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Summer School on Multidimensional Poverty Analysis

11–23 August 2014

Oxford Department of International Development
Queen Elizabeth House, University of Oxford

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Multidimensional Poverty and Distribution: Inequality among the poor and Disparity across regions

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20 August 2014

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Main Sources of this Lecture

- Seth S. and S. Alkire (2014), Measuring and Decomposing Inequality among the Multidimensionally Poor using Ordinal Data: A Counting Approach, Working Paper 68, Oxford Poverty & Human Development Initiative, University of Oxford.
- Alkire, S. and Foster, J. E. (2013), Evaluating Dimensional and Distributional Contributions to Multidimensional Poverty, *Mimeo*.
- Alkire S., J. E. Foster, S. Seth, S. Santos, J. M. Roche, P. Ballon, Multidimensional Poverty Measurement and Analysis, Oxford University Press, forthcoming, (Ch 10.1).

Motivation

- Poverty measurement tools affect policy design and policy incentive
  - Incidence, Intensity, Inequality
    - Distributional concerns (Sen 1976)
    - Three I's (Jenkins and Lambert 1997)

- How to incorporate distributional issues in the measurement of multidimensional poverty?

Can We Incorporate Inequality?

- Relevant properties:
  - Weak Transfer: An averaging of achievements among the poor reduces poverty
  - Deprivation Rearrangement (Substitutes): Decrease in association between dimensions decreases poverty
  - Converse Deprivation Rearrangement (Complements): Decrease in association between dimensions decreases poverty

Kolm (1977), Atkinson and Bourguignon (1982)

Can We Incorporate Inequality?

- Transformation for transfer (using bistochastic matrix)

$$Y = BX = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 3.5 & 4.5 & 3 \\ 3.5 & 4.5 & 3 \\ 8 & 6 & 3 \end{bmatrix}, z = [5 \;\; 6 \;\; 5]$$

- Transformation for rearrangement (among the poor)

$$Y = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 3 & 2 \\ 10 & 10 & 12 \end{bmatrix} \quad X = \begin{bmatrix} 3 & 4 & 8 \\ 2 & 3 & 2 \\ 10 & 10 & 8 \end{bmatrix} \quad z = \begin{bmatrix} 4 & 5 & 3 \end{bmatrix}$$

Can We Incorporate Inequality?

- Many measures in chapter 3 satisfies these properties
  - Requires *cardinal data* and not applicable for counting measures
- Dimensional Transfer: Poverty should fall whenever the total deprivations among the poor in each dimension are unchanged, but are reallocated according to an association decreasing rearrangement among the poor

Alkire and Foster (2013)

  - Capturing inequality by looking at the extent of joint deprivations

Example

- Dimensional transfer

$$g^0(Y) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad g^0(X) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

- Which one has larger inequality when dimensions are equally weighted (use union approach for illustration)?

Example: Inequality among the Poor

Initial Deprivation Count Vector (*k* = 0.3)

(0, 0.2, **0.4**, **0.6**, **0.8**, **1**)

Time Line 1

Time Line 2

(0, 0.2, 0.2, **0.3**, **0.8**, **1**)

(0, 0.2, 0.2, **0.5**, **0.7**, **0.9**)

Similar reductions in incidence and intensity

Incidence : 4/6 → 3/6

Intensity : 0.8 → 0.7

Inequality?

Two Practical Properties of M_0

The following two properties of M_0 are useful in practice:

- Ordinality allows the measure to be used with ordinal, binary, or ordered categorical data

- Dimensional Breakdown permits the dimensional composition of poverty to be seen easily

Alkire and Foster (2013)

An Impossibility Result

- There is no multidimensional counting poverty measure satisfying symmetry, dimensional breakdown and dimensional transfer

Alkire and Foster (2013)

- In other words, one has to choose measures that satisfy *either* one, *or* the other.

- How to proceed?

Two Possible Way Outs

1. Use a poverty measure that satisfies dimensional breakdown and use another poverty measure that satisfies dimensional transfer and ordinality

2. Use a poverty measure that satisfies dimensional breakdown and in addition analyze inequality among the poor separately

The First Approach

Additionally use a poverty measure that satisfies dimensional transfer and ordinality

- Jayaraj and Subramanian (2009), Bossert, Chakravarty and D'Ambrosio (2013), Alkire and Foster (2013)

