Which of the following distributions has more poverty?

- $x = (5,4,4,5)$; $y = (4,4,5,5)$; and $z = 10$
  - And why?

- Symmetry
  - If y is obtained from x by a permutation of incomes and z remains unchanged, then $P(y;z) = P(x;z)$

Example 2

- Which of the following distributions has more poverty?

- x = (9,9,15); y = (9,9,9,15,15,15,15,15); and z =10
  - And why?

- Replication invariance
  - If y is obtained from x by a replication and z remains unchanged, then $P(y;z) = P(x;z)$
  - Example: z = 10, x = (9,4,15,8), y = (9,9,4,4,15,15,8,8)

Example 3

- Which of the following distributions has more poverty?

- x = (\$2,\$4,\$6,\$12) and z_x = \$10
- y = (£3,£6,£9,£18) and z_y = £15 £1 = \$1.5
  - And why?

- Scale invariance
  - If all incomes in x and z are changed by the same proportion $\alpha > 0$, then $P(\alpha x; \alpha z)=P(x;z)$
  - Related properties: *Unit Consistency*, *Translation invariance*

Example 4

- Which of the following distributions has more poverty?

- x = (5,4,4,15); y = (5,4,4,25); and z =10
  - And why? Hint: Censor

- Focus
  - If y is obtained from x by an increment to a non-poor person's income and z remains unchanged, then $P(y;z) = P(x;z)$
  - Two types of focus properties in multidimensional context

Example 5

- Which of the following distributions has more poverty?

- x = (4,4,4,4); y = (4,4,4,5); and z =10
  - And why?

- Monotonicity
  - If y is obtained from x by a decrement of incomes among the poor and z remains unchanged, then $P(y,z) > P(x,z)$
- Weak monotonicity
  - If y is obtained from x by a decrement of incomes among the poor and z remains unchanged, then $P(y,z) \geq P(x,z)$

Example 6

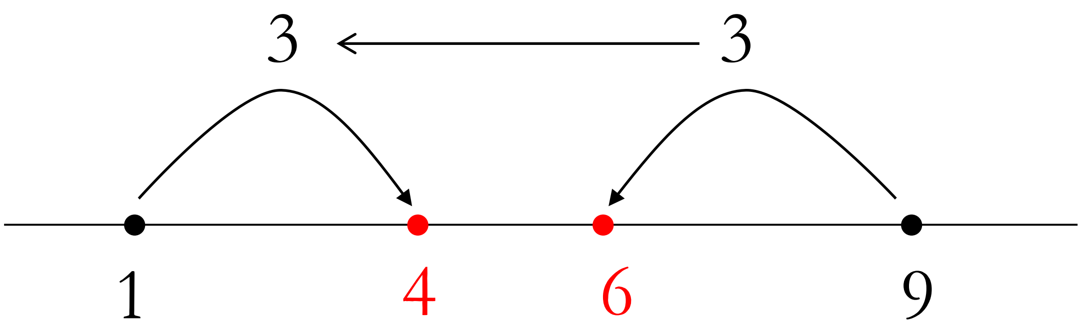
- Which of the following distributions has more poverty?

- x = (1,1,9,9); y = (4,4,6,6); and z =10
  - And why?

- Transfer principle
  - If y is obtained from x by a progressive transfer among the poor and z remains unchanged then $P(y;z) < P(x;z)$
- Weak transfer principle
  - If y is obtained from x by a progressive transfer among the poor and z remains unchanged then $P(y;z) \leq P(x;z)$

Donaldson and Weymark (1986)

Progressive Transfer



Transfer: Is there a limit on the amount of transfer in this property?

Yes. Post-transfer income cannot fall below the lower pre-transfer income

What is the implication of this axiom for non-transferable dimensions?

Example 7

- Let z=10, x=(9,4,15,8), x^1=(9,4), and x^2=(15,8)
  - y is obtained from x such that y^1=(**6**,4), while $y^2 = x^2$
  - Which distribution has more poverty: y or x?
  - Note $P(y^1;z) > P(x^1;z)$ by monotonicity, but $P(y^2;z) = P(x^2;z)$, and so it is consistent to have $P(y;z) > P(x;z)$

- <u>Subgroup Consistency</u>
  - If y is obtained from x, such that (i) $P(y^1;z) > P(x^1;z)$, (ii) $P(y^2;z) = P(x^2;z)$, and (iii) the population size of each group and z remains unchanged, then $P(y;z) > P(x;z)$

Subgroup Consistency

- For which practical reason is the subgroup consistency property important?
  - Evaluation of poverty reduction programs!
  - It can be seen as an extension of monotonicity
    - Monotonicity requires poverty to fall when one person's poverty level is reduced.
    - Subgroup consistency requires aggregate poverty to fall when one group's poverty level is reduced
    - However, a reduction in one group's poverty may be accompanied by both increase and fall in individual incomes (where Sub. Con. Differs from Mon.)

