Summer School on Multidimensional Poverty Analysis

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Alkire Foster Methodology

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Outline

- Motivation
- Multidimensional Data
- Identification
- Aggregation
- Useful Properties
- AF Class Measures
- General Case

Challenge

- A government would like to create an official multidimensional poverty index
- **Desiderata**
  - It must be understandable and easy to describe
  - It must conform to "common sense" notions of poverty
  - It must be able to target the poor, track changes, and guide policy
  - It must be technically solid
  - It must be operationally viable
  - It must be easily replicable
    - **What would you advise?**

Practical Steps

- **Select**
  - Purpose of the index (monitoring, targeting, etc.)
  - Unit of Analysis (persons, households, countries)
  - Dimensions
  - Specific variables or indicators for each dimension
  - Cutoff for each independent variable/dimension
  - Value of deprivation for each variable/dimension
  - Poverty cutoff
  - **Identification: who is poor?**
  - **Aggregation: how much poverty is there?**

This Presentation

- Assumes that the purpose, variables, dimensional cutoffs, values, etc. have been selected
- Focus here on the **methodology** for measuring poverty
  - **Identification**
  - **Aggregation**
- Note
  - Identification step is more challenging when there are many dimensions

AF Methodology: Overview

Identification of poor – Dual cutoffs

- Deprivation cutoffs - each deprivation counts
- Poverty cutoff - in terms of aggregate deprivation values

Aggregation across the poor – Adjusted FGT

- Reduces to FGT in single variable case

Key Measure: Adjusted headcount ratio $M_0 = HA$

- H is the share of the population identified as poor, or the *incidence*
- A is the average breadth of deprivations people suffer at the same time, or the *intensity*

Observations

- **Satisfies many desirable properties**
- **Decomposability** by sub-group
  - Key for targeting
- **Breakdown** by factor after identification
  - Key for policy coordination
- **Ordinality property**
  - Key for applicability

Traditional Unidimensional Methods

Variable – income

Identification – poverty line

Aggregation – Foster-Greer-Thorbecke (1984)

Example Incomes = (7,3,4,8) poverty line z = 5

Deprivation vector $g^0 = (0,1,1,0)$

Headcount ratio $P_0 = \mu(g^0) = 2/4$

Normalized gap vector $g^1 = (0, 2/5, 1/5, 0)$

Poverty gap = $P_1 = \mu(g^1) = 3/20$

Squared gap vector $g^2 = (0, 4/25, 1/25, 0)$

FGT Measure = $P_2 = \mu(g^2) = 5/100$

μ is a mean operator

Multidimensional Poverty

Suppose *many* variables or dimensions

Question

How to evaluate poverty?

Answer 1

If variables can be meaningfully combined into some overall resource or achievement variable, *traditional methods can be used*

Combining Variables

- Welfare aggregation (Overall Achievement)
  - Construct each person's welfare level
  - Set cutoff and apply traditional poverty index
- Problems
  - Many assumptions needed
  - Cardinal utility?
  - Comparability across people?
    - Alkire and Foster (2010) "Designing the Inequality-Adjusted Human Development Index"

Combining Variables

Price aggregation

- Construct each person's expenditure level
- Set cutoff and apply traditional poverty index

Problems

- Many assumptions needed
- Ordinal and nonmarket variables problematic
- Link to welfare tenuous (local and unidirectional)
  - Foster, Majumdar, Mitra (1990) "Inequality and Welfare in Market Economies" *JPubE*

Caveats

Note

- Even if an aggregate exists, it may not be the right approach

Idea

- Aggregate resource approach signals what *could be*
  - The budget constraint
- Does not indicate what *is*
  - The actual bundle purchased
- Example
  - Consumption poverty is falling rapidly in India
  - Yet 45% of kids malnourished

Problem

- Aggregating may hide policy relevant information can't retrieve

Multidimensional Poverty

Suppose *many* variables or dimensions

Question

How to evaluate poverty?

