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## Institute for International Economic Policy (IIEP)
## George Washington University
## Washington, DC

Tabita, Kenya — Rabiya, India — Stéphanie, Madagascar — Agathe, Madagascar — Dalma, Kenya — Ann-Sophia, Kenya — Valérie, Madagascar

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# Shapley Decomposition of Changes Over Time

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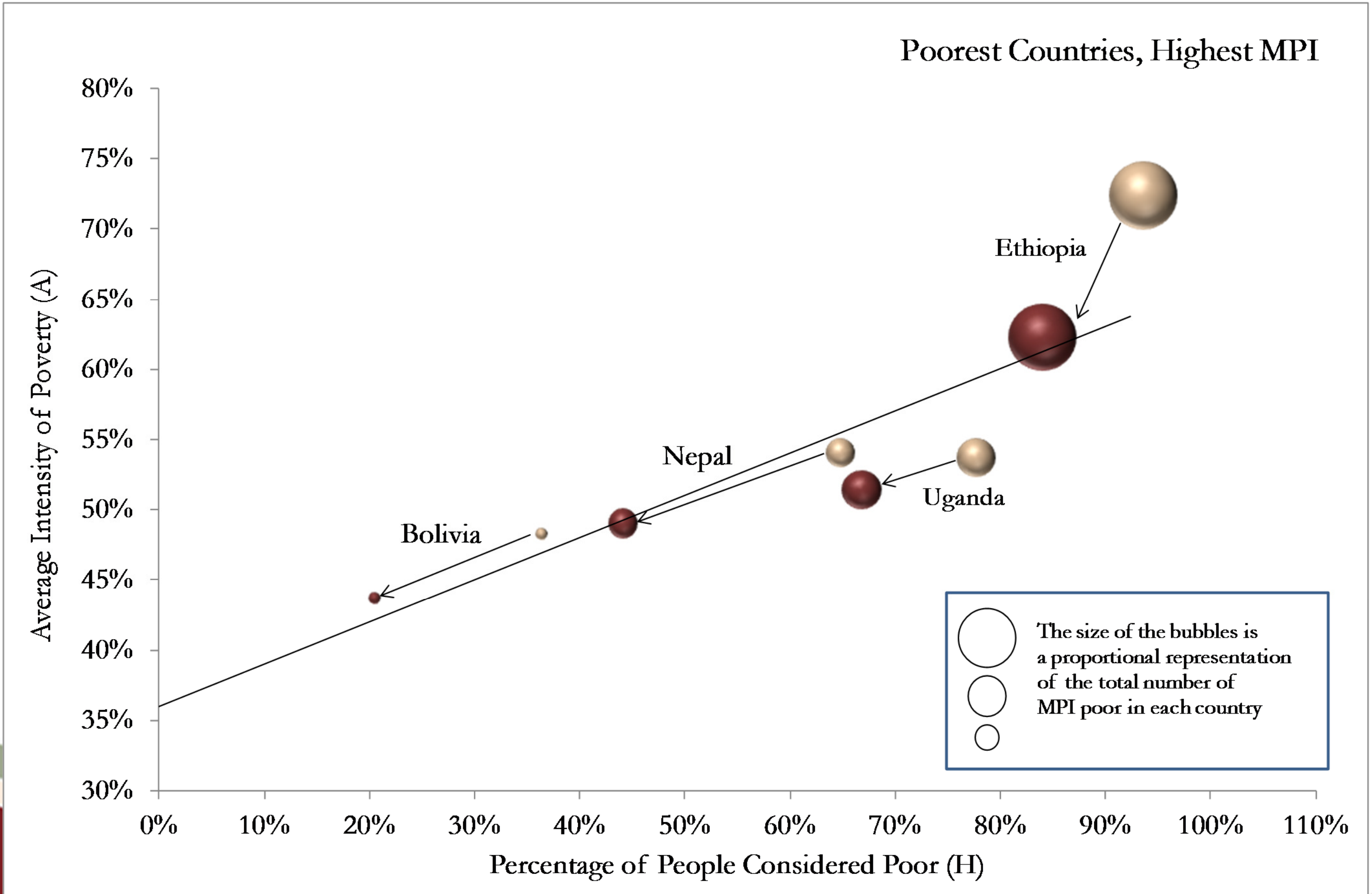
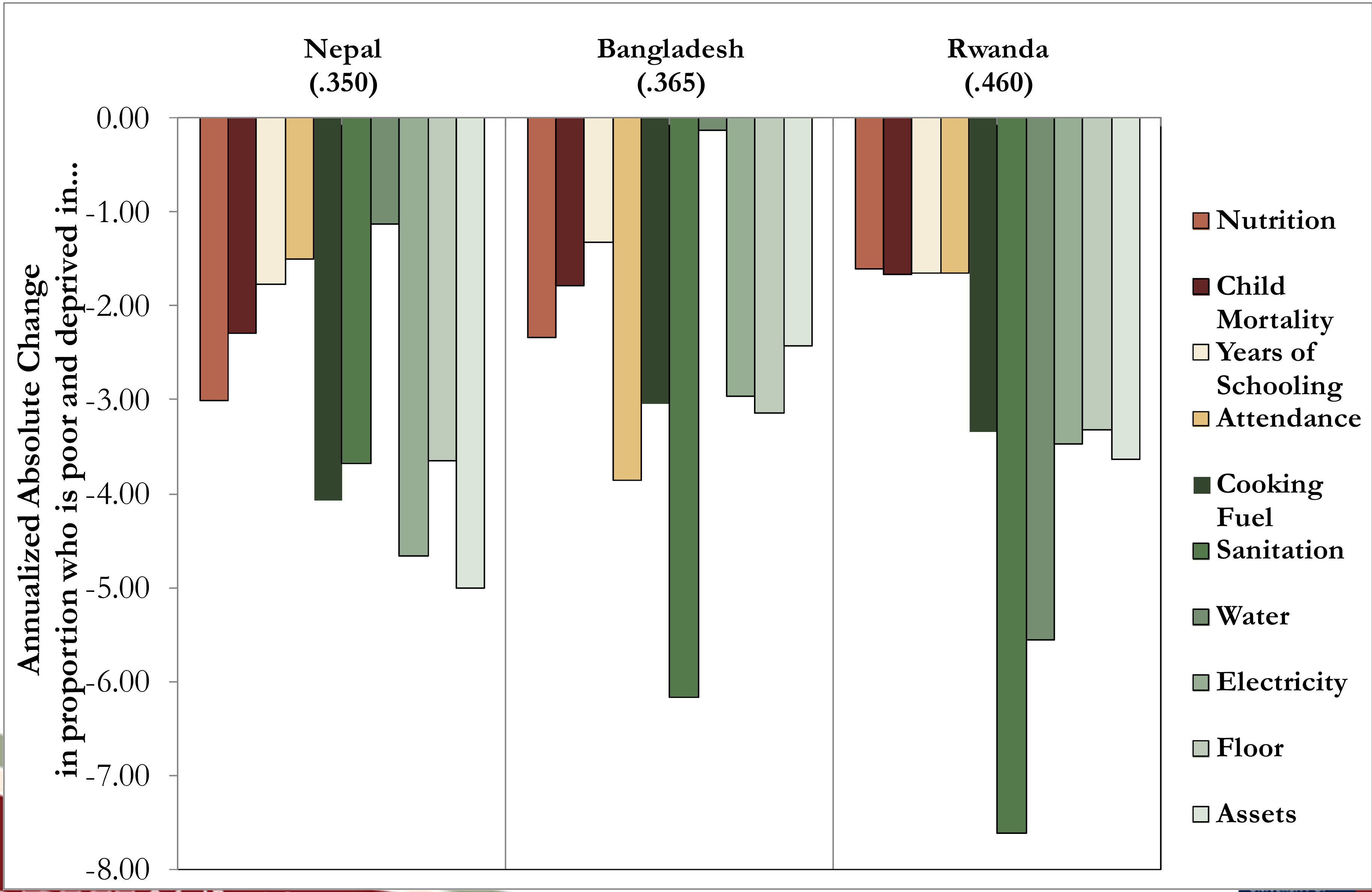
# Changes in Bolivia, Ethiopia, Nepal and Uganda



