Problem Set on Multidimensional Poverty Measures – Alkire and Foster's Measures

Answer Key

Paper-Based Problems:

Given the following matrix of distribution of three dimensions (income, self rated health, and years of education):

$$X = \begin{bmatrix} 4 & 1 & 5 \\ 8 & 4 & 6 \\ 12 & 1 & 11 \\ 3 & 4 & 6 \\ 15 & 1 & 9 \\ 12 & 5 & 12 \end{bmatrix}$$

a) Calculate H, M0, M1 and M2 using a cutoff value of k=2 and equal weights. Assume that the poverty lines are (10, 3 and 8 correspondingly).

Recall that:

$M_0 = \mu(g_0(k)) = HA$ where $A = |c(k)| / qd$ (please note that the bars $|c(k)|$ denote summation of the elements of a vector or the elements of a matrix. In this case we are adding up all the elements of the censored vector of deprivation counts.

$M_1 = \mu(g_1(k)) = HAG$ where $G = |g_1(k)| / |g_0(k)|$

$M_2 = \mu(g_2(k)) = HAS$ $S = |g_2(k)| / |g_0(k)|$

$$g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad c = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \quad c(k) = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} \quad g^0(k) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

H and M0:

Calculated as the mean of the matrix:

$M_0 = 7/18$ = 0.389

Also note that:

$H = 1/2$ (Half of the population is deprived in 2 or more dimensions)

$A = \left(\frac{3+2+2}{3}\right)\frac{1}{3} = \frac{7}{9}$ = 0.778 On average, those deprived in 2 or more dimensions, are deprived in 78% of the considered dimensions, which –in this case- means they are deprived, on average in 2.34 dimensions.)

You can verify that indeed: M0=HA= 0.5 * 0.778 = 0.389

M1:

$$g^1(k) = \begin{bmatrix} 0.6 & 0.666 & 0.375 \\ 0.2 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0.7 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Calculated as the mean of the matrix:

M_1 = 3.041 /18 = 0.169

Also note that:

$$G = \frac{(0.6 + 0.666 + 0.375 + 0.2 + 0.25 + 0.7 + 0.25)}{7} = 0.435$$

So it can be verified that: M_1 = HAG = 0.5*0.778*0.435 = 0.169

M2:

$$g^2(k) = \begin{bmatrix} 0.36 & 0.444 & 0.141 \\ 0.04 & 0 & 0.0625 \\ 0 & 0 & 0 \\ 0.49 & 0 & 0.0625 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Calculated as the mean of the matrix:

$M_2 =$ 1.6 / 18 = 0.089

Also note that:

$$S = \frac{(0.36 + 0.444 + 0.141 + 0.04 + 0.0625 + 0.49 + 0.0625)}{7} = 0.229$$

So it can be verified that: M_2 = HAS = 0.5*0.778*0.229 = 0.089

Which is the contribution of each dimension to M0?

The contribution of each dimension to overall Malpha is given by:

$$\left(\mu(g^{\alpha}_{\bullet j}(k))/d\right)/M_{\alpha}(y;z)$$

Income contribution=(3/6)(1/3)/(7/18) = 0.429

Health contribution=(1/6)(1/3)/(7/18) = 0.143

Education contribution=(3/6)(1/3)/(7/18) = 0.429

Obviously, the sum of the contribution of the dimensions needs to be 1.

Which is the contribution of the group of the first three individuals to overall M1?

$$M(x,y;z) = \frac{n_x}{n}M(x;z) + \frac{n_y}{n}M(y;z)$$

So the contribution of each group is: $C(x) = \left((n_x/n)M(x;z)\right)/M(x,y;z)$

In this case, group 'x' is composed of the first three people, so it is as taking a submatrix of the whole g1 matrix:

$$g^1{}_x(k) = \begin{bmatrix} 0.6 & 0.666 & 0.375 \\ 0.2 & 0 & 0.25 \\ 0 & 0 & 0 \end{bmatrix}$$

$M_1(x;z)$ = 2.091 / 9 = 0.232

And $n_x/n=1/2$

Therefore, the contribution is given by:

C(x) = (1/2) (0.232) / 0.169 = 0.686

This means that the group of the first two people contributes with 68% of overall multidimensional poverty measured by M1.#You can verify that the other three people contribute with the other 32%

I. What happens to each of the measures if individual 2 reported a health status of 2 instead of 4?

He becomes poor in one additional dimension. H will not change. M0, M1 and M2 will increase.

b) Calculate H, M0, M1 and M2 using nested weights: assigning a value of 2 to income, and 0.5 to health and education respectively.

$$g^0 = \begin{bmatrix} 2 & 0.5 & 0.5 \\ 2 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 2 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad c = \begin{bmatrix} 3 \\ 2.5 \\ 0.5 \\ 2.5 \\ 0.5 \\ 0 \end{bmatrix} \quad c(k) = \begin{bmatrix} 3 \\ 2.5 \\ 0 \\ 2.5 \\ 0 \\ 0 \end{bmatrix} \quad g^0(k) = \begin{bmatrix} 2 & 0.5 & 0.5 \\ 2 & 0 & 0.5 \\ 0 & 0 & 0 \\ 2 & 0 & 0.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Coincidentally, using these other weighting structure, the set of the poor does not change. But will not always be the case. Most likely, the set of the poor will change.

Following the same procedure as before:

H, M0:

M0=8/18 = 0.444

H=1/2

$$A = \left(\frac{3+2.5+2.5}{3}\right)\frac{1}{3} = 8/9$$

Indeed M0=HA=(1/2)*(8/9)=8/18

M1:

$$g^1(k) = \begin{bmatrix} 2(0.6) & 0.5(0.666) & 0.5(0.375) \\ 2(0.2) & 0 & 0.5(0.25) \\ 0 & 0 & 0 \\ 2(0.7) & 0 & 0.5(0.25) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

M1=3.77/18=0.21

$$G = \frac{2*(0.6+0.2+0.7)+0.5*(0.66+0.375+0.25+0.25)}{8} = 0.47$$

Indeed: M1=0.5*0.88*0.47=0.21

M2:

$$g^2(k) = \begin{bmatrix} 2(0.36) & 0.5(0.444) & 0.5(0.141) \\ 2(0.04) & 0 & 0.5(0.0625) \\ 0 & 0 & 0 \\ 2(0.49) & 0 & 0.5(0.0625) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

M1 = 2.135 / 18 = 0.119

$$S = \frac{2*(0.36 + 0.04 + 0.49) + 0.5*(0.444 + 0.141 + 0.0625 + 0.0625)}{8} = 0.266$$

Indeed: M2=0.5*0.88*0.266=0.118