Answer 2

If variables cannot be meaningfully aggregated into some overall resource or achievement variable, *new methods must be used*

Multidimensional Data

Matrix of well-being scores for n persons in d domains

Domains

$$X = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix}$$

Persons

Defining Deprivations (dichotomous)

- **Income:** "What is your income per capita in dollars a day?"
  - **$13 or above (bold is non-deprived)**
  - Below $13 (non-bold is deprived)
- **Schooling:** "How many years of schooling have you completed?"
  - **12 or more**
  - 1-11 years
- **Health:** "Would you say that in general your health is?"
  - **Excellent, very good or good**
  - Fair or poor
- **Social Service:** "Do you have access to social service?"
  - **Yes**
  - No

- For this illustration we will assume deprivations have equal value

Multidimensional Data

Matrix of well-being scores for n persons in d domains

Domains

$$X = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix}$$ Persons

$$z = (13 \quad 12 \quad 3 \quad 1)$$ Cutoffs

Multidimensional Data

Replace entries: 1 if deprived, 0 if not deprived

Domains

$$X = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix}$$ Persons

$$z = (13 \quad 12 \quad 3 \quad 1)$$ Cutoffs

These entries fall below cutoffs

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

Domains

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Persons

$$g_{ij}^0 = 1 \; if \; x_{ij} < z_j$$

$$g_{ij}^0 = 0 \; otherwise$$

Deprivation Matrix

Uncensored headcounts

Domains

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Deprivation Matrix

Uncensored headcounts ('raw')

Domains

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$h = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & \frac{2}{4} \end{bmatrix}$$

Identification – Weights

Deprivation Matrix

Domains

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix}$$

Weighted Deprivation Matrix

Domains

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & w_2 & 0 & w_4 \\ w_1 & w_2 & w_3 & w_4 \\ 0 & w_2 & 0 & 0 \end{bmatrix}$$

Identification – Counting Deprivations

Assuming equal weights and $\sum_{j=1}^{d} w_j = d$

	Domains				c	
$\bar{g}^0 =$	0	0	0	0	0	
	0	1	0	1	2	Persons
	1	1	1	1	4	
	0	1	0	0	1	

$$c_i = \sum_{j=1}^{d} w_j g_{ij}^0 = \sum_{j=1}^{d} \bar{g}_{ij}^0$$

Identification

Q/ Who is poor?

Domains — c

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix}$$

Persons

Identification – Union Approach

Q/ Who is poor?

A1/ Poor if deprived in any dimension $c_i \geq 1$

Domains $\qquad c$

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix}$$

Persons

Identification – Union Approach

Q/ Who is poor?

A1/ Poor if deprived in any dimension $c_i \geq 1$

Domains $\quad c$

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ \underline{1} \end{matrix}$$

Persons

Observations

Union approach often predicts very high numbers.

Charavarty et al '98, Tsui '02, Bourguignon & Chakravarty 2003 etc use the union approach

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

Domains $\qquad c$

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix}$$

Persons

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

$$
\begin{array}{c}
\text{Domains} \qquad\qquad c \\
\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ 2 \\ \underline{4} \\ 1 \end{matrix} \qquad \text{Persons}
\end{array}
$$

Observations

Demanding requirement (especially if d large)

Often identifies a very narrow slice of population

Atkinson 2003 first to apply these terms.

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $c_i \geq k$

Domains $\qquad c$

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix}$$

Persons

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $c_i \geq k$ **(Ex: $k = 2$)**

Domains $\qquad$ c

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{matrix}$$

Persons

Identification – Empirical Example

k =	H
Union 1	91.2%
2	75.5%
3	54.4%
4	33.3%
5	16.5%
6	6.3%
7	1.5%
8	0.2%
9	0.0%
Inters. 10	0.0%

Poverty in India for 10 dimensions

91% of population would be targeted using union

0% using intersection

We need something in the middle

(Alkire and Seth 2009)