# How the best countries reduced MPI



In this example we know how each individual have change over time, like in panel data, when using cross sectional data we do not have this level of detail.

## Time 1

$g_0(k>=2)$:

| $x_{i1}$ | $x_{i2}$ | $x_{i3}$ | $x_{i4}$ | $x_{i5}$ | $C_i(k>=2)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 2 |
| 1 | 1 | 1 | 0 | 1 | 4 |
| 1 | 1 | 1 | 1 | 1 | 5 |

CenH: [ 2/5 3/5 3/5 1/5 2/5 ]

H= 3/5

A= 11/5 * 1/3 =11/15

M0 = 3/5 * 11/5 = 11/25

## Time 2

$g_0(k>=2)$:

| $x_{i1}$ | $x_{i2}$ | $x_{i3}$ | $x_{i4}$ | $x_{i5}$ | $C_i(k>=2)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 4 |
| 1 | 0 | 0 | 1 | 0 | 2 |

CenH: [ 2/5 1/5 1/5 1/5 1/5 ]

H= 2/5

A= 6/5*1/2 = 6/10

M0 = 2/5 * 6/10 = 6/25

# Variation over time

Absolute Change:

$$\Delta X = (X^{t'} - X^{t})$$

Relative Change:

$$\%\Delta X = (X^{t'} - X^{t}) / X^{t}$$

If we would like to compare different periods

Annualized Absolute Change:

$$\Delta X = (X^{t'} - X^{t}) / (t' - t)$$

Annualized Relative Change:

$$\%\Delta X = (X^{t'} - X^{t}) / X^{t} (t' - t)$$


# Group decomposition of change in poverty

# Group decomposition of change in poverty

Since $M_0$ is 'decomposable' we know that it can be obtained from the population weighted average of the subgroup poverty levels:

$$M_{0_t} = \sum_{g=1}^{G} n_{gt} M_{0\,gt}$$

The variation of M0 can be expressed as: $\Delta M_0 = \sum_{g=1}^{G} \left( n_{g2} M_{0\,g2} - n_{g1} M_{0\,g1} \right)$

More intuitively...

| | n (t-1) | n (t=2) | M0 (t=1) | M0 (t=2) |
|---|---|---|---|---|
| Group 1 | 15% | 20% | 0.065 | 0.047 |
| Group 2 | 22% | 30% | 0.110 | 0.085 |
| Group 3 | 30% | 30% | 0.205 | 0.189 |
| Group 4 | 33% | 20% | 0.312 | 0.275 |
| Total | 100% | 100% | 0.198 | 0.147 |

There are changes in $M_{0g}$ and also in the population share $n_g$.

**Can we decompose each effect?**

# Group decomposition of change in poverty

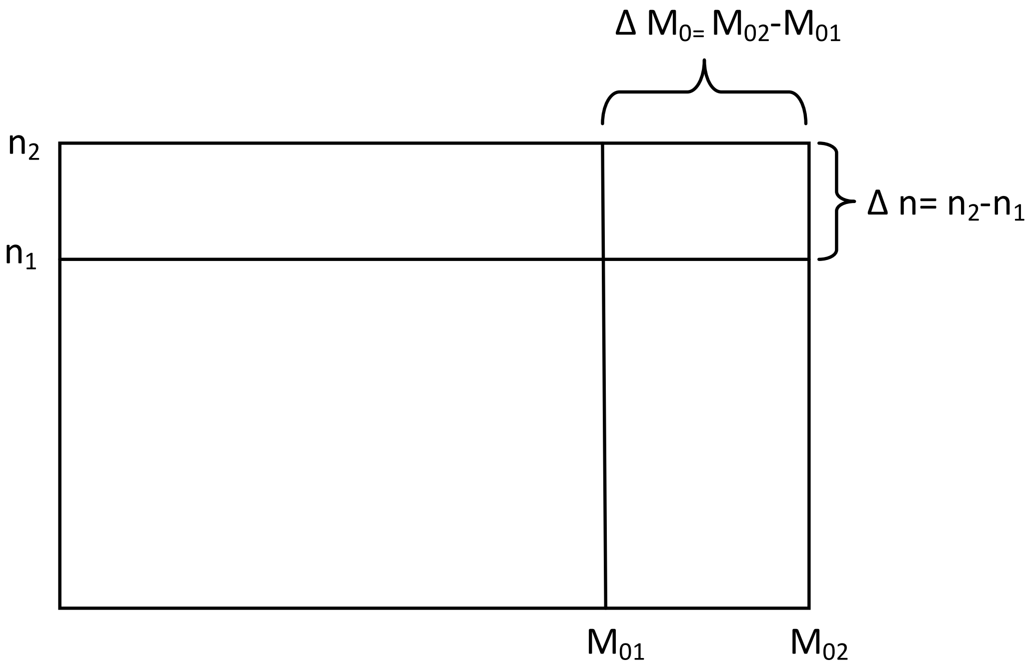
Following a similar decomposition of change in FGT income poverty measures (Ravallion and Huppi, 1991), the variation in poverty level can be broken down in three components:

1) changes due to intra-sectoral or within-group poverty effect,

2) changes due to demographic or inter-sectoral effect, and

3) the interaction effect which are changes due to the possible correlation between intra-sectoral and inter-sectoral.

# Group decomposition of change in poverty




The interaction effect is difficult to interpret and there is an arbitrariness in the period of reference

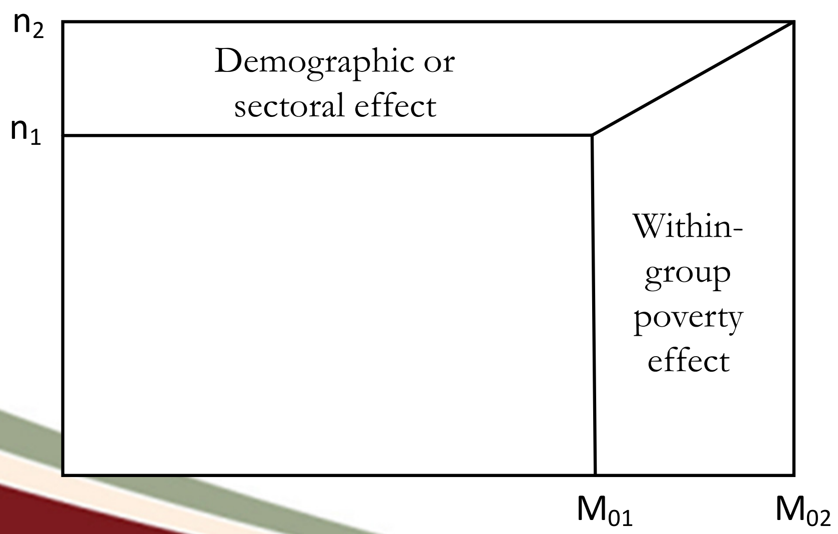
So the overall change in the adjusted headcount between two periods $t$ (1 and 2) can be express as follows:

$$\Delta M_0 = \underbrace{\sum_{g=1}^{G} n_{g1}\left(M_{0g2} - M_{0g1}\right)}_{\text{Within-group poverty effect}} + \underbrace{\sum_{g=1}^{G} M_{0g1}\left(n_{g2} - n_{g1}\right)}_{\text{Demographic or sectoral effect}} + \underbrace{\sum_{g=1}^{G} \left(M_{0g2} - M_{0g1}\right)\left(n_{g2} - n_{g1}\right)}_{\text{Interaction or error term (within-group * demographic)}}$$

# Group decomposition of change in poverty

Following Shorrocks (1999), after applying a Shapley decomposition approach we obtain:

$$\Delta M_0 = \underbrace{\sum_{g=1}^{G} \frac{(n_{g1} + n_{g2})}{2} (M_{0g2} - M_{0g1})}_{\text{Within-group poverty effect}} + \underbrace{\sum_{g=1}^{G} \frac{(M_{0g1} + M_{0g2})}{2} (n_{g2} - n_{g1})}_{\text{Demographic or sectoral effect}}$$




The contribution of a given factor is equal to its expected marginal contribution

# Group decomposition of change in poverty

| | Within-group Effect | Demographic effect | ΔM0 |
|---|---|---|---|
| Group 1 | 2.7% | 0.8% | 3.5% |
| Group 2 | 23.1% | 8.5% | 31.5% |
| Group 3 | 34.3% | -0.1% | 34.1% |
| Group 4 | 8.9% | 0.5% | 9.4% |
| Group 5 | 23.8% | -2.8% | 21.0% |
| Group 6 | 8.6% | -8.2% | 0.4% |
| Overall Population | 101.4% | -1.4% | 100% |

✓ Group 3 contributes the most to overall poverty reduction which is almost exclusively due to a within-group effect.

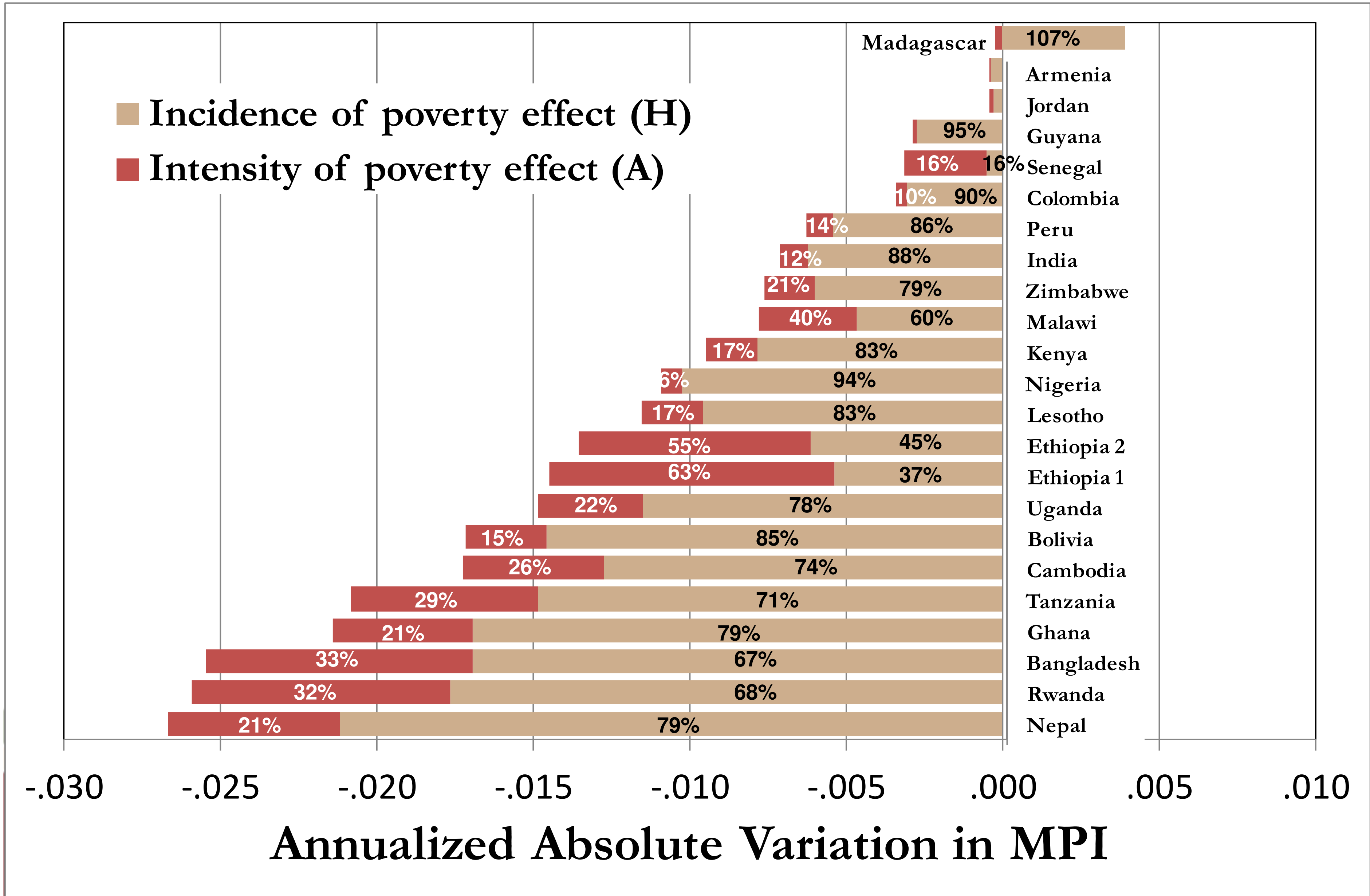
✓ Group 2 contributes nearly as much as group 3 but part of the effect is due to reducing the population share.

