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Counting and Accounting: Measuring the Effectiveness of Fiscal Policy in Multidimensional Poverty Reduction

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Abstract

In this paper we propose indicators of impact and spending effectiveness of fiscal interventions for multidimensional poverty reduction. We bring together CEQ's fiscal incidence methodology with OPHI's multidimensional poverty methodology, using an MPI with the M_0 structure as the metric for evaluation. The effectiveness indicators in the multidimensional case need to simultaneously consider the best allocation of money across dimensions (which deprivations to lift?) and across households (to whom should they be lifted?). In the impact effectiveness indicator, the observed poverty reduction is compared against the optimal reduction that could have been achieved. In turn, the spending effectiveness indicator compares the observed spent budget with the minimum budget that could have been spent to achieve the same poverty reduction had the money been allocated optimally. We consider two alternative criteria to find the optimal allocation: one that prioritizes reducing poverty (either incidence or intensity) to the biggest number of people – the MaxN-LNOB criterion – and another which prioritizes reducing poverty among poorest poor – the LNOB-MaxN criterion – which is a form of *prioritarianism*. When household sizes are ignored or poverty identification is done at the individual level, the two criteria coincide. The proposed methodology can be implemented using cross-sectional household survey (or census) data, alongside information on the cost of removing each deprivation at the household level, and information

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on the public spending the government has allocated or plans to allocate to the dimensions under analysis. The methodology can be implemented *ex-post*, as an effectiveness assessment, as well as *ex-ante*, to guide a multidimensional poverty reduction programme.

Keywords: fiscal incidence analysis, multidimensional poverty, impact effectiveness, spending effectiveness, optimal poverty reduction, Leave No One Behind.

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1. Introduction

For over a decade now there has been a burst of measures of multidimensional poverty to reflect more accurately the complexity of the phenomenon, which includes but exceeds income poverty. The most popular approach to measuring multidimensional poverty so far is the so-called *counting approach* (Atkinson, 2003). The Oxford Poverty and Human Development Initiative (OPHI)'s methodology, developed by Alkire and Foster (2007, 2011), outstands as the counting approach with most widespread application. The Global Multidimensional Poverty Index (global MPI) designed by OPHI in collaboration with the United Nations Development Programme (UNDP) in 2010 for the 20th Anniversary of the Human Development Report (HDR) (Alkire and Santos, 2010, 2014; UNDP, 2010) has gained wide recognition as a relevant development metric, and it is regularly updated for over 100 developing countries. Also, the Report of the Commission on Global Poverty (World Bank, 2017) recommended a Multidimensional Poverty Index (MPI) as a complementary indicator to income poverty, and the Sustainable Development Goal (SDG) 1.2.2 focuses specifically on the reduction of multidimensional poverty, according to national definitions. At the time of writing this paper there were 24 countries with an official national MPI, eleven of them in Latin America.

In most of its applications, multidimensional poverty measurement can be associated to the *direct method* to measure poverty (Sen, 1981), in that it evaluates whether people satisfy a set of specified basic needs, rights, or—in line with Sen's capability approach—functionings. This contrasts with the *income method*, which determines whether people's incomes fall below the poverty line—the income level at which some specified basic needs can be satisfied (Alkire and Santos, 2014).[1]

Contemporaneously with the development of OPHI's multidimensional poverty measurement methodology, the Commitment to Equity Institute (CEQ) developed a fiscal incidence analysis methodology in an internationally comparable way, in such a way that it can and has been implemented in over 60 countries so far. The methodology belongs to the so-called *accounting approach*. "The accounting approach consists of starting from an income concept and, depending on the fiscal intervention under study, allocating the proper amount of a tax or a transfer to each household or individual" (Lustig, 2018, p. 15). Most commonly the methodology entails adding (benefits) and subtracting (taxes) amounts from the pre-fiscal income to obtain the post fiscal one.[2] The difference in a poverty index computed over the

1 However, whenever MPIs include an income indicator, they become a hybrid form of poverty measurement.

2 When the data available is consumption (assimilated to disposable income), then the methodology works backwards (subtracting out benefits and adding taxes), to obtain the pre-fiscal income (market income).

pre-fiscal income and the one computed over the post-fiscal one indicates whether the fiscal intervention is poverty-reducing (whenever such difference is positive) or poverty-increasing (whenever such difference is negative). This approach does not consider behavioral or general equilibrium modelling (Lustig, 2018, p. 18). "…we do not claim that the pre-fiscal income obtained from this exercise equals the true counterfactual income in the absence of taxes and transfers. It is a first-order approximation (and in a variety of settings a first-order approximation is all one may need)" (Lustig, 2018, p. 18).

In an international development agenda in which reducing multidimensional poverty is an explicit priority, one natural question that emerges is -just like with income poverty- what is the impact of the fiscal interventions over multidimensional poverty? After more than a decade of methodological developments from both CEQ and OPHI -both institutions with high policy impact, in this paper we bring together OPHI's *counting* methodology of multidimensional poverty measurement with CEQ's *accounting* methodology of fiscal incidence analysis.

As a relevant previous related work, there is the study by Cuesta et al (2021), in which the authors perform a fiscal incidence analysis using CEQ's methodology for the case of Uganda using a multidimensional child poverty measure. Specifically, they identify "child-relevant" budget, that is the public spending and tax revenues that explicitly and directly target child (aged 0-17) well-being. Then, they evaluate the incidence of such fiscal intervention stratifying the children, rather than by income quantiles, by their multidimensional poverty intensity, i.e. the number of deprivations they experience. The approach followed in this paper is completely different. We intend to develop a methodology in which *both sides* of the analysis -not only the metric for evaluating fiscal incidence but also the fiscal intervention - are multidimensional. A related complementary work is that by Barbieri and Higgins (2015), who study with a political economy model how a multidimensional poverty measure can influence the allocation of resources across ministries.

However, the extension of the accounting methodology to the multidimensional poverty case is not straightforward. The information one can observe from household surveys data is the *post-fiscal* matrix of achievements, in which each row is a household or individual, and each column is an indicator. To measure the fiscal incidence over multidimensional poverty one needs to somehow construct an analogous to the *pre-fiscal* matrix of achievements, essentially a counterfactual. Such an exercise is far from obvious for two reasons. First, for most relevant dimensions, achievements consist of an access *vs.* no-access status, such as access to piped water, for example. That is, policy interventions in many dimensions in the multidimensional space entail a zero-to-one change, rather than marginal increments or decrements, as it happens with income. In other words, pre-fiscal achievements cannot be simulated by subtracting certain magnitudes to a continuous cardinal variable. We thus propose to use cross sectional data at two points in time, considering the achievements at the initial point in time as the pre-fiscal matrix, and the achievements at the final point in time as the post-fiscal one.

The second issue is that achievements in the multidimensional context may have been granted or facilitated by a fiscal intervention, but not necessarily. Thus, the selection of the indicators to include in the multidimensional poverty index (MPI) that will serve as a metric, needs to be carefully done and informed such that the reduction in deprivation rates in those indicators can be reasonably attributed to the fiscal action. Natural candidates for such analysis are access to public services such as water, sanitation sewage, natural gas and electricity. But other indicators may apply whenever there is information that a certain specific policy has taken place, say a social housing programme.