Identification – Dual Cutoff Approach

Identification function is $\rho_k(x_i; z)$ where

$\rho_k(x_{i\cdot}; z) = 1$ if $\mathbf{c_i \geq k}$ (in which case i is poor)

and

$\rho_k(x_{i\cdot}; z) = 0$ if $\mathbf{c_i < k}$ (in which case i is non-poor)

Aggregation

k = 2

Censor data of non-poor

Domains $\qquad\qquad c$

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & \textcircled{1} & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{matrix}$$

Persons

Aggregation

k = 2

Censored weighted deprivation matrix and censored deprivation score

Domains $\quad c(k)$

$$\bar{g}^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Aggregation – Headcount Ratio

k = 2

Censored weighted deprivation matrix

Domains

$$\bar{g}^0(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} c(2) \\ 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Two poor persons out of four: **H = 1/2**

Critique

Suppose the number of deprivations rises for person 2

Domains $\qquad c(2)$

$$\bar{g}^0(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{matrix}$$

Critique

Suppose the number of deprivations rises for person 2

$$
\begin{array}{cc}
 & \text{Domains} \qquad c(2) \\
\bar{g}^{0}(2) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{matrix}
\end{array}
$$

Two poor persons out of four: **H = ½**

No change!

Violates 'dimensional monotonicity'

Aggregation

Return to the original censored weighted deprivation matrix

Domains $\quad c(2)$

$$\bar{g}^0(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Aggregation - Intensity

Need to augment information

Deprivation shares among poor

	Domains	$c(k)$	$c(k)/d$

$$\bar{g}^0(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} \quad \begin{matrix} \\ 2/4 \\ 4/4 \\ \\ \end{matrix}$$

Aggregation - Intensity

Need to augment information

Deprivation shares among poor

	Domains				$c(k)$	$c(k)/d$
	0	0	0	0	0	
$\bar{g}^0(2) =$	0	1	0	1	$\underline{2}$	2/4
	1	1	1	1	$\underline{4}$	4/4
	0	0	0	0	0	

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA

	Domains				$c(k)$	$c(k)/d$	
$\bar{g}^0(2) =$	0	0	0	0	0		
	0	1	0	1	<u>2</u>	2/4	Persons
	1	1	1	1	<u>4</u>	4/4	
	0	0	0	0	0		

M_0 = HA = (1/2)*(3/4) = 0.375

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\mu(\bar{g}^0(k))$

	Domains				$c(k)$	$c(k)/d$	
$\bar{g}^0(2)=$	0	0	0	0	0		
	0	1	0	1	$\underline{2}$	2/4	Persons
	1	1	1	1	$\underline{4}$	4/4	
	0	0	0	0	0		

$$M_0 = \mu(g^0(k)) = 6/16 = .375$$

Aggregation – Adjusted Headcount Ratio

Suppose the number of deprivations rises for person 2

	Domains				$c(k)$	$c(k)/d$	
$\bar{g}^0(2)=$	0	0	0	0	0		
	$\underline{1}$	1	0	1	$\underline{3}$	3/4	Persons
	1	1	1	1	$\underline{4}$	4/4	
	0	0	0	0	0		

Aggregation – Adjusted Headcount Ratio

Suppose the number of deprivations rises for person 2

	Domains				$c(k)$	$c(k)/d$	
$\bar{g}^0(2) =$	0	0	0	0	0		
	$\underline{1}$	1	0	1	$\underline{3}$	3/4	Persons
	1	1	1	1	$\underline{4}$	4/4	
	0	0	0	0	0		

A = average deprivation share among poor = 7/8

M_0 changes! $M_0 = 7/16 = 0.4375$

Satisfies dimensional monotonicity

Methodology – Adjusted Headcount Ratio

- Interpretation: conveys information on deprivations
- Applicability: valid for ordinal data
- Simplicity: easy to compute
- Useful properties
  - Subgroup decomposition
  - Dimensional breakdown
- Expandable: If variables are all cardinal can go further