✓ Despite reducing poverty, group 6 had an almost cero overall effect because of an increase in population share

✓ The marginal figures shows how much the overall within-group effect would have been if we extract the demographic effect


# Decomposition by incidence and intensity

# Intensity and Incidence: both reduce MPI

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# Decomposition by incidence and intensity

Since the adjusted headcount can be expressed as the product of the incidence of poverty times the intensity of poverty, $M_{0_t} = H_t * A_t$ one might also want to decompose variation in the adjusted headcount by changes in these two components to obtain:

1) changes due to variation in the incidence of poverty, and

2) changes due to variation in the intensity of poverty

# Decomposition by incidence and intensity

Closely to Apablaza and Yalonetzky (2011) and following a Shapley decomposition (Shorrocks 1999), changes in the adjusted headcount can be decompose as follows:

$$\Delta M_0 = \underbrace{\frac{(A_1 + A_2)}{2}(H_2 - H_1)}_{\text{Incidence of poverty effect}} + \underbrace{\frac{(H_1 + H_2)}{2}(A_2 - A_1)}_{\text{Intensity of poverty effect}}$$

# Decomposition by incidence and intensity

| | Poverty incidence effect | Intensity of poverty effect | ΔM0 |
|---|---|---|---|
| Group 1 | 34.1% | 65.9% | 100% |
| Group 2 | 82.5% | 17.5% | 100% |
| Group 3 | 67% | 33% | 100% |
| Group 4 | 83.6% | 16.4% | 100% |
| Group 5 | 69.2% | 30.8% | 100% |
| Group 6 | 72.7% | 27.3% | 100% |
| Overall Population | 72.1% | 27.9% | 100% |

✓ Poverty reduction in Group 4 is mainly driven by a reduction in the incidence of poverty

✓ Poverty reduction in Group 1 is mainly driven by a reduction in the intensity of poverty

Tabita, Kenya Rabiya, India Stéphanie, Madagascar Agathe, Madagascar Dalma, Kenya Ann-Sophia, Kenya Valérie, Madagascar

# Decomposition of the variation in intensity of poverty by dimension

# Variation in $M_0$ and its components

## (figures from Roche 2013, Child Poverty)

The raw and censored headcount tells us about the reduction in each dimension and its relation to multidimensional poverty reduction.

**We can compute the contribution of each dimension to changes in intensity**

| | 1997 | 2000 | Absolute Variation 1997-2000 |
|---|---|---|---|
| $M_0$ | 0.555 | 0.495 | -6%*** |
| H | 82.9% | 75.8% | -7.1%*** |
| A | 66.9% | 65.3% | -1.6%*** |
| **Raw Headcount ratio** | | | |
| Health | 43.5% | 39.8% | -3.7%** |
| Nutrition | 74.3% | 62.2% | -12.1%*** |
| Water | 4.7% | 3.6% | -1.1% |
| Sanitation | 72.5% | 68.4% | -4.1%** |
| Shelter | 95.9% | 94.1% | -1.8%** |
| Information | 68.5% | 65.3% | -3.2%* |
| **Censored Headcount ratio (** | | | |
| Health | 41.3% | 37.1% | -4.1%** |
| Nutrition | 68.4% | 56.0% | -12.5%*** |
| Water | 4.6% | 3.4% | -1.2% |
| Sanitation | 69.8% | 63.8% | -6.0%*** |
| Shelter | 82.6% | 75.4% | -7.3%*** |
| Information | 66.0% | 61.3% | -4.7%*** |