Additive Decomposability

- A poverty measure is additive decomposable if

$$P(x) = \frac{n^1}{n} P(x^1) + \frac{n^2}{n} P(x^2)$$

- One may compute the contribution of each group to overall poverty as

$$\frac{n^\ell}{n} \frac{P(x^\ell)}{P(x)}; \ell = 1, 2$$

  – Note: Additive decomposability implies subgroup consistency, but the converse does not hold

Some Technical Properties

- Normalization
  - A poverty measure should be bounded between 0 and 1
    - 0 for no poverty and 1 for highest possible poverty
  - Example 1: $z = 2$ and $x = (9,4,15,8)$, then $P(x;z) = 0$
  - Example 2: $z = 16$ and $x = (9,4,15,8)$, then $P(x;z) = 1$

- Continuity
  - A poverty measure should be continuous on the achievements
  - This property prevents any sudden change in a poverty measure

Classification of Properties

- Invariance Properties
  - Ensure that poverty measures should not change under certain transformations of the achievement matrix
- Dominance Properties
  - Ensure that poverty measures should increase or decrease due to certain transformations in the achievement matrix
- Subgroup Properties
  - Relate overall poverty to either groups of people or groups of dimensions
- Technical Properties
  - Guarantee that measures behave within certain usual, convenient parameters

Poverty Measures

- Basic Measures
  - Headcount Ratio
  - Income Gap Ratio
  - Poverty Gap Ratio

- Advanced Measures
  - Squared Poverty Gap (Foster-Greer-Thorbecke)
  - Sen-Shorrocks-Thon Measure
  - Watts Measure
  - Clark-Hemming-Ulph-Chakravarty Class of Measures

Poverty Measures

- The Headcount Ratio (H)
  - The most commonly and widely used measure of poverty
  - It reports the proportion of the population that is poor
  - It ranges between 0 and 1
  - If q is the number of poor in vector x with population size n, then $H = q/n$
  - Example: Let $z = 10$ and $x = (9,4,15,8)$, then $H = 3/4$

Poverty Measures

- The Headcount Ratio (H)
  - What properties does this measure satisfy?
  - H satisfies symmetry, replication invariance, scale invariance, focus, normalization, and subgroup consistency
  - H does not satisfy monotonicity, transfer, continuity
  - Policy Implication?
    - Encourages a policy maker, with limited budget, to assist the marginally poor only instead of the severely poor

Poverty Measures

- Poverty Gap Ratio (PG)
  - This measure repairs some of the problems of the headcount ratio
  - It reports the average normalized income shortfall from the poverty line using the *censored* distribution x^*
  - The average normalized shortfall of the i^{th} person is

$$g_i^* = (z - x_i^*)/z.$$

  - The average income normalized shortfall is

$$PG = (1/n)\Sigma_i\, g_i^* = (z - \mu^*)/z = \mathbf{H \times I}$$

$$I = (1/q)\Sigma_i\,(z - x_i)/z$$

Poverty Measures

- Poverty Gap Ratio (PG)
  - Example: $x=(9,4,15,8)$; $z=10$
  - Then $x^*=(9,4,\mathbf{10},8)$ and $g^* = (0.1,0.6,0,0.2)$
  - So, PG = 0.9/4 = 0.225
  - Alternatively, μ^* = average of elements in x^*.
  - So, $\mu^* = 7.75$. Thus, PG = (10 – 7.75)/10 = 0.225
  - PG ranges between 0 and 1

Poverty Measures

- <u>Poverty Gap Ratio (PG)</u>
  - What axioms does this measure satisfy?
  - It satisfies - symmetry, replication invariance, scale invariance, focus, normalization, monotonicity, continuity and subgroup consistency
  - It does not satisfy –transfer

  - Policy Implication?
  - It does not encourage a policy maker to distinguish between a marginally poor and severely poor while assisting