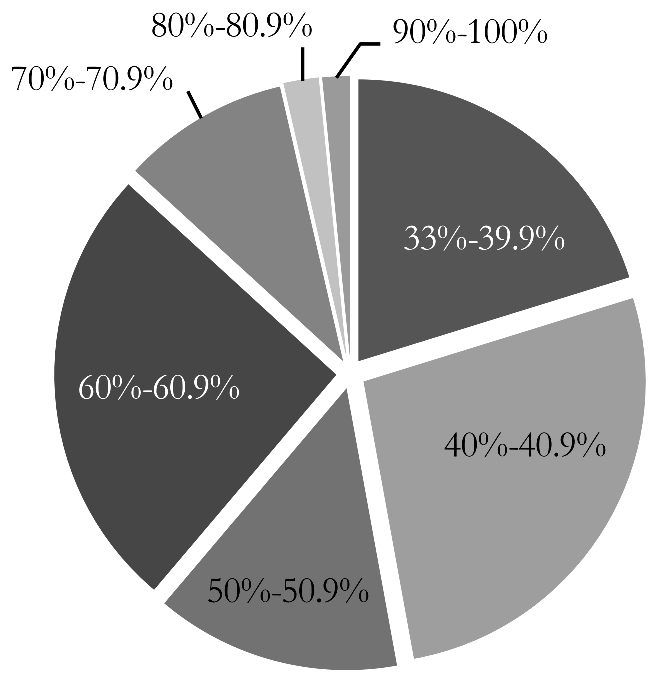
Other Practical Limitations:

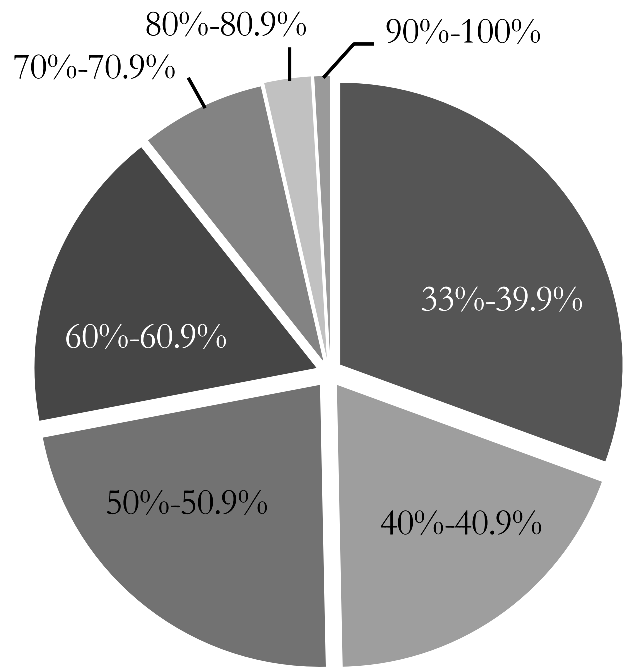
1. Mainly used for ordering
2. The final figure obtained may not be intuitive and may lack usefulness for inter-temporal analysis
3. Does not pay attention to subgroup disparity in poverty

The Second Approach

- Conduct analysis on inequality separately
- An example: Use of *standard deviation* in child poverty
  - Delamonica and Minujin (2007), Roche (2013)
- How may this approach be useful?
  - Additional information besides incidence and intensity
  - Can be used with a poverty measure that respects ordinality property and satisfies dimensional breakdown
  - If decomposable, can observe inequality decomposition within and between population subgroup

Example: A Descriptive Tool





Madagascar (2009)
MPI = 0.357, H = 67%, A = 53%

Rwanda (2010)
MPI = 0.350, H = 69% A = 50.8%

The Second Approach

- Use a separate inequality measure to capture inequality among the poor
- Which inequality measure?
  - Depends on value judgments
  - Consider an example

Example (Use Union Approach)

- Suppose vector $\underline{c}$ is obtained from vector c over time
  - $c = (0, 0.4, 0.4, 1, 1)$ and $\underline{c} = (0, 0.1, 0.1, 0.3, 0.3)$

- Has poverty gone down?
  - Indeed according to any poverty measure (integrated as well) that satisfies *dimensional monotonicity*
    - The property requires that if a poor person remains poor but becomes non-deprived in a dimension in which the person was deprived earlier, poverty should fall

Example (Use Union Approach)

- Suppose vector $\underline{c}$ is obtained from vector c over time
  - $c = (0, 0.4, 0.4, 1, 1)$ and $\underline{c} = (0, 0.1, 0.1, 0.3, 0.3)$

- How has poverty gone down?
  - Incidence? No
  - Intensity? Yes
  - Inequality among the poor?
    - Inequality of what?
    - Depends on value judgments!
    - Absolute (translation invariance) or relative (scale invariance)?
    - Across deprivations scores or across attainment scores?

Example (Use Union Approach)

- Suppose vector $\underline{c}$ is obtained from vector c over time
  - $c = (0, 0.4, 0.4, 1, 1)$ and $\underline{c} = (0, 0.1, 0.1, 0.3, 0.3)$

- Inequality of what?
  - Inequality (relative) across <u>deprivation scores</u> of the poor?
  - Then inequality has gone up
  - $GE(c,2) = 0.092$ and $GE(\underline{c},2) = 0.125$
    - Approach followed by Rippin (2011)
  - Is measuring inequality across the <u>deprivation scores</u> right?