**Note:** *** statistically significant at α=0.01, ** statistically significant at α=0.05, * statistically significant at α=0.10

# Raw and Censored Headcount Ratios

From previous sessions we know that…

**Raw headcount:** The raw headcount of dimension *j* represents the proportion of deprived people in dimension *j*, given by

$$H_j = |g_j^0| / n$$

**Censored headcount:** The censored headcount represents the proportion deprived and poor people in dimension *j*. It is computed from the censored deprivation matrix by

$$Ch_j = |g_j^0(k)| / n$$

**Intensity of poverty:** The intensity of poverty is define as the average deprivations shared across the poor and is given by

$$A = |c(k)| / (q)$$

# Decomposition of the variation in intensity of poverty by dimension

Following Apablaza and Yalonetzky (2011), we know that when dimensional weight is constant across period, the absolute change in intensity can be decomposed as follows

$\Delta A = \sum_{d=1}^{D} w_d (A_{2d} - A_{1d})$ where $w_{td}$ denotes the dimensional weight and $A_{td}$ the shared of the poor that are deprived in dimension $d$ at time $t$

Since $A_{td} = Ch_{td} / H_t$ the same decomposition can be expressed in terms of censored headcount as

$$\Delta A = \sum_{d=1}^{D} w_d \left(\frac{Ch_{2d}}{H_2} - \frac{Ch_{1d}}{H_1}\right)$$

# Decomposition by incidence and intensity

## (figures from Roche 2013, Child Poverty)

| | Contribution |
|---|---|
| $M_0$ | 100% |
| H | 72% |
| A | 28% |
| ΔA | 100% |
| Health | 63% |
| Nutrition | 24% |
| Water | 3% |
| Sanitation | 14% |
| Shelter | 0% |
| Information | -3% |

The contribution helps to understand the relation between changes in multidimensional poverty and changes in raw and censored headcount. It helps to analyse this together as it is mediated by the identification step

| | 1997 | 2000 | Absolute Variation 1997-2000 |
|---|---|---|---|
| $M_0$ | 0.555 | 0.495 | -6%*** |
| H | 82.9% | 75.8% | -7.1%*** |
| A | 66.9% | 65.3% | -1.6%*** |
| **Raw Headcount ratio** | | | |
| Health | 43.5% | 39.8% | -3.7%** |
| Nutrition | 74.3% | 62.2% | -12.1%*** |
| Water | 4.7% | 3.6% | -1.1% |
| Sanitation | 72.5% | 68.4% | -4.1%** |
| Shelter | 95.9% | 94.1% | -1.8%** |
| Information | 68.5% | 65.3% | -3.2%* |
| **Censored Headcount ratio** | | | |
| Health | 41.3% | 37.1% | -4.1%** |
| Nutrition | 68.4% | 56.0% | -12.5%*** |
| Water | 4.6% | 3.4% | -1.2% |
| Sanitation | 69.8% | 63.8% | -6.0%*** |
| Shelter | 82.6% | 75.4% | -7.3%*** |
| Information | 66.0% | 61.3% | -4.7%*** |

**Note:** *** statistically significant at α=0.01, ** statistically significant at α=0.05, * statistically significant at α=0.10

# Decomposition can also be undertaken simultaneously

## (figures from Roche 2013, Child Poverty)

We can analyze simultaneously subgroup contribution to reduction in M0, while also looking at the contribution of reduction in incidence and intensity as well as of each dimension to reduction in intensity

| | Barisal | Chittag | | | | | |
|---|---|---|---|---|---|---|---|
| **% Contribution (based on 2007 figures):** | | | | | | | |
| Population | 6.5% | 21.1% | 31.4% | 10.0% | 22.1% | [illegible] | 100% |
| Multidimensional Headcount ratio (H) | 7.5% | 19.9% | 31.3% | 8.8% | 22.9% | 9.6% | 100% |
| Multidimensional Child Poverty Index (M0) | 7.6% | 20.3% | 31.1% | 8.6% | 22.2% | 10.3% | 100% |
| **Decomposition variation in Multidimensinal Child Poverty (Period 1997/2000)** | | | | | | | |
| Total % contribution (ΔM0 for Bagladesh = 100) | 3.5% | 31.5% | 34.1% | 9.4% | 21.0% | 0.4% | 100% |
| → Demographic effect | 0.8% | 8.5% | -0.1% | 0.5% | -2.8% | -8.2% | -1.4% |
| → Within-group effect: | 2.7% | 23.1% | 34.3% | 8.9% | 23.8% | 8.6% | 101.4% |
| → Incidence of poverty effect (H) | 0.9% | 19.0% | 22.9% | 7.4% | 16.5% | 6.2% | 73.0% |
| → Intensity of poverty effect (A): | 1.8% | 4.0% | 11.4% | 1.5% | 7.3% | 2.3% | 28.4% |
| → Health effect (in reducing intensity) | 0.8% | 3.6% | 6.1% | 1.5% | 4.3% | 1.1% | 17.3% |
| → Nutrition (in reducing intensity) | 0.7% | 1.2% | 1.9% | 0.9% | 1.2% | 0.8% | 6.6% |
| → Water (in reducing intensity) | 0.5% | -0.7% | 0.6% | -0.5% | 0.8% | 0.4% | 1.1% |
| → Sanitation (in reducing intensity) | 0.1% | 0.2% | 2.1% | -0.4% | 1.6% | 0.4% | 4.0% |
| → Shelter (in reducing intensity) | 0.0% | 0.1% | 0.0% | 0.0% | -0.1% | 0.0% | -0.1% |
| → Information (in reducing intensity) | -0.3% | -0.3% | 0.8% | 0.0% | -0.4% | -0.4% | -0.7% |