Advance Poverty Measures

- Squared Poverty Gap (SG)
  - It reports the average of squared normalized income short-falls from the poverty line using the *censored* distribution x^*. Also, known as *Foster-Greer-Thorbecke* (FGT) measure
  - The average normalized shortfall of the i^{th} person is

$$g_i^* = (z - x_i^*)/z.$$

  - The average of squared normalized income shortfalls

$$SG = (1/n)\Sigma_i (g_i^*)^2$$

Advance Poverty Measures

- Squared Poverty Gap (SG)
  - Example: $x=(9,4,15,8)$; $z=10$.
  - Then $x^*=(9,4,\mathbf{10},8)$ and $g^* = (0.1,0.6,0,0.2)$
  - Squares of poverty gap are $sg^* = (0.1^2,0.6^2,0^2,0.2^2) = (0.01,0.36,0,0.04)$
  - Thus, SG = 0.41/4 = 0.102
  - SG ranges between 0 and 1.

Advance Poverty Measures

- Squared Poverty Gap (SG)
  - What axioms does this measure satisfy?
  - It satisfies symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, transfer, and subgroup consistency
  - This measure can be presented as: $SG = H[I^2 + (1 - I)^2 \times C_p^{\,2}]$
    - C_p is the coefficient of variation of income across the poor.
    - Policy implication: care for the severe poor first

Foster-Greer-Thorbecke (FGT) Class

The FGT class of measures is defined as

$$FGT_{\alpha} = (1/n)\Sigma_i (g_i^*)^{\alpha}$$

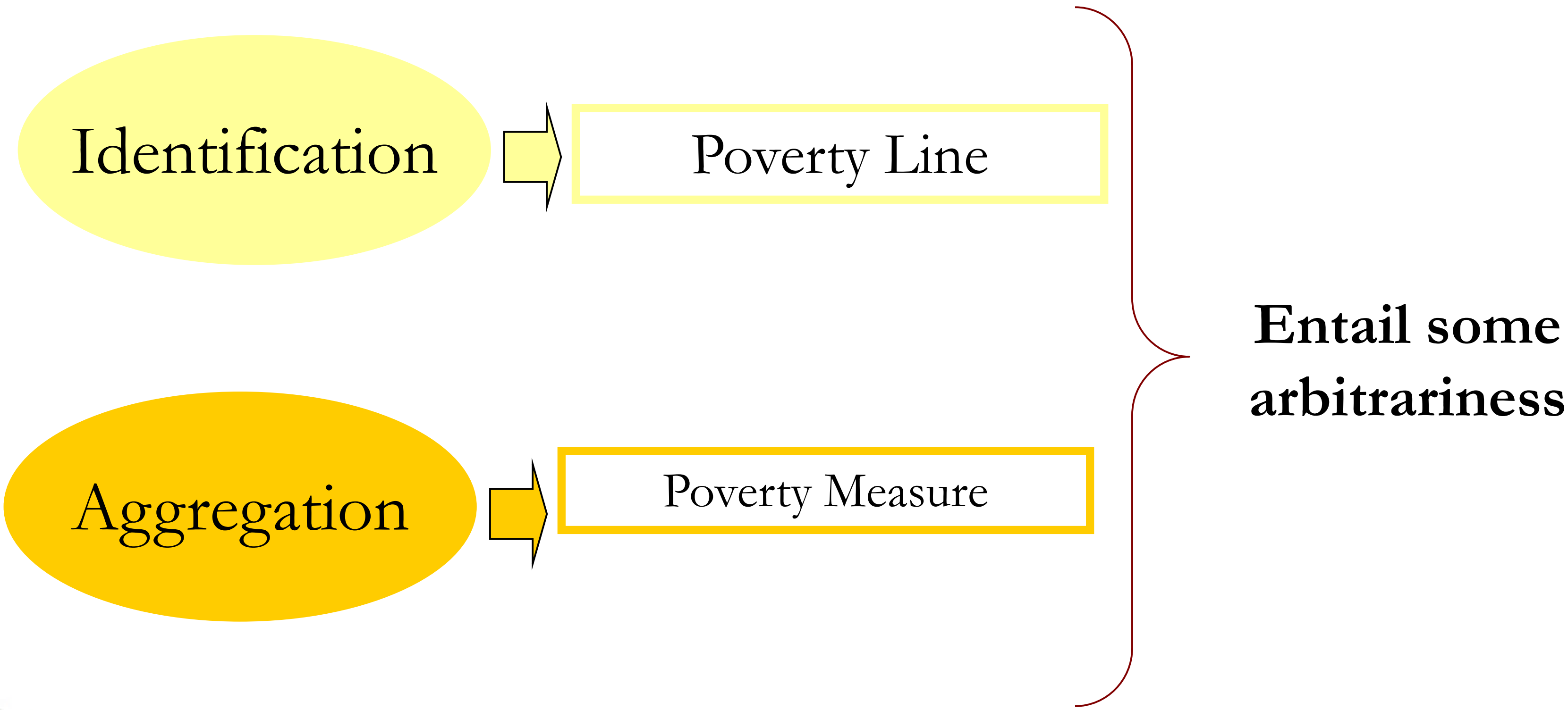
where α is a parameter and g_i^* is the normalized income gap of the i^{th} person in x^*