The methodology proposed in this paper includes three indicators. First, following Alkire et al (2015), we note that the change in one of the MPI's related indexes, namely the *change* over time in the so-called censored headcount ratios, can be interpreted as the marginal dimensional contribution to changes in the multidimensional poverty measure. Whenever this change can be reasonably attributed to the fiscal action, this indicator can be interpreted as an analogue of CEQ's Marginal Contribution Indicator in the multidimensional case. Second, we propose analogues of Enami (2018)'s impact and spending effectiveness indicators for the multidimensional poverty context. The impact effectiveness indicator is a tool for assessing how well has a certain budget been allocated to reduce multidimensional poverty, whereas the spending effectiveness indicator allows identifying the minimum budget that would have achieved the observed poverty reduction between two points in time. Interestingly, however, different alternative criteria can be considered to define what the optimal distribution should be, distribution against which the actual distribution is compared to measure effectiveness. We consider two alternative criteria which emerge from the fact that deprivations are removed at the household level but households have different sizes. One criterion prioritizes maximising the MPI reduction, the other prioritizes reducing the deprivations among the most intensely poor. When poverty is identified at the individual level or if household sizes are ignored, the two criteria coincide. It is noteworthy that the optimal distribution defined for evaluating *ex-post* the observed allocation of public budget can also be used prospectively as a policy tool for allocating budget in future across dimensions and across households.

The paper is structured as follows. In Section 2 we introduce the general notational framework and the methodology for poverty measurement. Section 3 briefly presents CEQ's indicators of marginal contribution, impact effectiveness and spending effectiveness. Section 4, especially Section 4.3, contains the main value added of this paper, presenting the proposed indicators for fiscal incidence analysis in the multidimensional poverty context. Section 5 presents numerical examples to illustrate the methodology with equal weights. Section 6 comments on the issue of reranking, quite frequently unavoidable in the multidimensional case. Section 7 details how the methodology can be implemented with real data. Finally, Section 8 concludes. The Appendix offers a numerical example with unequal weights.

2. The Measurement Framework

We present the notational framework, in line with Alkire and Foster's (2011) (AF hereafter) notation. However, in this presentation we will make explicit the fact that the unit of identification is the household. Although the AF measurement methodology can be implemented at the individual level, the usual practice so far has been that households are the unit of identification, and all members of households identified as poor are considered poor. While the unit of identification is the household, the statistics are presented in terms of population.[3] We make this practice explicit in the notational framework, as this will facilitate the presentation of the indicators proposed in Section 4.

At each period of time t, there are $i = 1, \ldots, n_t$ people who live in $h = 1, \ldots, T_t$ households. The relevant information is contained in an $T_t x d$ matrix $\boldsymbol{x_t} = [x_{hjt}]$ where each entry $x_{hjt} \in \mathbb{R}^+$ is the achievement of household h in indicator $j = 1, \ldots d$, at time $t = 0{,}1$. Each row vector x_{ht} contains the achievements of household h in each of the d indicators at time t. Deprivation cutoffs are summarized in a $1xd$ vector $z = [z_j]$, and indicators' weights in an $1xd$ vector $w = [w_j]$, where $\sum_{j=1}^{d} w_j = 1$. We assume the z and w vectors to be time invariant, that is, the minimum thresholds do not change over time, nor does the weight attached to each indicator. This assures consistency over time in the fiscal incidence analysis.

In multidimensional poverty analysis, variables are typically of ordinal nature, and are then converted into a dichotomy of deprived and non-deprived. Household h is identified as deprived in each j-indicator, in each t period, whenever $x_{ht} < z_j$. The deprivation of household h in indicator j can be defined as $g_{hjt}^0 = 1$ whenever $x_{hjt} < z_j$, and $g_{hjt}^0 = 0$ otherwise, and these can be collected in an $T_t x d$ deprivation matrix $\boldsymbol{g_t^0} = [g_{hjt}^0]$. We refer to achievements that fall below their corresponding cutoff value as *deprived achievements*. Next, a deprivation score is computed for each household at each time period, defined as the weighted sum of deprivations $c_{ht} = \sum_{j=1}^{d} w_j g_{hjt}^0$, which can be collected in a $T_t x 1$ vector of deprivation counts c_t.

[3] Note that this is also the usual practice in monetary poverty measurement, either using the income per capita or the income per equivalent adult in the household, which are compared against the poverty line.

2.1 Multidimensional Poverty Measures for ordinal variables

2.1.1 Identification

Poverty measurement first requires identifying the poor (Sen, 1976). In the AF framework identification is done comparing the deprivation score with a poverty cut-off k, which represents the proportion of minimum deprivations a household must experience to be identified as poor. Formally, household h is poor when $c_h \geq k$, and it is non-poor when $c_h < k$. The use of a set of deprivation cutoffs z *and* a poverty cutoff k is what makes the AF methodology a *dual-cutoff* approach. The use of the poverty cutoff also frames the AF methodology within *counting* approaches because the poor are identified by counting their deprivations, represented in the deprivation score c_h. This identification step considers the *joint distribution of deprivations*, a distinctive feature of multidimensional poverty measurement.

The poverty cutoff k can take values within the range: $\min(w_j) \leq k \leq 1$. When k is equal to the minimum weight assigned to the set of indicators, it corresponds to the *union criterion,* which implies that anyone in a household with at least one deprivation will be counted as multidimensionally poor. When k is equal to 1, it requires people belonging to households deprived in all the indicators to be counted as multidimensionally poor, which corresponds to the *intersection criterion*. Most commonly, intermediate k values are used.

Once the identification step has been completed, to proceed to the next step of aggregation, and to satisfy the poverty focus axiom, the deprivations of the non-poor need to be censored, analogously to the censoring of the incomes above the poverty line when measuring income poverty. Censoring implies that the deprivations of those not identified as poor are ignored, i.e. replaced by zeroes. The censored deprivation matrix is defined as $\boldsymbol{g_t}(\boldsymbol{k})^0 = [g^0_{hjt}(k)]$ such that each element is $g^0_{hjt}(k) = g^0_{hjt}$ when $c_{ht} \geq k$ and $g^0_{hjt}(k) = 0$ otherwise. The censored deprivation score is defined as $c_{ht}(k) = \sum_{j=1}^{d} w_j g^0_{hjt}(k)$, and these scores are collected in the $c_t(k)$ vector.

Next, one can proceed with the second poverty measurement step, which is aggregation (Sen, 1976). Given that in multidimensional poverty measurement exercises the presence of dichotomous, ordinal and categorical variables prevails, we focus here on the M_0 measure, with its two partial sub-indices: incidence and intensity.

2.1.2 Aggregation with M_0

The M_0 measure (Alkire and Foster, 2011) has given the mathematical structure to the global MPI as well as to most national and regional MPIs. As we have defined deprivations in terms of households, to obtain the aggregate poverty measure in population terms, we need to consider the household size s_h of each $h = 1, \dots, T_t$ household. M_0 is given by:

$$MPI_t = M_{0t}(\boldsymbol{x_t}; z) = \frac{1}{n_t}\sum_{h=1}^{T_t} s_h \sum_{j=1}^{d} w_j g_{hjt}^0(k) = \frac{1}{n_t}\sum_{h=1}^{T_t} s_h c_{ht}(k) \quad (1)$$ [4]

If the M_0 measure is computed with household survey data, which typically includes a survey weight variable p_{ht_0} that indicates how many households each h household in the sample represents. Then expression (1) would be:

$$MPI_t = M_{0t}(\boldsymbol{x_t}; z) = \frac{1}{n_t}\sum_{h=1}^{T_t} s_h p_{ht} c_h(k) \quad (1')$$

where $\sum_{h=1}^{T_t} s_h p_{ht} = n_t$. For simplicity, we continue ignoring the survey weight variable, but all the formulas can incorporate this variable.

It can be verified that M_0 is the product of two very relevant sub-indices which provide distinct and complementary information: the headcount ratio of multidimensional poverty H, and the average intensity of poverty among the poor A. This is why M_0 is called the *adjusted* headcount ratio: it is incidence adjusted by intensity.

