Decomposing changes in multidimensional poverty in 10 countries.*

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Abstract

Among the burgeoning literature on multidimensional poverty indices, the Alkire-Foster (AF) measure stands out for its resilience to identify the multidimensionally poor with cut-off criteria that cover the spectrum from the union approach to the intersection approach. The intuitive and easy applicability of the identification and aggregation methods used by the index are reflected in ongoing adoption of the AF measure to different applications including topics related and unrelated to poverty measurement. This paper shows intuitive ways to monitor changes in multidimensional poverty across time using the AF measure for cross-sectional data. The empirical applications track changes in poverty for a group of countries using DHS datasets. We find that....

1 Introduction

The insufficiency of an exclusive income approach to poverty has been well established in the literature[1]. This acknowledgment has led, among other things, to consider and quantify poverty as deprivation in multiple dimensions of well-being. Among the methodological approaches used, the counting approach proposes indices based on counting the number of

*This paper draws from the substantial contributions made by Sabina Alkire and Maria Emma Santos in defining the criteria of selection of dimensions, checking the datasets and writing the code for estimating AF measures. These contributions were made to generate the Multdimensional Poverty Index (MPI) rankings of the 2010 Human Development Report.

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[1] See for instance, Sen (2001, chapter 4), and Sen (2009, chapter 12).

dimensions in which people are deprived.[2] The approach has gained recent popularity with the Alkire-Foster (AF) family of poverty indices(Alkire and Foster, 2009). These indices identify the multidimensionally poor by counting the number of dimensions in which they are deprived, which relies on dimension-specific poverty lines, and comparing the number against a multidimensional-deprivation cut-off.[3] By changing the cut-off from 1 to the total number of dimensions, the AF family can adopt any identification criterion ranging from the union to the intersection approach.[4] The AF measures are a function the headcount of multidimensionally poverty, and of the average number of deprivations to the poor (and the average poverty gaps for continuous variables). The intuitive and easy applicability of their identification and aggregation methods are reflected in the ongoing adoption of the AF measure to different applications including topics related and unrelated to poverty measurement.[5]

In this paper we show basic ways to decompose and monitor changes in multdimensional poverty over time, when applying the AF measure to cross-sectional data. The focus on cross-sectional data means that, at this stage, we are neither identifying people in terms of chronic versus transient poverty,[6] nor looking at transitions into and out of poverty.[7] The methodological section develops results that are applicable to discrete variables. Therefore it works with the M0 measure of the AF family, i.e. a measure that is the product of the multidimensional headcount (named H) and the average number of deprivations of the poor (named A).[8] The decomposability of M0 allows for decomposing its changes over time in changes in H, changes in A, and a multiplicative factor. In turn, changes in H and changes in A can further be decomposed into changes in their subcomponents; e.g. changes in subgroup headcounts or changes in the proportion of the poor deprived in a specific dimension. This easy decomposition is appealing for policy purposes, e.g. evaluating poverty impacts of certain programs. [9]

In this paper we document anual rates of changes in the M0 for 10 countries, using DHS data. The experiences are different both in time covered and time period considered. Therefore the presentation of the trends is complemented by country background

[2] For a comparative discussion of approaches to measuring multidimensional poverty, see Atkinson (2003). For a stochastic dominance approach to multidimensional poverty see Duclos, Sahn, and Younger (2006, 2007).

[3] For instance, considering 10 dimensions of wellbeing, a multidimensional-deprivation cut-off of 5 means that a person is considered multidimensionally poor if the person is deprived on 5 or more of the 10 dimensions.

[4] According to the union approach, any person deprived in at least one dimension is considered multidimensionally poor. On the other extreme, the intersection approach demands considering as multidimensionally poor only people who are deprived in every dimension.

[5] For instance, Batana (2008), Santos and Ura (????), Alkire and Seth (????), Battiston, Cruces, Lopez-Calva, Lugo, and Santos (2009), Foster, Horowitz, and Mendez (2009), Azevedo and Robles (2009), Singh (2009), Trafton (2009) and Roche (2009).

[6] The burgeoning chronic poverty literature includes, e.g. Hoy and Zheng (2007); Foster (2007); Calvo and Dercon (2007); Chakravarty and D'Ambrosio (2008); Porter and Quinn (2008); Foster and Santos (2009).

[7] The literature on poverty transitions is very broad. For a recent review see Dercon and Shapiro (2007)

[8] For continuous variables, special decompositions can be derived for members of the AF family that have an average deprivation gap element.