For $\alpha = 0$, FGT is the Headcount Ratio

For $\alpha = 1$, FGT is the Poverty Gap Ratio

For $\alpha = 2$, FGT is the Squared Poverty Gap

Arbitrariness in Poverty Measurement



Which Poverty Line?

- Does the choice of poverty line alter the ranking of distributions? Yes!

  – Example: Consider two distributions

  $$x = (4,8,9,15) \text{ and } y = (3,6,12,17)$$

  – Which distribution has more poverty by H, if z =7?

    - $H(x;z) = 1/4$ and $H(y;z) = 2/4$

  – Which distribution has more poverty by H, if z =10?

    - $H(x;z) =3/4$ and $H(y;z) =2/4$

Which Measure?

- Does all measures rank any two distributions in the same manner? No!

  - *Example*: Consider the same two distributions
    x = (4,8,9,15) and y = (3,6,12,17)

  - Which distribution has more poverty by H, if z =10?
    - H(x;z) = 3/4 and H(y;z) = 2/4

  - Which distribution has more poverty by PG, if z =10?
    - PG(x;z) = 0.225 and PG(y;z) = 0.275

Dominance Approach

- To verify if one distribution dominates another distribution, do we need to calculate all measures for all possible poverty lines?
  - This would be a tedious task!

- Is there any useful tool for these purposes?
  - Yes. A tool known as **Stochastic Dominance**
  - This is closely linked to poverty ordering, where we rank different distributions

Dominance Approach

Two main types of poverty orderings:

1. Variable-line poverty orderings (focus on the identification step)

2. Variable-measure poverty orderings (address aggregation).

Variable-Line Poverty Orderings

Main procedure:

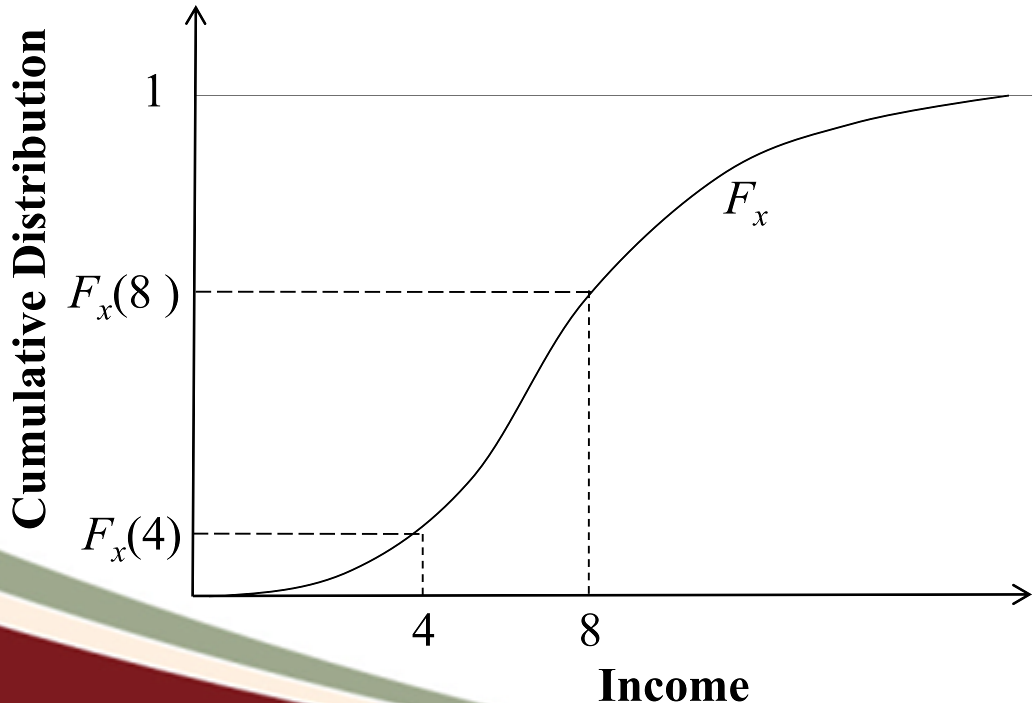
1. Choose a measure

2. Identify the condition that two distributions must satisfy so as to be able to say that one has more poverty than the other.
   - (Foster and Shorrocks, 1988)