The headcount ratio of multidimensional poverty can be expressed as:

$$H_t = \frac{1}{n_t}\sum_{h=1}^{T_t} s_h I(c_{ht} \geq k) = \frac{q_t}{n_t} \quad (2)$$

where $I(c_{ht} \geq k)$ is an indicator function that takes value 1 when the condition inside the parenthesis holds, and 0 otherwise, and q_t is the number of the poor in period t.

In turn, poverty intensity is the average deprivation score among the poor, which is defined as:

$$A_t = \frac{1}{q_t}\left[\sum_{h=1}^{T_t} s_h \sum_{j=1}^{d} w_j g_{hjt}^0(k)\right] = \frac{1}{q_t}\left[\sum_{h=1}^{T_t} s_h c_{ht}(k)\right] \quad (3)$$ [5]

The M_0 measure is a member of a broader class of measures, the M_α class, but the other members of the family are less applicable as they require all indicators to be cardinal and thus are not presented here.

The M_0 measure satisfies several convenient properties that make it suitable for wide applicability. Four of such properties outstand. First, it satisfies *ordinality*, meaning that it can be computed with a mix of

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[4] If the deprivations are defined at the individual level, $g_{ijt}^0(k)$, then the formulas are simplified to: $MPI_t = M_{0t}(\boldsymbol{x_t}; z) = \frac{1}{n_t}\sum_{i=1}^{n_t}\sum_{j=1}^{d} w_j g_{ijt}^0(k) = \frac{1}{n_t}\sum_{i=1}^{n_t}\sum_{j=1}^{d} c_{it}(k)$.

[5] Again, if the deprivations are defined at the individual level, the formula of A is simplified to $A_t = \frac{\sum_{i=1}^{n_t} c_{it}(k)}{q_t}$.

cardinal and ordinal indicators -a recurrent case in multidimensional poverty measurement- in a robust way. As achievements are dichotomized into 'deprived' and 'non-deprived', the poverty value does not change whenever the scaling of an ordinal variable (say, sanitation) changes.

Second, M_0 satisfies *dimensional monotonicity*. Suppose two distributions A and B, with the same poverty headcount ratio, say 20%, but such that in A the poor are deprived -on average- in 30% of the considered indicators, whereas in B the poor are deprived -on average- in 50% of the considered indicators. Dimensional monotonicity means that distribution B will have an M_0 value of 0.10, higher than the M_0 value of A, which will be 0.06, reflecting that B has a higher intensity of multidimensional poverty.

Third, M_0 satisfies *population subgroup decomposability*, which means that the overall poverty value can be expressed as a weighted sum of the poverty values of mutually exclusive and collectively exhaustive population subgroups $p = 1, \dots, P$, such that $\sum_{p=1}^{n_{pt}} n_{pt} = n_t$, where the weights are the subgroups' population shares $\frac{n_{pt}}{n_t}$. Let $M_0^p(\boldsymbol{x_t}; z)$ be the poverty value of each subgroup p, then M_0 can be expressed as:

$$M_{0t}(\boldsymbol{x_t}; z) = \sum_{r=1}^{P} \frac{n_{pt}}{n_t} M_{0t}^p(\boldsymbol{x_t}; z) \qquad (4)$$

From there, one can compute the contribution of each p subgroup to total poverty as:

$$C_{pt} = \frac{n_{pt}}{n_t} \frac{M_{0t}^p(\boldsymbol{x_t}; z)}{M_{0t}(\boldsymbol{x_t}; z)} \qquad (5)$$

and compare this with its population share. This enables decompositions by gender, ethnicity, age groups or regions that are quite relevant for fiscal incidence analysis.

Fourth, M_0 satisfies *dimensional breakdown*. This means that the overall poverty value can be expressed as a weighted sum of post-identification dimensional values, where the weights are the indicators' weights.[6] The expression is given by:

$$M_{0t}(\boldsymbol{x_t}; z) = \sum_{j=1}^{d} w_j \left(\frac{\sum_{h=1}^{T_t} s_h g_{hjt}^0(k)}{n_t} \right) \qquad (6)$$

The expression in parenthesis either in formula (6) is called the *censored headcount ratio* CH_j, defined as the proportion of the total population in households which have been identified as poor *and* are deprived in indicator j. In this way, one can compute the contribution of the deprivation in each indicator *j* to total poverty as:

6 The property is called break-down rather than decomposability because it holds post-identification of the poor.

$$C_{jt} = \frac{w_j\left(\sum_{h=1}^{T_t} s_h g_{hjt}^0(k)/n_t\right)}{M_{0t}(\boldsymbol{x};z)} \qquad (7)$$

While the M_0 measure is very convenient for the four mentioned properties, it has one drawback, which is that it is not sensitive to inequality among the poor.

Distributional properties can be weak or strong. Weak versions of distributional properties guarantee the measure not to move in the 'wrong' direction. Strong versions of distributional properties require the poverty measure to strictly change in a particular direction in response to the transformation. Another important point to consider in distributional properties is whether they require the set of the poor to remain unchanged after the transformation.[7]

One kind of distributional properties relevant for ordinal data refer to rearrangements among the poor, which are transformations in which achievements are *switched* between poor people, modifying the association of achievements among the poor. For example, an association-decreasing rearrangement among the poor occurs when achievements are switched between two poor people in such a way that, while before the transfer one person had no lower achievement than the other in all dimensions and strictly higher in at least one dimension, after the transfer, this does not longer hold. This rearrangement is typically defined in such a way that it requires the number of the poor not to change.

The M_0 measure satisfies *weak* rearrangement given that the property requires the set of the poor not to change with the transformation, i.e. the measure does not change under an association-decreasing rearrangement among the poor. However, M_0 does not satisfy *strong* rearrangement. Moreover, if the transformation was broadened to allow a household (or person) to be lifted from poverty because of an association increasing rearrangement among the poor, M_0 would not even satisfy a weak version of the property, as the measure could decrease in such case, while others would argue that it should always increase as a result of a higher concentration of deprivations among the poor.[8]

However, it must be noted that the strong form of the rearrangement property is incompatible with the dimensional breakdown property, admittedly quite relevant for policy purposes (AF, 2016, 2019).[9] The M_0 measure has been extended to a distributional-sensitive measure: the M_0^2, but at the cost of renouncing to

[7] For more comprehensive discussion on distributional properties see Seth and Santos (2019) and Santos (2023).

[8] See Rippin (2013) and Datt (2019), for example.

[9] Note also that the requirement that poverty should increase under such transformation implicitly assumes achievements to be substitutes. However, the converse case in which achievements are complements and thus poverty should decrease under such transformation, has also been considered (Bourguignon and Chakravarty, 2003).

dimensional break-down.[10] Barbieri and Higgins (2015) have emphasized the importance of the dimensional break-down property from a political economy point of view. In this paper we develop the methodology using the M_0 measure because it is the one with wide applicability at country levels, and because sensitivity to the poorest poor can also be incorporated by using higher poverty cutoffs, as it exemplified in Section 5.

3. Three distinguished CEQ's indicators

CEQ's fiscal incidence analysis relies on the computation of different income concepts. The pre-fiscal income concept is the market income, namely, wages and salaries, income from capital plus: private transfers, imputed rent and own production before: taxes, social security contributions and government transfers. Market income also includes contributory social-insurance old-age pensions (or excludes contributions to social insurance old-age pensions) whenever contributory pensions are treated as deferred income. From that income concept, different post-fiscal income concepts are constructed: disposable income, consumable income and final income (Lustig et al., 2018).

The computation of the different income concepts is performed at one point in time. It is certainly the most challenging and core task of the fiscal incidence analysis. CEQ's methodology most commonly needs to implement a variety of tools which combines direct identification (the survey tells) with inference, imputation, simulation, prediction or matching techniques to bring information from other data sources (Lustig et al., 2018, ch. 6). Once the different income concepts have been defined, the CEQ methodology computes different indicators which allow answering key questions on the distributional impact of the fiscal system and of specific components. We consider here three of these indicators, of which we propose extensions to the multidimensional case in Section 4.