[9] By contrast, other indices, e.g. that of Bourguignon and Chakravarty (2003) can not be decomposed in a similar manner.

macroeconomic information. Most of this paper's countries experienced reductions in multidimensional poverty for the indicators considered. Moreover, in most countries these reductions occur across the range of possible values for the multidimensional-deprivation cut-off. MENTION SOMETHING ABOUT STATISTICAL SIGNIFICANCE (ALSO REGARDING COUNTRIES WHOSE TRENDS DEPEND MORE ON K).

Concerning decompositions of changes in H, we find reductions in the periods and countries covered, which, in turn, tend to be driven mostly by reductions in the multidimensional headcount of rural populations. The decomposition of changes in A yields very interesting results: most countries experienced reductions in the average deprivations of the poor. However, in most countries, the latter took place notwithstanding significant increases in the percentage of the multidimensionally poor who were deprived either in educational, health or living standard dimensions. This results highlights the importance of decomposing and disaggregating the trends in multidimensionally poverty in order to pinpoint the main drivers behind them.

The next section presents the methodology, followed by a presentation of the data. Then the results are presented and discussed. The paper finishes with some concluding remarks.

2 Decomposition of changes in the MPI from one period to another

2.1 Notation

For cross-sectional datasets the information consists of matrices, X^t, for different periods in time. In every period, a matrix X^t has N^t rows representing the sample size in period t. The number of columns is the number of dimensions, D, and it is assumed to be constant across time. A typical attainment element of the matrix in period t is: $x_{nd} \, (\in \mathbb{R})$, that is, the attainment of individual n in dimension d.

In the identification stage, the dimension-specific cut-offs are denoted by z_d; and for the second identification stage dimensions are weighted by weights w_d such that: $w_d \in \mathbb{R}_+ \wedge \sum_d^D w_d = D$. For simplicity of exposition, we also consider the weights: $\theta_d = \frac{w_d}{D}$. The matrix of deprivations is formed by replacing x_{nd} with a deprivation gap g_{nd} such that:

$$\begin{aligned} g_{nd}(k) &= \frac{z_d - x_{nd}}{z_d} \text{ if } z_d > x_{nd} \wedge c_n \geq k \\ g_{nd}(k) &= 0 \text{ otherwise} \end{aligned} \tag{1}$$

where $k \leq D$ is the multidimensional-deprivation cut-off and c_n is the weighted number of deprivations of individual n. If $c_n > k$ individual n is said, and identified, to be multidimensionally poor. $c_n = \sum_{d=1}^{D} w_d I\left(z_d > x_{nd}\right)$ [10]

Now the multidimensional headcount in period t can be defined:

[10] $I\,()$ is an indicator that takes the value of 1 if the expression in parenthesis is true. Otherwise it takes the value of 0.

$$H\left(X^t;Z\right)=\frac{1}{N^t}\sum_{n=1}^{N^t}\left[\sum_{d=1}^{D}w_d g_{nd}\left(k\right)\right]^0$$ [11] (2)

Also the average number of deprivations of the multidimensionally poor in the same period is defined:

$$A\left(X^t;Z\right)=\frac{\sum_{n=1}^{N^t}\sum_{d=1}^{D}w_d\left[g_{nd}\left(k\right)\right]^0}{D\sum_{n=1}^{N^t}\left[\sum_{d=1}^{D}w_d g_{nd}\left(k\right)\right]^0} \quad (3)$$

Definitions (2) and (3) suffice to derive results for $M^0\left(X^t;Z\right)=H^tA^t$.[12] More results, following the ones presented below, can be derived for M^1 and other members of the Alkire-Foster class.

2.2 General results

The following results apply to any dataset but the cross-sectional notation is used for simplicity. Denoting $\Delta\%_a Y\left(t\right)\equiv\frac{Y(t)-Y(t-a)}{Y(t-a)}$ and simplifying notation a bit, the first straight forward result is the following:

$$\Delta\%_a M^0\left(t\right)=\Delta\%_a H\left(t\right)+\Delta\%_a A\left(t\right)+\Delta\%_a H\left(t\right)\Delta\%_a A\left(t\right) \quad (4)$$

In other words, a percentage change in M0 can be decomposed into a percentage change in the number of multidimensionally poor, a percentage change in the average number of deprivation of the multidimensionally poor, and a multiplicative effect. Note that $\Delta\%_a H\left(t\right)$ and $\Delta\%_a A\left(t\right)$ are not independent, but there are circumstances in which a change in one may not necessarily produce a change in the other. For instance, in the extreme case of identifying the poor by the intersection approach $\Delta\%_a A\left(t\right)=0$ and so $\Delta\%_a M^0\left(t\right)=\Delta\%_a H\left(t\right)$.