Poverty Ordering Based on H

Recall the Concept of CDF

For a society with large population size, a typical cdf looks like



What does a CDF tell us?

It tells us the share of the population having income less than a particular income level

e.g., $F_x(4)$ is the share of the population having income less than $4

Poverty Ordering Based on H

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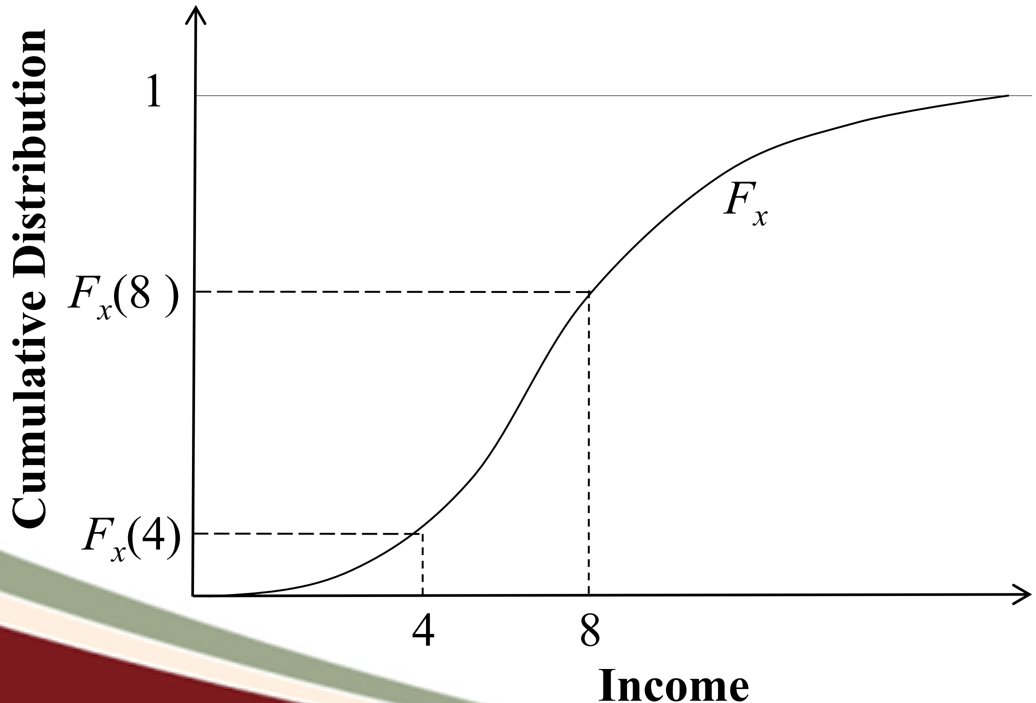


If the poverty line is *z*=4, then what is H?