3.1 CEQ's Marginal Contribution Indicator

The marginal contribution of a specific tax or any combination of taxes (T) or a specific transfer or combination of transfers (B) to changes in the overall level of poverty or inequality is a fundamental indicator in CEQ's framework. This indicator is given by the inequality or poverty indicator computed over the income distribution without the tax/es (T) or transfer/s (B) under analysis minus the inequality or poverty indicator computed over the income distribution with the tax or transfer under analysis.

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10 The M_0^2 measure was introduced at the end of Alkire and Foster (2011) paper, and further elaborated in Alkire and Foster (2016, 2019).

$$MC_{TorB}^{End\ Income} = Index_{End\ Income\ without\ T\ or\ B} - Index_{End\ Income} \quad (8)$$

Whenever $MC_{TorB}^{End\ Income} > 0$ the fiscal intervention is equalizing or poverty-reducing, and whenever $MC_{TorB}^{End\ Income} < 0$, the fiscal intervention is unequalising or poverty-increasing (Lustig et al., 2018, p. 36–37). Naturally, taxes can only increase poverty.

3.2 CEQ's Impact Effectiveness Indicator

The Impact Effectiveness (IE hereafter) indicator (Enami, 2018) intends to determine how effective taxes and government spending are in reducing inequality and poverty, that is whether a transfer generates as much poverty or inequality reduction as it could potentially do given a certain budget. This indicator is defined as:

$$IE = \frac{\textbf{Observed}\ Marginal\ contribution\ of\ T\ or\ B}{\textbf{Optimal}\ Marginal\ contribution\ of\ T\ or\ B} \quad (9)$$

The key element contained in this indicator is given by the allocation of taxes or benefits that produces the *optimal distribution*, which in turn produces the optimal contribution of that tax or benefit. Because taxes can only increase poverty, the poverty-reduction indicator is only defined for benefits and combined tax-transfer systems that have a positive marginal contribution (Enami, 2018).

Consider the case of benefits. Given a certain total observed benefit allocated to the poor, the aim is to produce an allocation such that it reduces poverty (or inequality) the most, and thus can be considered optimal. The optimal distribution is based on Fellman et al. (1999). The procedure is to order individuals from poorest to richest and increase the income of the poorest poor individual with a benefit until her income becomes equal to the income of the second poorest poor. Next, the incomes of both these two poorest poor are raised, through the benefit, to the income of the third poorest poor, and so on. In other words, the total budget is allocated among the poor in such a way that a certain number (say *J*) of the poorest poor receive each a certain amount of benefit such that *all their incomes are equalized*, preserving the original income ranking. Note that the cardinality and continuity of the income variable enables the total available budget to be divided into infinitesimal parts if necessary to produce the optimal allocation, which reduces inequality among those who receive the benefit to zero and it is rank preserving.[11] The interpretation of the IE indicator is straightforward: a value of 0.60 of the IE indicator means that the transfer has accomplished 60% of its potential in reducing poverty.

11 In the case of a tax, to maximize the inequality-reducing impact of a tax of a given size, the richest person is taxed until her pretax income equals the pretax income of the second richest person; then, both would be taxed until their pretax income equals the pretax income of the third richest person, and so on until there is no more of the tax to be allocated.

3.3 CEQ's Spending Effectiveness Indicator

A twin indicator of the Impact Effectiveness indicator is the Spending Effectiveness (SE hereafter) indicator. The aim of this indicator is to determine the lowest amount of benefit, i.e. the smallest fiscal budget, with which the observed inequality or poverty reduction could have been achieved. The Spending Effectiveness indicator is given by:

$$SE = \frac{\textbf{Optimal Amount} \; of\ T\ or\ B\ that\ achieves\ the\ observed\ MC}{\textbf{Observed Amount} \; of\ T\ or\ B} \quad (10)$$

In this case the key element is to look for the optimal allocation of a benefit (or a tax) that will give the minimum amount of benefit (or tax) needed to achieve the observed reduction in poverty or inequality. The definition of the optimal allocation is exactly the same as the one described for the IE indicator: benefits will be allocated in such a way they equalize income among a certain J number of the poorest poor, in this case, until they produce the same observed poverty or inequality reduction. Note that the difference is that while in the IE indicator one looks for the biggest reduction in a poverty or inequality measure given a budget, in the SE indicator one looks for the smallest budget that would achieve the given reduction in poverty or inequality. Again, because income benefits are a cardinal and continuous variable, the exercise allows dividing benefits into infinitesimal parts if necessary to produce the optimal allocation.

4. Extending CEQ's measures to the multidimensional context

4.1 Specificities of the multidimensional context

Extending CEQ's framework of fiscal incidence analysis to the multidimensional case requires considering the specificities of the multidimensional context, which condition the way in which to think analogue indicators. We first point such specificities and then propose analogue indicators within these methodological constraints.

4.1.1 Defining the pre-fiscal distribution

One first natural question is: what is the analogue to the pre-fiscal income in the multidimensional space? Defining a pre-fiscal matrix of achievements does not seem obvious. Fiscal interventions in the multidimensional space take the form of in-kind interventions, such as connecting households to the electricity grid, extending the network of piped water, sanitation sewage, electricity or natural gas, or building more public schools. These policies aim at switching a deprived achievement into a non-deprived one, and thus they are not reflected into additions to a quantitative variable such as income.

One practical way in which the pre-fiscal matrix of achievements can be inferred is by taking advantage of repeated cross-sectional household survey data. Given the matrix of achievements at two points in time $\boldsymbol{x_{t_0}} = [x_{hjt_0}]$ and $\boldsymbol{x_{t_1}} = [x_{hjt_1}]$, we may understand the $\boldsymbol{x_{t_0}}$ matrix of achievements as the pre-fiscal matrix of achievements of the $\boldsymbol{x_{t_1}}$ matrix, the post-fiscal matrix of achievements. Alternatively, the same can be done using panel data, but this is not a requirement for implementing the proposed methodology.

Naturally, the plausibility of assuming the initial achievement matrix as the pre-fiscal matrix relies on the specific $j = 1, \dots d$ indicators that compose the matrix. They need to be indicators that must have been influenced by fiscal action, such as the development of infrastructure. In other words, they need to be indicators such that their change between t_0 and t_1 can only be reasonably attributed to the intervention of the State. Access to basic services, such as water, sanitation sewage, natural gas and electricity are natural options to consider.

4.1.2 Joint deprivations

The fact that multidimensional poverty looks at the *joint distribution* of deprivations brings complexities into the fiscal incidence analysis. Different combinations of deprivations may produce the same deprivation score and reducing it may be achieved by lifting different combinations of deprivations which in turn imply different fiscal costs. This has direct implications for constructing the impact and spending effectiveness indicators.

Also, when an intermediate k-poverty cutoff is used, rather than a union one, the censored distribution of deprivations is used. The censoring of deprivations of households which are not poor is reasonable for focusing fiscal efforts on the poor, but it brings some technical difficulties that need to be considered in the search of the optimal distributions, as it will be explained below.

4.1.3 Indivisibilities and Discontinuities

The multidimensional context also has one characteristic that imposes restrictions: there are many indivisibilities which create discontinuities.

The first indivisibility is given by the deprivation score c_h. Whenever $w_j = 1/d \; \forall j$, the deprivation score changes in steps of $1/d$ and - unlike \$1- there is no way to 'divide' those values, which represent having vs. not having a deprivation. Other weighting schemes have other steps, but still indivisible. Equalizing a certain group of deprivation scores among the poorest, analogously to CEQ's IE and SE indicators, is less applicable than one could think a priori. Lifting a certain deprivation may reduce a household's deprivation score in more or in less than one would need to equalize it to the score of other households, depending on the weights and the combination of deprivation this and other households have.