# Suggestion

1. The starting point is the simple analysis of variation of changes in $M_0$ and its elements – often it is informative to analyze both absolute and relative variation
2. Check for statistical significance of differences that are key for your analysis. Reporting the SE and/or Confidence Interval is a good practice so the reader can make other comparison
3. Undertake robustness test of the main findings (by range of weights, deprivation cut-off and poverty cut-offs)
4. The analysis of changes in $M_0$ should be undertaken integrated with changes in its elements: incidence, intensity, and dimensional changes (it is useful to analyze both raw and censored headcount)
5. It is important for policy to differentiate the within group effect and demographic effect when analysing the contribution of each subgroup to overall change in Multidimensional Poverty. The demographic factors can further be studied with demographic data regarding population growth and migration.


# Thank you

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# APPENDIX:
# The AF method and two points in time – variation in $M_0$ and its constitutive elements

# The AF method and two points in time

## Time 1

| | $x_{i1}$ | $x_{i2}$ | $x_{i3}$ | $x_{i4}$ | $x_{i5}$ | $c_i$ |
|---|---|---|---|---|---|---|
| $g_0$: | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 1 | 0 | 0 | 0 | 1 |
| | 0 | 1 | 1 | 0 | 0 | 2 |
| | 1 | 1 | 1 | 0 | 1 | 4 |
| | 1 | 1 | 1 | 1 | 1 | 5 |
| RawH: | 2/5 | 4/5 | 3/5 | 1/5 | 2/5 | |

# The AF method and two points in time

## Time 1

$g_0(k>=2)$:

| $x_{i1}$ | $x_{i2}$ | $x_{i3}$ | $x_{i4}$ | $x_{i5}$ | $C_i(k>=2)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | <u>0</u> | 0 | 0 | 0 | <u>0</u> |
| 0 | 1 | 1 | 0 | 0 | 2 |
| 1 | 1 | 1 | 0 | 1 | 4 |
| 1 | 1 | 1 | 1 | 1 | 5 |

CenH: [ 2/5 <u>3/5</u> 3/5 1/5 2/5 ]

H= 3/5

A= 11/5 * 1/3 =11/15

M0 = 3/5 * 11/5 = 11/25

# The AF method and two points in time

## Time 1

| | $x_{i1}$ | $x_{i2}$ | $x_{i3}$ | $x_{i4}$ | $x_{i5}$ | $c_i$ |
|---|---|---|---|---|---|---|
| $g_0$: | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 1 | 0 | 0 | 0 | 1 |
| | 0 | 1 | 1 | 0 | 0 | 2 |
| | 1 | 1 | 1 | 0 | 1 | 4 |
| | 1 | 1 | 1 | 1 | 1 | 5 |
| RawH: | 2/5 | 4/5 | 3/5 | 1/5 | 2/5 | |

## Time 2

| | $x_{i1}$ | $x_{i2}$ | $x_{i3}$ | $x_{i4}$ | $x_{i5}$ | $c_i$ |
|---|---|---|---|---|---|---|
| $g_0$: | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 1 | 0 | 0 | 1 |
| | 1 | 1 | 1 | 0 | 1 | 4 |
| | 1 | 0 | 0 | 1 | 0 | 2 |
| RawH: | 2/5 | 1/5 | 2/5 | 1/5 | 1/5 | |

In this example we know how each individual have change over time, like in panel data, when using cross sectional data we do not have this level of detail.