Another circumstance in which a change in one element may not necessarily produce a change in the other element, is when the proportion of the multidimensionally poor remains the same, but their number of deprivations increases. For this to happen it is necessary that $k<D$.

Result (4) can be further expanded by decomposing both $\Delta\%_a H\left(t\right)$ and $\Delta\%_a A\left(t\right)$. In the case of changes in the headcount it may be of interest to decompose it in terms of changes in headcount for different groups of society. We do this by partitioning society in G non-overlapping groups recalling that:

[11] Alternatively: $H^t=\frac{1}{N^t}\sum_{n=1}^{N^t}I\left(c_n>k\right)$

[12] Z is a vector of dimension D, containing all the z_d.

$$
\begin{aligned}
H\left(X^{t};Z\right) &= \sum_{i=1}^{G}\varphi_{i}^{t}H^{i}\left(X_{i}^{t};Z\right), \qquad (5)\\
H^{i}\left(X_{i}^{t};Z\right) &\equiv \frac{1}{N_{i}^{t}}\sum_{n=1}^{N^{t}}\left[\sum_{d=1}^{D}w_{d}g_{nd}\left(k\right)\right]^{0} I(\text{individual } n \text{ belongs to group } G)\\
\varphi_{i}^{t} &= \frac{N_{i}^{t}}{N^{t}}
\end{aligned}
$$

where N_i^t is the number of individuals belonging to group i in period t. From (5) it is clear that :

$$
\begin{aligned}
\Delta\%_{a}H\left(t\right) &= \sum_{i=1}^{G}\Delta\%_{a}\left[\varphi_{i}^{t}H^{i}\left(X_{i}^{t};Z\right)\right]r_{i}\left(t-a\right) \qquad (6)\\
&= \sum_{i=1}^{G}r_{i}\left(t-a\right)\left[\Delta\%_{a}\varphi_{i}^{t}+\Delta\%_{a}H^{i}\left(X_{i}^{t};Z\right)+\Delta\%_{a}\varphi_{i}^{t}\Delta\%_{a}H^{i}\left(X_{i}^{t};Z\right)\right]
\end{aligned}
$$

Result (6) indicates that the percentage change in the multidimensional headcount can be decomposed into changes in the composition of the population, changes in the percentage of the multidimensionally poor within each group and a multiplicative effect. The relative impact of such changes depends on the initial contributions of every group headcount to the total, i.e. they depend on $r_i\left(t-a\right) \equiv \frac{\varphi_i^{t-a}H^i\left(X_i^{t-a};Z\right)}{H\left(X^{t-a};Z\right)}$.

Similarly $\Delta\%_a A\left(t\right)$ can also be decomposed by noting that $A\left(t\right)=\sum_{d=1}^{D}\theta_d A_d\left(t\right)$ where $A_d\left(t\right)$ is the percentage of the multidimensionally poor deprived in dimension d. Using a similar decomposition as in (5) and (6), the following decomposition is also derived:

$$
\Delta\%_{a}A\left(t\right)=\sum_{d=1}^{D}\Delta\%_{a}\left[\theta_{d}A_{d}\left(X^{t};Z\right)\right]s_{d}\left(t-a\right)=\sum_{d=1}^{D}s_{d}\left(t-a\right)\Delta\%_{a}A_{d}\left(X^{t};Z\right) \qquad (7)
$$

where $s_d\left(t-a\right)=\frac{\theta_d A_d\left(X^{t-a};Z\right)}{A\left(X^{t-a};Z\right)}$. Notice that $\Delta\%_a A\left(t\right)$ is only affected by $\Delta\%_a A_d\left(X^t;Z\right)$, by mediation of the $s_d\left(t-a\right)$, because we keep the dimensional weights constant. Otherwise (7) would look like (6).