Example (Use Union Approach)

- Suppose vector $\underline{c}$ is obtained from vector c over time
  - $c = (0, 0.4, 0.4, 1, 1)$ and $\underline{c} = (0, 0.1, 0.1, 0.3, 0.3)$

- Inequality of what?
  - What if we capture inequality across <u>attainment scores</u>?
    - $c_a = (1.01, 0.61, 0.61, 0.01, 0.01)$ and $\underline{c}_a = (1.01, 0.91, 0.91, 0.71, 0.71)$
  - Then inequality (relative) among the poor has gone down
  - $GE(c_a, 2) = 0.468$ and $GE(\underline{c}_a, 2) = 0.008$

Inequality across Attainment Scores

- Is this then the right way to reflect inequality?

- Example (Attainment scores only among the poor)
  - $c_a = (0.51,0.51,0.11,0.11)$ and $\underline{c}_a = (0.91,0.91,0.21,0.21)$
  - Least improvement among the poorest (in red)
  - What should happen to inequality among the poor?
    - $GE(c_a,2) = 0.208$ and $GE(\underline{c}_a,2) = 0.195$
  - Value judgment?
    - Hard to argue that poverty has fallen by improving the situation of the poorest

The Second Approach

- Which inequality measure to use?
  - Depends on value judgments or properties

Properties

- Same level of inequality should be reflected whether across deprivation scores or across attainment scores

The Second Approach

- Which inequality measure to use?
  - Depends on value judgments or properties

Properties

- We also want to capture disparity across population subgroups
  - Between-group inequality
- Additive Decomposability: Total inequality should be presented as a sum of two components: a within-group component and a between group component

A Policy Relevant Property

- Within-group Mean Independence: Total within-group component does not change if there is no change in inequality within any subgroup
  - Analogous to path independence (Foster and Shneyerov, 2000)
  - The weight attached to each within-group inequality component is the population share of the group

Within-group Mean Independence

Consider c and two subgroups c^1 and c^2

$$\text{Additive Decomp.: } I(c) = \underbrace{\omega_1 I(c^1) + \omega_2 I(c^2)}_{\text{Total within-group}} + \text{Bet}(c^1, c^2)$$

Suppose for $\underline{c} \neq c$, $I(\underline{c}^1) = I(c_1)$ and $I(\underline{c}_2) = I(c_2)$

- Unchanged population size in two subgroups

Q: Should the total <u>within-group inequality</u> be different in c and $\underline{c}$?

Path independence (Foster and Shneyerov, 1999)

The Inequality Measure

The inequality measure that we use:

$$I(y) = \frac{\tilde{\beta}}{t} \sum_{i=1}^{t} [y_i - \mu(y)]^2$$

$I(y)$: positive multiple of variance of distribution y

$\mu(y)$: mean of distribution y

t: Number of elements in y

$\tilde{\beta} > 0$

Chakravarty (2001)

Inequality Decomposition

- $I(y)$ can be decomposed across subgroups as follows

$$I(y) = \sum_{\ell=1}^{m} \frac{t^{\ell}}{t} I(y^{\ell}) + \tilde{\beta} \sum_{\ell=1}^{m} \frac{t^{\ell}}{t} \left(\mu(y^{\ell}) - \mu(y)\right)^2$$

Total within-group — Between-group

  - Number of subgroups: m
  - Score vector of subgroup ℓ: y^{ℓ}
  - The population share of subgroup ℓ: t^{ℓ}/t

Inequality among the Poor

- Two applications:
  - Inequality among the poor

$$I^q = \frac{\tilde{\beta}}{q}\sum_{i=1}^{q}[c_i(k) - A]^2$$

  - Inequality across population subgroups

$$I^n = \tilde{\beta}\sum_{\ell=1}^{m}\frac{n^{\ell}}{n}\left(M_0\left(X^{\ell}\right) - M_0\right)^2$$

Disparity in Poverty across Subgroup

- A valid question: Has the national reduction in MPI been uniform across subgroups?
  - Analogous to horizontal inequality (Stewart 2010)
  - Sub-national disparity (Alkire, Roche and Seth 2011)

- It may be possible that the overall inequality and within group inequalities in all subgroups fall but still disparity in poverty increases

Example ($\tilde{\beta} = 4$)

Country	Year	M_0	A	H	Inequality Among the Poor (I^q)	Disparity Between MPIs (I^n)	Number of Regions
Yemen	2006	0.283	53.90%	52.50%	0.122	0.052	21
India	2005	0.283	52.70%	53.70%	0.104	0.05	29
Togo	2010	0.25	50.30%	49.80%	0.086	0.042	6
Bangladesh	2011	0.253	49.50%	51.20%	0.084	0.005	7

Source: Seth and Alkire (2014)

- Yemen and India: Same MPI, different inequality
- Bangladesh and Togo: Similar inequality but different sub-national disparity (with similar number of regions)

Concluding Remarks

- Integrated approaches to poverty measurement in counting approach are appropriate for ordering, but may not be intuitive with strong policy implications
- Various important properties conflicts with each other
- An alternative approach is to study inequality among the poor using a separate inequality measure

Concluding Remarks

- Added advantage: The inequality measure reflects same level of inequality whether deprivations are counted or achievement counterparts

- The tool can also be used to assess and monitor disparity in poverty across population subgroups