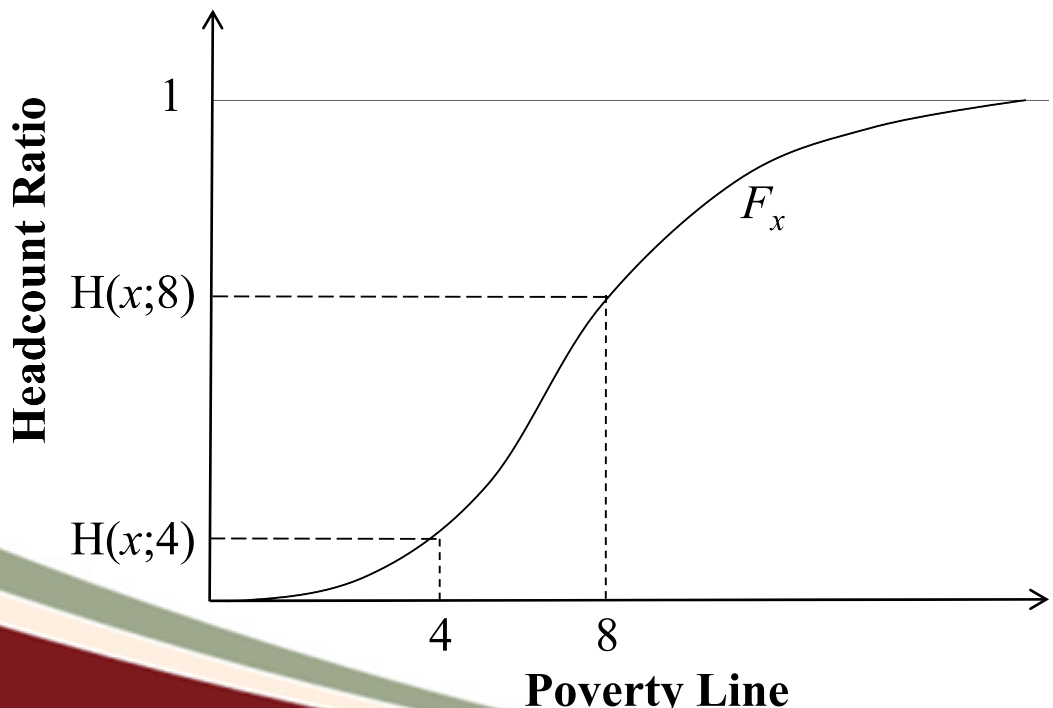
H is equal to $F_x(4)$ in this situation.

The diagram to the left may be presented as:

Poverty Ordering Based on H

Recall the Concept of CDF

For a society with large population size, a typical cdf looks like



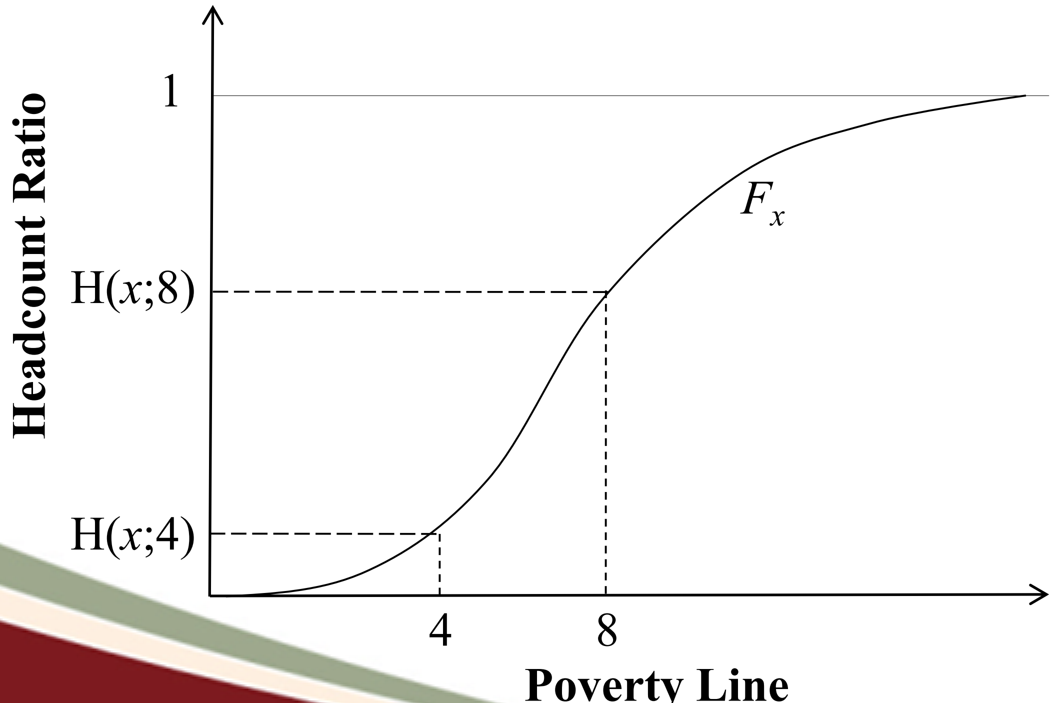
If the poverty line is *z*=4, then what is H?

H is equal to $F_x(4)$ in this situation.