The second indivisibility is given by the cost of removing each deprivation, typically expressed in a per household cost. In the income space, the fiscal effort is given by a certain budget that can be divided until the very last cent to be distributed across individuals so that incomes are equalized, generating the optimal distribution against which the actual distribution can be compared. In the multidimensional case, in contrast, the fiscal effort is also given by a certain spent budget, but this can only be discretely divided into -for example- a certain number of 'connections' to public services such budget can achieve. Bringing water or sewage sanitation to a household has a certain cost, and such benefit cannot be delivered in parts. It is either given, and such deprivation is lifted, or not given.

The third indivisibility comes from the fact that people cohabitate in households, which have different sizes which, again, are indivisible. Many of the deprivations considered in the multidimensional context, especially those related to services, are equally experienced by all household members. When one of these deprivations is removed, for example, bringing running water to the household, it is removed to all household members together. One cannot remove this kind of deprivations only to certain household members. It is precisely because of this indivisibility that we have explicitly incorporated in the notation the fact that households are the unit of identification.

As we will see, these three indivisibilities impose restrictions to the search of an optimal allocation of a certain fiscal budget.

4.2 A Marginal Contribution Indicator in the multidimensional case

Suppose the matrix of achievements at two points in time: $\boldsymbol{x_{t_0}}$ and $\boldsymbol{x_{t_1}}$ and the corresponding values of the multidimensional poverty index $M_{0t_0}(\boldsymbol{x}; z)$ and $M_{0t_1}(\boldsymbol{x}; z)$. Interpreting matrix $\boldsymbol{x_{t_0}}$ as the pre-fiscal distribution of $\boldsymbol{x_{t_1}}$, a natural and simple way to think of an analogue of CEQ's MC indicator of a certain benefit (or tax) is to think of a *Dimensional* Marginal Contribution indicator. This indicator would indicate to what extent the fiscal action in reducing deprivation in one dimension has contributed to the reduction in total multidimensional poverty.

Following Alkire et al. (2015, chapter 9), given that the M_0 measure satisfies dimensional breakdown, marginal contributions can be directly equated to *changes* in the censored headcount ratios defined in equation (6) as a proportion of the total change in M_0. Changes in the censored headcount ratios are given by:

$$\Delta CH_j = \frac{\sum_{h=1}^{T_{t_1}} s_h g_{hjt_1}^0(k)}{n_1} - \frac{\sum_{h=1}^{T_{t_0}} s_h g_{hjt_0}^0(k)}{n_0} \quad (11)$$

It can also be verified that:

$$\Delta M_{0,t_0-t_1} = M_0(x_{t_0}) - M_0(x_{t_1}) = \sum_{j=1}^{d} w_j \Delta CH_j \quad (12)$$

That is, the weighted sum of the changes in the censored headcount ratios (the ΔCH_j) equals the total change in the M_0. Then, the expression of the marginal contribution of dimension j to total poverty reduction is given by:

$$MC_j^{MPI} = \frac{w_j \Delta CH_j}{\left(M_0(x_{t_0};z) - M_0(x_{t_1};z)\right)} \text{ for any } k \quad (13)$$

such that $\sum_{j=1}^{d} MC_j^{MPI} = 1$. In words, expression (13) registers the change in the proportion of the total population who has been identified as poor *and* is deprived in indicator j between t_0 and t_1, weighted by the indicator's weight, as a proportion of total poverty change, and thus it can be interpreted as a marginal contribution to multidimensional poverty.

However, whenever the identification criterion departs from the union approach and an intermediate poverty cutoff is used, the interpretation of expression (13) as a dimensional marginal contribution should be done with caution. The reason is that the reduction of a certain deprivation j between t_0 and t_1, may have lifted some households from multidimensional poverty, even when some deprivations remain. In such case, the deprivations of the no-longer poor households in t_1 will be censored, and thus, these deprivations will not be counted in the censored headcount ratios in t_1. In consequence, the contribution of the reduction in the deprivation/s that lifted these households from poverty can be under-estimated, underplaying the fiscal effort done in some dimension, and the contribution of other dimensions can be overestimated. Yet, one can resort to evaluate the change in the uncensored headcount ratios, as a complementary information, as recommended by Alkire et al. (2015, chapter 9). This point is further clarified with an example in Section 5.3.

4.3 An Impact and a Spending Effectiveness Indicator in the multidimensional case[12]

We now introduce an analogue of CEQ's IE indicator for the multidimensional case. Assume that poverty is evaluated with an MPI, with the M_0 mathematical structure composed of d indicators that can be influenced by the intervention of the State. Consider the matrix of achievements at two points in time: $\boldsymbol{x}_{t_0}$ and $\boldsymbol{x}_{t_1}$, and their corresponding multidimensional poverty index values $M_0(\boldsymbol{x}_{t_0}; z)$ and $M_0(\boldsymbol{x}_{t_1}; z)$. Let's assume there has been a reduction in multidimensional poverty as measured by this MPI. Let's also assume

12 We must emphasize that the word 'impact' in these indicators does not have the meaning of the impact evaluation literature. We are within an accounting framework and our counterfactual (the achievement matrix in t_0) depends on strong assumptions.

that there is information on the fiscal cost the State has incurred to remove one or more of the d deprivations to certain parts of the population, such as a program to bring piped water and sewage sanitation to shantytowns in urban areas. Has the fiscal investment done its best at reducing poverty? Or could the reductions in deprivations have been allocated differently to produce a more effective poverty reduction?

Let MP denote a measure of multidimensional poverty. The change of such multidimensional poverty measure at two points in time can be noted as $\Delta MP_{t_0-t_1}$. Thus, the IE indicator in the multidimensional case, name it IE^{MP}, will be defined as:

$$IE^{MP} = \frac{\textbf{Observed}\ \Delta MP_{t_0-t_1}}{\textbf{Optimal}\ \Delta MP_{t_0-t_1}} \quad (14)$$

The twin indicator to the impact effectiveness indicator is the spending effectiveness indicator. Analogously to CEQ's SP indicator, the spending effectiveness in the multidimensional case compares the B observed fiscal cost incurred to produce the observed poverty reduction $\Delta MP_{t_0-t_1}$with the minimum fiscal effort B^* that could have been spent to produce the same (or higher) poverty reduction. The expression of the Spending Effectiveness indicator in the multidimensional case SE^{MP} is thus given by:

$$SE^{MP} = \frac{\textbf{Optimal}\ B^*\ to\ achieve\ observed\ \Delta MP_{t_0-t_1}}{\textbf{Observed}\ \mathrm{B}} \quad (15)$$

It is important to note that there is one fundamental difference between CEQ's IE and SE indicators and the $IE^{MP}$and SE^{MP}proposed here. CEQ's indicators look at how a given fiscal budget is allocated among a set of individuals to increase their incomes and reduce their distance to the poverty line. In contrast, the IE^{MP} and SE^{MP} indicators will be looking at how a given fiscal budget is allocated among a set of individuals to convert a deprived achievement into a non-deprived one, but each of these has a different cost. That is, effectiveness in the multidimensional case needs to simultaneously consider the best allocation of money across dimensions (which deprivations to lift?) and across households (to whom should these deprivations bee lifted?). Additionally, as explained in Section 4.1.3, individuals are tied together in households, and deprivations are lifted in most cases at the household level.

The first element to define the optimal allocation is to select a poverty measure. In the IE^{MP} indicator, the optimal allocation of deprivation reductions will be such that the selected poverty measure reduction is maximized given the fiscal effort that has been observed between t_0 and t_1. In the SE^{MP} indicator the optimal allocation of deprivation reductions will be such that the observed reduction in the selected poverty measure is achieved with the minimum budget possible.

