Problem Set on Unidimensional Poverty Measurement

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Paper-Based Problems

1. Consider a poverty measure P that satisfies symmetry, replication invariance, scale invariance, focus, monotonicity, and transfer. Please use the axioms to rank the following pair of distributions given the corresponding poverty lines. Please state which axioms and what logic you use.

a. x = (3, 1, 12, 6) with z = 8 and y = (3, 9, 2, 6) with z = 8

Ans: x^{ord} = (1, 3, 6, 12) and y^{ord} = (2, 3, 6, 9). Censoring them we get: x^* = (1, 3, 6, 8) and y^* = (2, 3, 6, 8). x has more poverty by *focus* and *monotonicity*.

b. x = (2, 4, 10) with z =15 and y = (2, 10, 10, 4, 2, 4) with z = 15

Ans. x^{ord} **=** (2, 4, 10) and y^{ord} = (2, 2, 4, 4, 10, 10, 10). Censoring them we get x^* **=** (2, 4, 10) and y^* = (2, 2, 4, 4, 10, 10, 10). Same poverty by *replication invariance*.

c. x = (3, 6, 12) with z = 10 and y = (12, 4, 5) with z = 10

Ans. x^{ord} = (3, 6, 12) and y^{ord} = (4, 5, 12). Censoring them we get x^* = (3, 6, 10) and y^* = (4, 5, 10). x has more poverty by *progressive transfer*.

2. Consider the distribution x = (6, 12, 18, 40). Suppose the income of the third person increases by 5 units over time and distribution y = (6, 12, 23, 40) is obtained.

a. If the poverty line is z = 20, should poverty increase or decrease over time? Why?

Ans. Poverty should decrease by *monotonicity* axiom.

b. Please calculate the Income Gap Ratio (I) of both x and y. What do you find?

Ans. Previously $\mu_p(x)$ = (6+12+18)/3 = 12. So, I(x) = (20 – 12)/20 = 8/20. Later, $\mu_p(y)$ = (6+12)/2 = 9. So, I(y) = (20 – 9)/20 = 11/20. Poverty has actually increased by the income gap ratio.

c. Now calculate the Poverty Gap Ratio (PG). What do you find?

Ans. First find the censored distribution of x: x^* = (6, 12, 18, 20). Then $\mu^*(x)$ = (6 + 12 + 18 + 20)/4 = 14 and PG(x) = (20 – 14)/20 = 6/20. Then, first find the censored distribution of y: y^* = (6, 12, 20, 20). Then $\mu^*(y)$ = (6 + 12 + 20 + 20)/4 = 14.5 and PG(y) = (20 – 14.5)/20 = 5.5/20. The poverty has decreased.

3. Consider the following three distributions: x=(3, 6, 9, 30), y=(3, 8, 9, 31), and u=(4, 5, 9, 30).

a. Calculate the headcount ratios for x, y, and u if z = 10. Are they different: yes or no? Should the headcount ratios be different for x and y? Why? Should the headcount ratios be different for x and u? Why?

Ans. In all cases, the headcount ratio is ¾. Yes, they should be different. y should have less poverty than x by monotonicity. u should have less poverty than x by transfer.

b. Calculate the poverty gap ratios for x, y, and u if z = 10. Are they different: yes or no? Why? How do your results differ from the results of part a.?

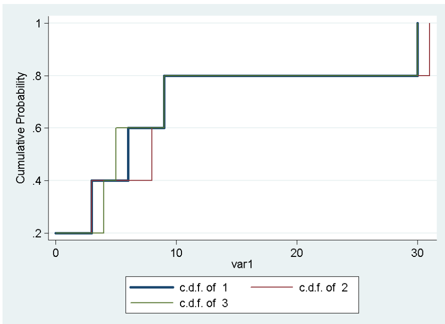
Ans. PG(x) = PG(u) = 0.3 and PG(y) = 0.25. Yes, they are different partly. x and y are different as PG satisfies monotonicity, but x and u are not as it does not satisfy transfer.

c. Calculate the squared poverty gaps for x, y, and u if z = 10. Are they different: yes or no? Why? How do your results differ from the results of part a. and part b.?

Ans. SG(x) = 0.165, SG(y) = 0.135, and SG(u) = 0.155. Poverty in both y and u are lower than that in x because SG satisfies both monotonicity and transfer.

d. Can you state if any of these three distributions has unambiguously higher poverty than others when poverty is measured by the headcount ratio. [Hint: Draw the cumulative distribution functions (CDF).]

Ans. As can be seen that distribution y (cdf 2 in red) FSD x (cdf 1 in bold blue). Other two pairs do to FSD as they cross each other.



Extra Question (for Interested Students)

4. Consider the following two distributions: x = (3, 6, 9, 12) and y = (3, 6, 8, 10, 12). Can you verify if one distribution has unambiguously higher poverty than another distribution if poverty is assessed by a measure that satisfies symmetry, scale invariance, replication invariance, and focus?

Ans. As we see that the cdfs of the two vectors intersect and so a poverty index satisfying only these axioms cannot rank these two distributions unambiguously.