Subgroup Decomposition

Suppose following scenario (k = 2 and equal weighting):

$$\bar{g}^0(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{Region 1} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{Region 2} \end{matrix}$$

$M_0 = 8/20 = 2/5$

Subgroup Decomposition

Suppose following scenario (k = 2 and equal weighting):

$$\bar{g}^0(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Region 1 (rows 1–3): M_0 (Region 1)= 1/3

Region 2 (rows 4–5): M_0 (Region 2) = 1/2

Subgroup Decomposition

Overall poverty is a population-share weighted sum of subgroup poverty levels.

$$\bar{g}^0(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Rows 1–3: Region 1 — M_0 (Region 1) $= 1/3$

Rows 4–5: Region 2 — M_0 (Region 2) $= 1/2$

$M_0 = (n^1/n) * M_0$ (Region 1) $+ (n^2/n) * M_0$ (Region 2)

$M_0 = \quad (3/5) * (1/3) \quad + \quad (2/5) * (1/2)$

Dimensional Breakdown

Censored Headcount Ratio of a dimension is the % of population who are both multidimensionally poor **and** deprived in that dimension.

Domains

$$\bar{g}^0(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Dimensional Breakdown

Censored Headcount Ratios

Domains

$$\bar{g}^0(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{h}_j(\boldsymbol{k}) = \frac{\mu(\bar{\boldsymbol{g}}^0_{\bullet j}(\boldsymbol{k}))}{\boldsymbol{w}_j}$$

$$h_j(2) = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

Dimensional Breakdown

M_0 can be expressed as a the weighted sum of the censored headcount ratios, where the weight on dimension *j* is the <u>relative weight</u> assigned to that dimension.

$$M_0 = \sum_{j=1}^{d} \frac{w_j}{d} h_j(k)$$

Domains

$$\boldsymbol{h}_j(2) = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

$$w = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

AF Class Measures

- When we have cardinal data we can compute:
  - The Adjusted Poverty Gap (M_1); and
  - The Adjusted Square Gap (M_2).

- The identification step proceeds in exactly the same way as with M_0. The difference is in the aggregation step.

Censored Normalized Gap Matrix

Censored Normalized gap = $(z_j - x_{ij})/z_j$ if deprived, 0 if not deprived

Domains

$$x = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix}$$

Persons

$$\boldsymbol{z = (13 \quad 12 \quad 3 \quad 1)}$$

Cutoffs

Censored Normalized Gap Matrix

Normalized gap = $(z_j - x_{ij})/z_j$ if deprived, 0 if not deprived

Domains

$$\bar{g}^1(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = M_1 = M_0G = HAG

Domains

$$\bar{g}^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Average **gap** across all deprived dimensions of the poor:

G = (0.04+0.42+0.17+0.67+1+1)/6

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = $M_1 = M_0 G = HAG = \mu(\bar{g}^1(k))$

Domains

$$\bar{g}^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = $M_1 = M_0 G$ = HAG = $\mu(\bar{g}^1(k))$

Domains

$$\bar{g}^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Obviously, if in a deprived dimension, a poor person becomes even more deprived, then M_1 will rise.

Satisfies monotonicity

Censored Squared Gap Matrix

Squared gap = $[(z_j - x_{ij})/z_j]^2$ if deprived, 0 if not deprived

Domains

$$\bar{g}^2(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Persons}$$

Aggregation: Adjusted Square Gap

Adjusted Square Gap = M_2 = M_0S = HAS = $\mu(\bar{g}^2(k))$

Domains

$$\bar{g}^2(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Persons

Properties Reviewed

- This methodology satisfies a number of typical properties of multidimensional poverty measures:

Symmetry	*Scale invariance*
Normalization	*Replication invariance*
Poverty Focus	*Weak Monotonicity*
Deprivation Focus	*Weak Deprivation Re-arrangement*

- M_0, M_1 and M_2 satisfy *Dimensional Monotonicity, Decomposability*

- M_1 and M_2 satisfy *Monotonicity* (for $\alpha > 0$) – that is, they are sensitive to changes in the depth of deprivation in all domains with cardinal data.