In this paper we propose using an MPI, that is, a measure with the structure of $M_0(\boldsymbol{x}; z)$, as used in most national, regional, and global MPIs so far. As M_0 is the incidence of multidimensional poverty *adjusted* by the intensity, using this measure avoids the perverse incentives of prioritizing the least intensely poor, which is in line with Sen's (1976) warning for the unidimensional case. Reducing poverty intensity also reduces M_0. Using M_0 also avoids the somehow opposite perverse incentives of using poverty intensity A as the sole indicator: reduce intensity as long as no one leaves poverty (q is in the denominator of the measure).

While being sensitive to intensity, it is important to note that because M_0 is not sensitive to inequality among the poor, maximizing its reduction does not guarantee that the poorest poor households are lifted deprivations. Poverty intensity A is an average, and given two households of equal size, A and thus M_0 will be equally reduced either if we lift a certain number of deprivations to a household with a higher deprivation score or to a household with a lower deprivation score. Moreover, given two households $h = 1,2$, such that one is more intensely poor but it is of smaller size than the other (i.e. $c_1(k) > c_2(k)$, but $s_1 < s_2$), A and thus M_0 will be reduced more if a deprivation is lifted to the bigger household than if the same deprivation is lifted to the smaller household, even when it is a poorer one.

We present two alternative criteria to define the optimal distribution for the IE^{MP} and SE^{MP} indicators subsequently. In both criteria the costing of removing each considered deprivation naturally plays a critical role. Thus, in the next section we first address the issue of costing. Next, we present the two criteria for the optimal allocation. The first criterion for the IE^{MP} indicator consists of maximizing the reduction in M_0 given a certain budget, that is looking for the most cost-effective removal of deprivation such that these reach the greatest possible reduction in M_0, in other words, to the greatest possible number of people. As a second guiding principle, deprivations are removed in decreasing order of poverty intensity, considering the Leave No One Behind (LNOB) pledge of the 2030 Agenda. We name this criterion the 'MaxN-LNOB' criterion. The dual exercise for the SE^{MP} indicator is to look for the minimum fiscal effort that would achieve the observed reduction in MPI, allocating it with the same guiding principle of reducing M_0 the most first, and prioritizing the poorest poor next. One non-trivial issue is that of censoring. When the union criterion is used, i.e., when all deprivations are counted, finding the optimal allocation can be solved as a linear programming problem. However, when an intermediate poverty cutoff is used, such that the deprivations of the non-poor need to be censored, the optimization process needs to be implemented iteratively.

Considering that maximizing the reduction in the MPI a priori does not guarantee reducing deprivations among the poorest poor, and to fully embody the Leave No One Behind Pledge, the second criterion for the indicators inverts the order of the guiding principles: it first looks for the most cost-effective removal of deprivation such that these reach the most intensely poor households. As a second guiding principle, deprivations are removed in decreasing order of household size, i.e. maximizing the number of people to whom deprivations are removed. We name this criterion the 'LNOB-MaxN' criterion. This optimization procedure needs to be implemented iteratively, regardless of the deprivation cutoff used.

It is however worth noting that if the identification of the poor was done at the individual level, and the cost of removing the deprivations was also at the individual level, the two criteria produce the same optimal distribution. Analogously if, despite identifying the poor and removing deprivations at the household level, household sizes are ignored, then the two criteria also produce the same results.

4.3.1 Costing of removing deprivations

A fundamental piece of information for the effectiveness indicators is the costing of removing each kind of deprivation. In this paper we are thinking in terms of indicators that reflect shared deprivations for the households, for example lack of access to basic services networks such as water, sanitation, natural gas and electricity. Housing indicators (for example, overcrowding) may also be considered.

In the first place, let's assume that there is a per household cost of removing each j deprivation which we denote phc_j, i.e. the cost 'per connection'.[13] The ratio between the cost of removing each deprivation and the MPI's weight of each deprivation phc_j/w_j suggests an ordering of cost-effectiveness of each dimension, as presented in Table 1. In the case in which dimensions are equally weighted, cost-effectiveness is simply given by the cost. However, the ordering given by the phc_j/w_j ratio is only suggestive because the cost-effectiveness of removing a deprivation will also be influenced by the size of the poor households which are deprived in that indicator. It may be optimal to reduce deprivations in an indicator with a higher cost-weight ratio but also with a higher deprivation rate, i.e. more widespread deprivation. This will be exemplified in Section 5.1.1.

Table 1. Cost-Weight ratios

phc *of each* j *dimension*	*Weighting in the* M_0 *of each* j *dimension*	*Cost-Effectiveness Ratios*	*Suggestive Priorities*
phc_1	w_1	phc_1/w_1	*From lowest cost-effectiveness ratio to highest*
phc_2	w_2	phc_2/w_2	
…	…		
phc_d	w_d	phc_d/w_d	

Presented in this way, the costing of removing each deprivation is assumed to be a) independent and constant across households, and b) independent between deprivations. However, for many public services

[13] This framework can also be adapted to costs expressed *per capita*.

this -a priori- may not sound accurate. As it is well known, expanding the water, sanitation, gas, or electricity network most typically entails very high fixed costs, which in turn imply that the households' connection cost is decreasing in the number of households to be connected. In such case, the phc_j for one household would not be independent of the cost of other households. Also, certain services such as water and sewage sanitation, have technical complementarities. For example, the extension of the sanitation network first requires the extension of the water network. Thus, if an area already has the water network, the cost of bringing sanitation is lower than the cost of bringing sanitation to an area in which there is no piped water. Nevertheless, these considerations can be incorporated in the analysis.

First, while it is true that expanding the access to a public service may entail a significant infrastructure investment, this is not always the case. Many urban areas in developing countries already have a network for these services and yet not all neighborhoods are connected to them. For example, Galiani et al (2009), evaluate the impact of a program of expansion of the water network in urban shantytowns in Argentina which was precisely focused on extending secondary connections, not the primary water network.[14] In such cases, the assumption of independent costs is not unrealistic. Second, the cost of removing deprivations can be discriminated by geographic area in the optimization process such that all kinds of specificities can be included. Remote areas that require a big infrastructure investment will have a higher connection cost than urban areas which have the primary network nearby. In such case, the per household cost would be $ph_a c_j$, depending on the area a where the household is located. Also, in the cases in which a significant investment in infrastructure was required to bring connections to certain locations in a country, it should be possible to estimate the minimum number of households to connect to the service so that a certain per household connection cost would be achieved, and such minimum number of connections can be incorporated as a restriction into the optimization process.

4.3.2 Defining the optimal allocation under the MaxN-LNOB criterion for the Impact Effectiveness Indicator

4.3.2.1 Impact Effectiveness under the MaxN-LNOB criterion using a union poverty cutoff

For simplicity in the exposition of the methodology, we will present it as if one had panel data. However, the implementation of the methodology does not require panel data. Assume that between t_0 and t_1 there has been a certain multidimensional poverty reduction of size $\Delta M_{0_{t_0-t_1}} = DM$. For the moment, assume that the total population has not changed $n_0 = n_1 = n$, nor the number of households $T_1 = T_0 = T$. For presenting the optimization problem it is useful to consider the change in the poverty measure. This is

14 In fact, a technical condition for the shantytown to access the program was that the neighborhood had to be less than two hundred meters away from the main water network.

straightforward in the case of the union approach. With n invariant over time, the change in M_0, with $k = \min(w_j)$ can be expressed as:

$$\Delta M_{0_{t_0-t_1}}(k = \min(w_j)) = M_{0_{t_0}}(k = \min(w_j)) - M_{0_{t_1}}(k = \min(w_j)) =$$
$$\frac{1}{n}\left[\sum_{j=1}^{d} w_j \sum_{h=1}^{T} s_h\left(g_{hjt_0}^0 - g_{hjt_1}^0\right)\right] = DM \quad (16)$$

In words, under the union approach, the change in M_0 can be expressed as the dimensionally weighted sum of the number of people (expressed in turn as the weighted sum of households, with weights being their corresponding sizes s_h) that have stopped being poor and deprived in each dimension. This does not hold for k poverty cutoffs other than union because the remaining deprivations in t_1 of those who have stopped being poor are censored. This is further explained below.