DISCUSS THE ANNUALIZATION OF RATES AND THE ISSUE OF TIME IN GENERAL

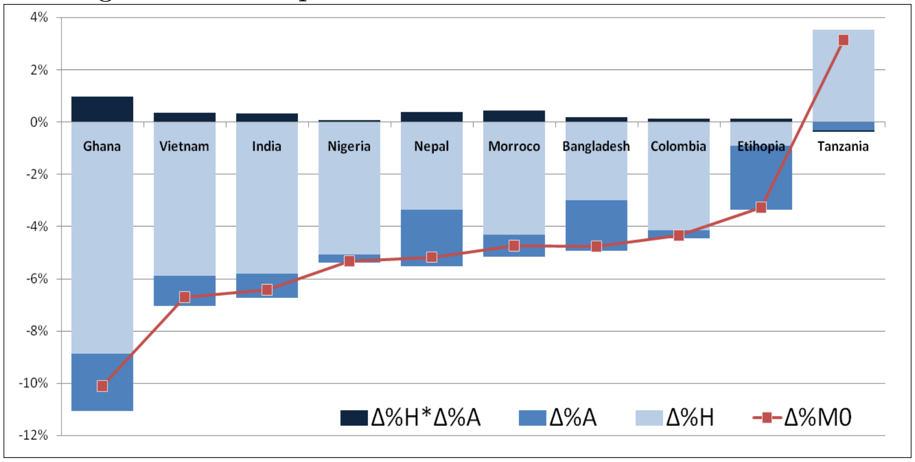
3 Data

Discuss the Datasets, the countries chosen, the years, the indicators chosen.

4 Results

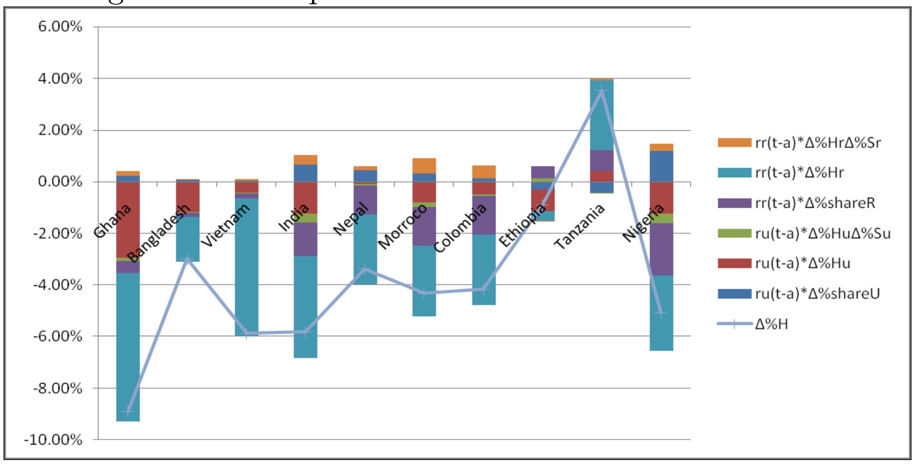
4.1 General decomposition results

Figure 1: Decomposition of M0 for 10 countries and k=3



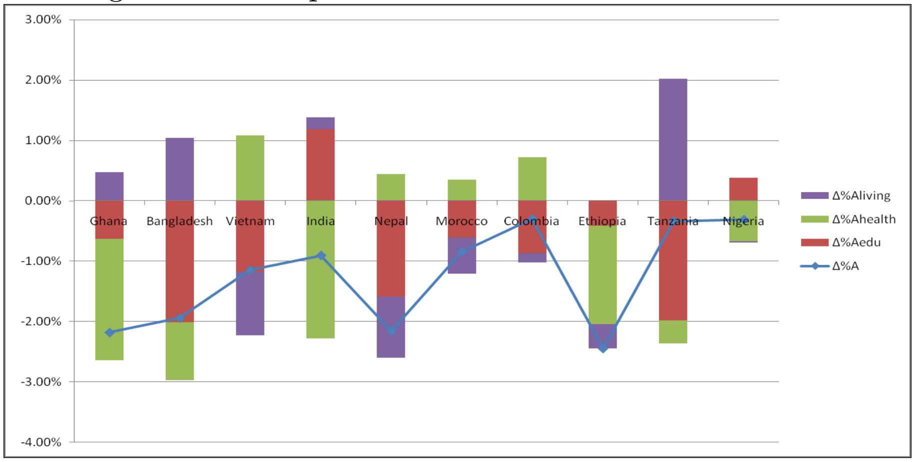
4.2 Specific decomposition results for H

Figure 2: Decomposition of H for 10 countries and k=3



4.3 Specific decomposition results for A

Figure 3: Decomposition of A for 10 countries and k=3



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