The diagram to the left may be presented as:

Poverty Ordering Based on H

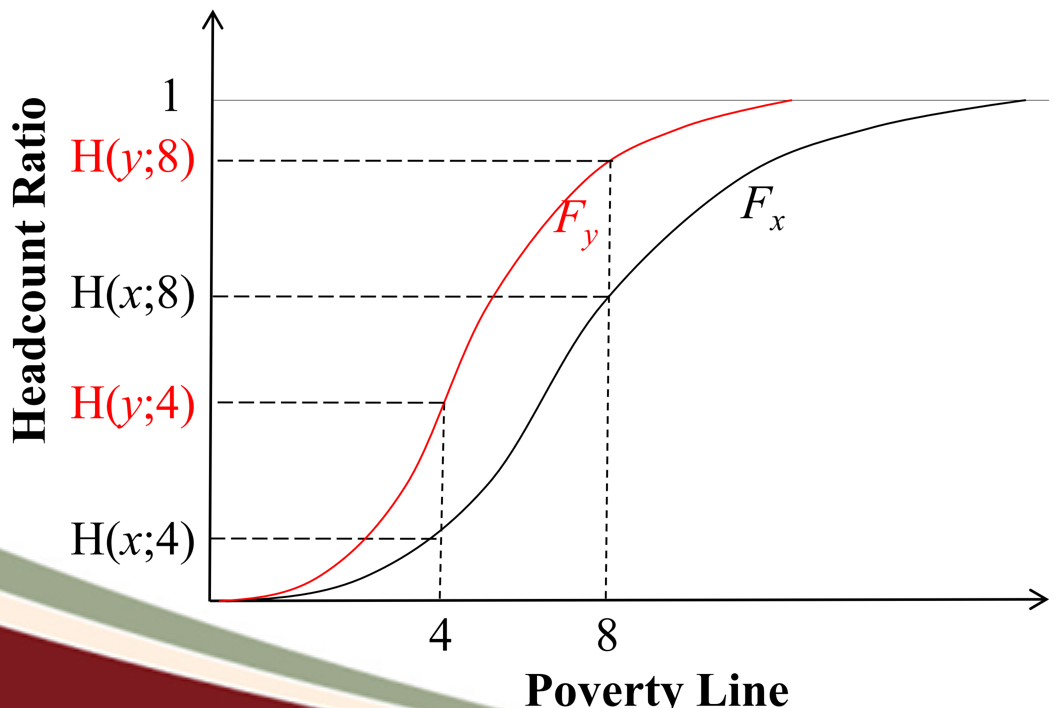
For any poverty line *z*, the CDF of *x* gives the headcount ratio



If the cdf of another distribution *y*, F_y, lies no where to the right of the cdf of x, then *y* has no lower headcount ratio than x for each and every poverty line

Poverty Ordering Based on H

For any poverty line *z*, the CDF of *x* gives the headcount ratio



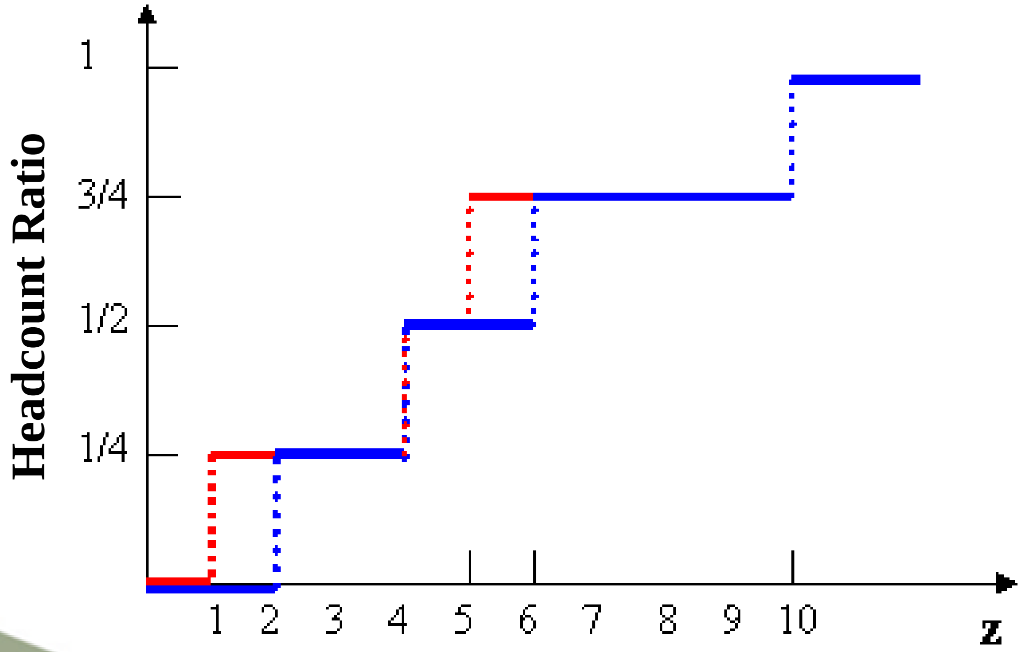
If the cdf of another distribution *y*, F_y, lies no where to the right of the cdf of x, then *y* has no lower headcount ratio than x for each and every poverty line