Now assume that the poverty reduction DM has been achieved with a fiscal budget of amount B. Let's assume that the cost of removing each j deprivation is given by the *per household cost* phc_j, with the considerations done in Section 4.3.1, such that these may be refined taking different values across geographical areas. The optimization problem consists of finding the sets of households $R_j = \{h: (g_{hjt_0}^0 - g_{hjt^*}^0) = 1\}$, with size $|R_j| = T_j^R$, (i.e., T_j^R is the total number of households to which deprivation j is removed between t_0 and an optimal distribution t^*), for $j = 1, \ldots d$,such that:

1) $\frac{1}{n}\left[\sum_{j=1}^{d} w_j \sum_{h=1}^{T_0} s_h\left(g_{hjt_0}^0 - g_{hjt^*}^0\right)\right]$ is maximum, subject to:

2) $\sum_{j=1}^{d} phc_j T_j^R \leq B$ *(the cost of reducing poverty must be within the observed fiscal budget)*

As explained in Section 4.3.1, if needed, a restriction on the required minimum number of connections to a service that need to be achieved, let it be denoted by e, can be incorporated as a further restriction: $e \leq T_j^R$ with $e \in \mathbb{R}^{++}$.[15]

Such a linear programming problem can be easily solved with a software like Mathematica. Moreover, the above optimization problem, which maximizes the reduction in MPI, and thus prioritizes bigger households, can be implemented within an algorithm that has, as a second guiding principle, the LNOB criterion. That is, given two households $h = 1,2$, of equal size $s_1 = s_2$, such that it is optimal that a certain deprivation is removed to one of them, an algorithm which starts from the solution values of the linear

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[15] If the exercise is implemented with household survey data which contains a survey weight variable p_{ht_0}, the change in the poverty measure is given by: $\Delta M_{0_{t_0-t_1}}(k = \min(w_j)) = \sum_{j=1}^{d}\sum_{h=1}^{T_0} s_h p_{ht_0}\left(g_{hjt_0}^0 - g_{hjt_1}^0\right)/n$, and the budget constraint is expressed as $\sum_{j=1}^{d} p_{ht_0} phc_j T_j^R \leq B$.

programing problem can select the household with the highest deprivation score. If $c_1(k) > c_2(k)$, then the deprivation will be removed to household 1.

While this procedure can be relatively easily implemented, it is not general enough, as it does not hold for the case of poverty cutoffs which are not the union criterion. We address that case next.

4.3.2.2 Impact Effectiveness under the MaxN-LNOB criterion in the general case: allowing an intermediate poverty cutoff

Whenever the poverty cutoff k is not the one of the union criterion, the removal of a deprivation can lift a household from poverty, according to that k-cutoff, even when other deprivations remain. The remaining deprivations must be censored and should thus 'disappear' from the objective function of expression (16). Thus, the optimization in this case, cannot be solved with a linear programing problem. It must be solved as an iterative optimization problem, that re-identifies the poor after each removal of deprivations and censors the deprivations of those who have stopped being poor.

To present the optimization process under the MaxN-LNOB criterion for the IE^{MP} indicator in the general case of an *any* poverty cutoff, including an intermediate or even intersection one, it is useful to define a cost-effectiveness matrix $\boldsymbol{CE^{IE}}$, which constitutes the decision tool. Whenever the k poverty cutoff is different from the one corresponding to the union criterion ($k > \min(w_j)$), the $\boldsymbol{CE^{IE}}$ matrix will change in each iteration.

Denote with v the iteration number, with $v = 1, \ldots, V$. Iteration number V is such that an optimal distribution has been found. Let $\_v$ denote the current iteration, and thus $\_v - 1$ as the previous one. The cost-effectiveness matrix for the IE^{MP} indicator in each iteration, is given by $\boldsymbol{CE^{IE}_{\_v}} = \left[ce^{IE}_{hj\_v}\right]$, such that:

$$ce^{IE}_{hj\_v} = \frac{(s_h/n)\, w^*_{j\_v}\, g^0_{hj(\_v-1)}(k)}{phc_j} \quad (23)$$

where:

$w^*_{j\_v} = \sum_{j=1}^{d} w_j g^0_{hj(\_v-1)}$ if $(c_{h(\_v-1)} - w_j) < k$ (is lifted from poverty)

$w^*_{j\_v} = w_j$ if $(c_{h(\_v-1)} - w_j) \geq k$ (remains poor)

With $c_{h(\_v-1)} = c_{ht_0}$ for $v = 1$.

In words, the numerator of each $ce^{IE}_{hj\_v}$ element indicates by how much the MPI would be reduced if deprivation j was lifted to household h. This general formula accounts for the case in which an intermediate poverty cutoff is used and thus the need to define the w^*_j parameter. When the weight of deprivation j, w_j, is such that removing that deprivation would lift that household from poverty, that is, when $(c_{h(\_v-1)} - w_j) < k$, then the full impact of removing that deprivation in the MPI reduction should not only consider the *j*-th deprivation's weight w_j, but rather the sum of the weights of all the deprivations still experienced by that household up to the previous iteration $(\_v-1)$ ($w^*_{j\_v} = \sum_{j=1}^{d} w_j g^0_{hj(\_v-1)}$), as the remaining deprivations will be censored once the household is lifted up from poverty. Otherwise, when removing deprivation *j* is not sufficient for removing that household from poverty, i.e. when $(c_{h(\_v-1)} - w_j) \geq k$, then only the *j*-th deprivation weight matters in the account of poverty reduction, and thus $w^*_{j\_v} = w_j$. In turn, the denominator of each ce_{hj} element indicates the cost of removing deprivation *j*. Altogether, each $ce^{IE}_{hj\_v}$ element of matrix $\boldsymbol{CE^{IE}}$ indicates the reduction in M_0 produced by one monetary unit spent in eliminating the *j*-th deprivation for household h.

It is also necessary to define the accumulated cost of removing deprivations up to each $\_v$ iteration, which is given by:

$$AccCost_{\_v} = \sum_{v=1}^{\_v} \sum_{j=1}^{d} phc_j \sum_{h=1}^{T} \left(g^0_{hj(\_v-1)}(k) - g^0_{hj\_v}(k) \right) \quad (24)$$

with $g^0_{hj(\_v-1)} = g^0_{hjt_0}$ for $v = 1$.

Expression (24) indicates that the accumulated cost up to iteration $\_v$ is given by the change of household h from being deprived in indicator j in the previous iteration to being non- deprived in the current iteration, in which case $g^0_{hj(_v-1}(k) - g^0_{hj_v}(k) = 1$, multiplied by the cost of removing deprivation j, phc_j, adding across all T households and d dimensions, accumulating all the iterations up to the current one.

Now we can describe the optimization algorithm, written in Mathematica, which iteratively proceeds in this way. In each current $\_v$ iteration:

1) It finds the maximum value(s) of the $\boldsymbol{CE^{IE}_{\_v}}$ matrix.
2) It verifies that the accumulated cost of removing that deprivation j from household h is within the budget, ie.: $AccCost_{\_v} \leq B$.
   a. If this is not the case, the algorithm discards removing that *j* deprivation to that household.
   b. If the condition holds, whenever there are two equal cost-effectiveness values, the algorithm selects the household which has the highest deprivation score and removes that deprivation from that household.
3) The algorithm re-identifies the poor (according to the k value) and censors the deprivations of the non-poor. It computes the $\boldsymbol{CE^{IE}_{(\_v+1)}}$ matrix (i.e. the cost-effectiveness matrix for the next iteration).