- M_2 satisfies *Weak Transfer* (for $\alpha > 1$).

General Case – Unequal weights

Now, let's allow weights to be general: $w_j > 0$

This affects Identification and Aggregation steps.

1) Poverty cutoff k is compared to deprivation score or sum of the weighted deprivations
2) Aggregation matrix now has columns weighted by deprivation values, and measures are found by taking mean of matrix

Identification – Weights

Assuming equal weights

Domains

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Persons

Weighting vector $w = (1 \quad 1 \quad 1 \quad 1)$

Identification –Weights

Assuming unequal weights

Deprivation Matrix

Domains

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$w = (.5 \quad 2 \quad 1 \quad .5)$$

$$\sum_{j=1}^{d} w_j = d$$

Weighted Deprivation Matrix

Domains

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

Identification – Normalized Weights

Assuming unequal weights

Deprivation Matrix

Domains

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$w = (.125 \ .5 \ .25 \ .125)$$

$$\sum_{j=1}^{d} w_j = 1$$

Weighted Deprivation Matrix

Domains

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & .5 & 0 & .125 \\ .125 & .5 & .25 & .125 \\ 0 & .5 & 0 & 0 \end{bmatrix}$$

Identification – Weights

Weighted Deprivation Matrix

Domains

$$\bar{g}^{0} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

Persons

$$w = (.5 \quad 2 \quad 1 \quad .5)$$

Identification – Weights

Deprivation Count Vector

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} \quad \begin{matrix} c_i \\ 0 \\ 2.5 \\ 4 \\ 2 \end{matrix}$$

Domains

Persons

$$w = (.5 \quad 2 \quad 1 \quad .5)$$

Identification – Weights

Who is poor?

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} \quad \begin{matrix} c_i \\ 0 \\ 2.5 \\ 4 \\ 2 \end{matrix}$$

Domains (columns); Persons (rows)

$w = (.5 \quad 2 \quad 1 \quad .5)$

Identification – Weights

Who is poor?

Let k = 2

Domains c_i

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2.5} \\ \underline{4} \\ \underline{2} \end{matrix}$$

Persons

$$w = (.5 \quad 2 \quad 1 \quad .5)$$

Identification – Weights

Who is poor?

Let k = 2

$$\bar{g}^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} \quad \begin{matrix} c_i \\ 0 \\ \underline{2.5} \\ \underline{4} \\ \underline{2} \end{matrix}$$

Domains (columns); Persons (rows)

w = (.5 2 1 .5)

Note: Impact on Identification

Aggregation – Weights

How much poverty? $M_0 = HA$

Let k = 2

$$
\begin{array}{c}
\text{Domains} \\
\bar{g}^{0}(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix}
\end{array}
\quad
\begin{array}{c}
c_i(2) \\ 0 \\ \underline{2.5} \\ \underline{4} \\ \underline{2}
\end{array}
\quad
\begin{array}{c}
c_i(2)/d \\ \\ \underline{2.5/4} \\ \underline{4/4} \\ \underline{2/4}
\end{array}
$$

$$w = (.5 \quad 2 \quad 1 \quad .5)$$

$$H = 3/4 \quad A = 8.5/12$$

Informal Glossary of Terms

Deprivation: if $y_{id} < z$ person *i* is **deprived** in y_d
Poverty: if $c_i \geq k$ person *i* is poor.
Deprivation cutoffs: the *z* cutoffs for each dimension
Poverty cutoff: the overall cutoff *k*
Dimension: for AF – a column in the matrix having its own deprivation cutoff (sometimes called an 'indicator')
Joint distribution: showing the simultaneous or coupled deprivations a person/hh has

Tabita, Kenya | Rabiya, India | Stéphanie, Madagascar | Agathe, Madagascar | Dalma, Kenya | Ann-Sophia, Kenya | Valérie, Madagascar

Thank you