## Time 1

$g_0(k>=2)$:

| | $x_{i1}$ | $x_{i2}$ | $x_{i3}$ | $x_{i4}$ | $x_{i5}$ | $C_i(k>=2)$ |
|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 1 | 1 | 0 | 0 | 2 |
| | 1 | 1 | 1 | 0 | 1 | 4 |
| | 1 | 1 | 1 | 1 | 1 | 5 |
| CenH: | 2/5 | 3/5 | 3/5 | 1/5 | 2/5 | |

H= 3/5

A= 11/5 * 1/3 =11/15

M0 = 3/5 * 11/5 = 11/25

## Time 2

$g_0(k>=2)$:

| | $x_{i1}$ | $x_{i2}$ | $x_{i3}$ | $x_{i4}$ | $x_{i5}$ | $C_i(k>=2)$ |
|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 1 | 1 | 0 | 1 | 4 |
| | 1 | 0 | 0 | 1 | 0 | 2 |
| CenH: | 2/5 | 1/5 | 1/5 | 1/5 | 1/5 | |

H= 2/5

A= 6/5*1/2 = 6/10

M0 = 2/5 * 6/10 = 6/25

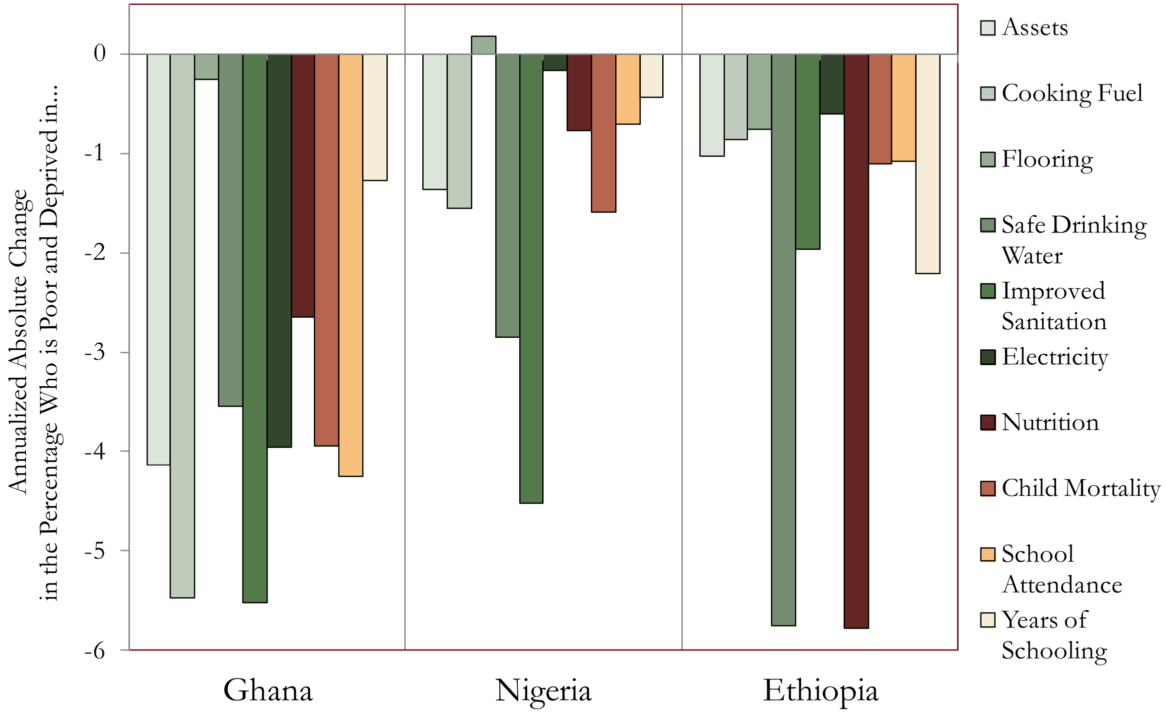
# The AF method and two points in time: Consider $c_i$ vector or CH vector

| Time 1 | Time 2 | Variation |
|---|---|---|
| RawH: [ 2/5 4/5 3/5 1/5 2/5 ] | RawH: [ 2/5 1/5 2/5 1/5 1/5 ] | ΔRawH: [ 0 -3/5 -1/5 0 -1/5 ] |
| CenH: [ 2/5 3/5 3/5 1/5 2/5 ] | CenH: [ 2/5 1/5 1/5 1/5 1/5 ] | ΔCenH: [ 0 -2/5 -1/5 0 -1/5 ] |
| H= 3/5 | H= 2/5 | ΔH= -1/5 |
| A= 11/5 * 1/3 =11/15 | A= 6/5*1/2 = 6/10 | ΔA= -2/15 |
| M0 = 3/5 * 11/5 = 11/25 | M0 = 2/5 * 6/10 = 6/25 | ΔM0 = -5/15 |

It won't stop here – we could also perform further analysis on inequality among the poor based on the $c_i$ vector or assessing changes in association or joint distribution

# Reporting changes in Censored Heacount

| Time 1 | Time 2 | Variation |
|---|---|---|
| CenH: [ 2/5 3/5 3/5 1/5 2/5 ] | CenH: [ 2/5 1/5 1/5 1/5 1/5 ] | ΔCenH: [ 0 -2/5 -1/5 0 -1/5 ] |




*Computing change*

$$\Delta X=(X_2-X_1)$$