First order Stochastic Dominance (FSD)

x FSD *y*

Poverty Ordering Based on H

Example of FSD: Let x=(2,4,6,10) and y=(1,4,5,10)

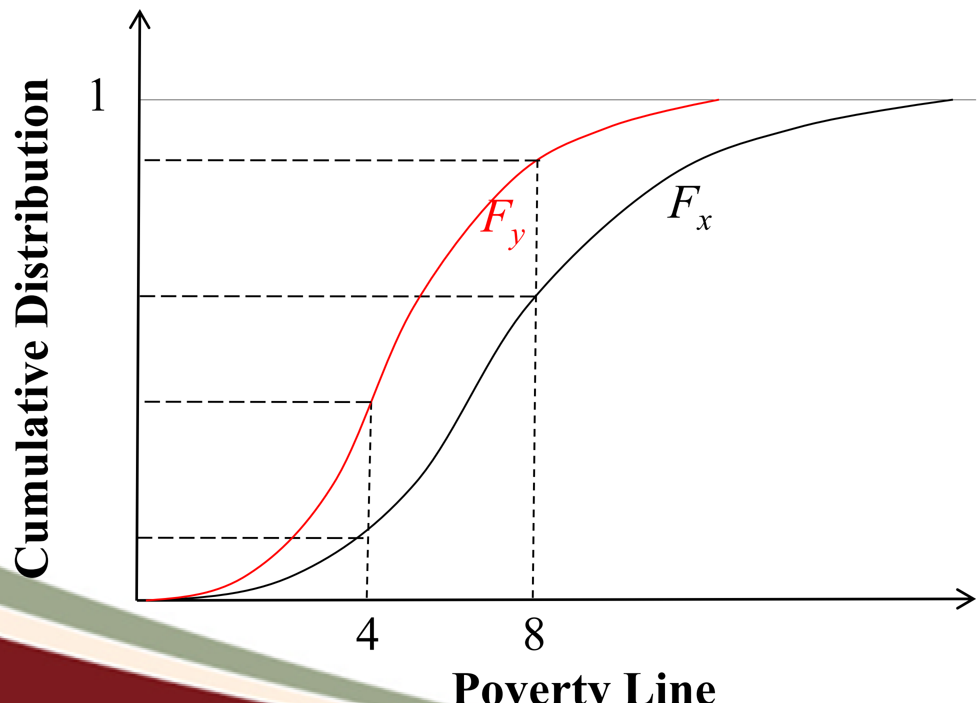


No part of y lies to the right of y

Thus, x FSD y in this case, which means x has unambiguously less poverty than y according to H

Definition of FSD

For distributions x and y, x FSD y if and only if $F_x(b) \leq F_y(b)$ for all b and $F_x(b) < F_y(b)$ for some b, where b is income



How strong is the FSD result?

If FSD holds, then there is agreement for *all* continuous poverty measures satisfying symmetry, focus, scale and replication invariance and monotonicity for all z.

What Happens When CDFs Cross?

- In this case, FSD does not hold for all *z* and H no more provides an unambiguous ranking
  - Can distributions be ranked in this situation based of other measures for all *z*? Foster & Shorrocks (1988)
  - *Second order stochastic dominance* (SSD)
    - This requires comparing the area under the CDFs, which is closely linked to **PG**
  - *Third order stochastic dominance* (TSD)
    - This requires comparing the area under the SSD curves used for checking SSD, which is closely linked to **SG**

Limited Range Poverty Orderings

While deciding the precise value of the poverty line may be difficult, agreement is likely to occur on an interval **z**.

So now the poverty ordering would be defined as

$$P(y;z) \geq P(x;z) \text{ for all } z \text{ in } \mathbf{z}$$

$$\text{and } > \text{ for some } z \text{ in } \mathbf{z}$$

By restricting the values of **z**, the obtained comparisons will be "more complete" than the comparisons based on the entire poverty line

Thank you. Questions.