4) Steps 1-3 are repeated until the budget limit is reached, i.e. until

$$AccCost_{\_v} = B$$ [16]

or until the remaining budget is not enough to remove any other deprivation.

Implementing this algorithm is equivalent to solving the linear programming problem detailed before, but in the general case in which such problem needs to be solved iteratively. The MaxN-LNOB criterion is exemplified in Section 5.1.1.

The algorithm can be implemented with household survey data, with a survey weight variable. Note that the values of the $\boldsymbol{CE^{IE}}$ matrix remain the same as if there were no survey weights, because the survey weight p_h multiplies both the numerator (because the survey weight affects the MPI reduction) and the denominator (because the per household cost of removing deprivation j to an h household needs to be multiplied for as many households that household represents). However, both the computation of the poverty reduction DM as well as the accumulated cost, incorporate the survey weight variable in their expressions, as detailed in footnote 18. Thus, the selected deprivations to be removed to which households, will naturally differ from the case in which there are no survey weights.

It is also worth noting that the algorithm can be adapted to include additional constrains, like (for instance) the restriction that deprivations must be removed for a minimum number of households for technical reasons related to costing. Similarly, the per household costs of removing each deprivation need not be the same across all households. Variations according to their geographical location can also be incorporated.

4.3.3 Defining the optimal allocation under the MaxN-LNOB criterion for the Spending Effectiveness Indicator

We now present the same criterion but for the SE^{MP} indicator.

4.3.3.1 Spending Effectiveness under the MaxN-LNOB criterion using a union poverty cutoff

Looking for the optimal distribution under the MaxN-LNOB criterion for the SE^{MP} indicator with a union poverty cutoff consists of solving the dual of the linear programming problem set for the IE^{MP} indicator, and this can be stated as follows.

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[16] Equivalently, until $\sum_{j=1}^{d} phc_j T_j^R = B$.

Find the sets of households $R_j = \{h: (g^0_{hjt_0} - g^0_{hjt^*}) = 1\}$, with size $|R_j| = T^R_j$, for $j = 1, \dots d$, such that:

$$\sum_{j=1}^{d} phc_j T^R_j \text{ is minimum, subject to:}$$

$$\frac{1}{n}\left[\sum_{j=1}^{d} w_j \sum_{h=1}^{T_0} s_h\left(g^0_{hjt_0}(k) - g^0_{hjt^*}(k)\right)\right] \geq DM$$

Like for the SE^{MP}, given the solution to the linear programming problem, for each household size to which a certain deprivation must be removed, one can select the households with the highest deprivation scores, such that the second guiding principle is the LNOB criterion.

As with the SE^{MP} indicator, to generalize the optimization process to any poverty cutoff, we need to define an iterative optimization process analogous to the one defined above.

4.3.3.2 Spending Effectiveness under the MaxN-LNOB criterion in the general case: allowing an intermediate poverty cutoff

Continuing with the same notation, we now define the cost-effectiveness matrix for the SE^{MP} indicator. For that, it is necessary to define the accumulated poverty reduction until the iteration before the current $\_v$ iteration, which is given by:

$$AccDM_{(\_v-1)} = \frac{1}{n}\sum_{v=1}^{\_v-1}\sum_{j=1}^{d} w_j \sum_{h=1}^{T} s_h\left(g^0_{hj\_v-2}(k) - g^0_{hj(\_v-1)}(k)\right) \quad (25)$$

Then, the cost-effectiveness matrix for the SE^{MP} indicator $\boldsymbol{CE^{SE}_{\_v}} = \left[ce^{SE}_{hj\_v}\right]$, is such that:

$$ce^{SE}_{hj\_v} = \frac{Min\left(DM - AccDM_{(\_v-1)},\ (s_h/n)w^*_{jv} g^0_{hjt\_v}(k)\right)}{phc_j} \quad (26)$$

where $w^*_{j\_v}$ is the same as defined in (23). Expression (26) indicates that, for each iteration $\_v$, the cost-effectiveness coefficients of the $\boldsymbol{CE^{SE}_{\_v}}$ matrix are the same coefficients of the $\boldsymbol{CE^{IE}_{\_v}}$ matrix whenever these are smaller than the difference between the target poverty reduction and the accumulated poverty reduction up to the iteration previous to the current one. This will surely be the case for the first iterations. As poverty reduction progresses by lifting deprivations, the gap between the target poverty reduction and the already achieved one, i.e. $DM - AccDM_{(\_v-1)}$, will narrow and, at some advanced iteration, for at least one (h, j), such distance will become smaller than the expression $s_h\ w^*_{j\_v}\ g^0_{hj(\_v-1)}(k)$, which gives the reduction in poverty that can be achieved by lifting deprivation j, to household h. Then, the minimum value divided by the cost of removing that deprivation will be the $ce^{\boldsymbol{SE}}_{hj\_v}$ coefficient. The intuition is that, because the optimal distribution for the spending effectiveness indicator is a minimization exercise, as the optimal allocation of the budget approaches meeting the target, the last selected deprivations to be removed do not need to be those with the biggest poverty reduction impact but rather those that just meet the target. Otherwise, poverty reduction might exceed what is required by the exercise, and thus the budget would not be minimized.

Using the $\boldsymbol{CE^{SE}}$ matrix, the optimization algorithm, developed in Mathematica, iteratively proceeds in this way:

1) It finds the maximum value(s) of the $\boldsymbol{CE^{SE}_{\_v}}$ matrix.
    a. Whenever there are two equal cost-effectiveness values, it selects the household which has the highest deprivation score and it removes that deprivation from that household.
2) It verifies that the accumulated poverty reduction has not yet reached the poverty reduction target DM, i.e. that $AccDM_{(\_v)} < DM$
3) It re-identifies the poor (according to the k value) and censors the deprivations of the non-poor. It computes the $\boldsymbol{CE^{SE}_{(\_v+1)}}$ matrix (i.e. the cost-effectiveness matrix for the next iteration).
4) Steps 1-3 are repeated until the accumulated decrease in M_0 is at least DM, i.e. until $AccDM_{(\_v)} \geq DM$.

This algorithm is exemplified in Section 5.1.2. The algorithm can be implemented with household survey data, with a survey weight variable.

As explained above, the MaxN-LNOB criterion does not guarantee that the poorest poor households are lifted deprivations, but rather that the most cost-effective deprivations are lifted to the greatest number of people. In the above algorithms larger households will be prioritized and, as it will be exemplified in Section 5, in the optimal allocation under this criterion, the poorest households may be left as poor as they were initially, or with little change. While empirically it is frequently the case that the poorer households tend to be larger, this may not always hold. Then, it is reasonable to consider an optimization criterion that explicitly prioritizes the poorest poor, embodying the philosophical principle of *prioritarianism,* in the spirit of the Leave No One Behind claim of the 2030 Agenda. This criterion is detailed in what follows.

Note however, that if deprivations were removed to individuals and not households, or if household sizes were ignored, the MaxN-LNOB criterion would coincide with the next proposed criterion, the LNOB-MaxN. In fact, in such case, the elements of the $\boldsymbol{CE^{IE}}$ and $\boldsymbol{CE^{SE}}$ matrices would not have the household size variable s_h, and thus the most cost-effective indicators would be lifted to the poorest poor.

4.3.4 Defining the optimal allocation under the LNOB-MaxN criterion for the Impact Effectiveness Indicator

Under the LNOB-MaxN criterion the optimal allocation is such that deprivations are removed to the poorest poor households in the first place, even when this does not imply removing deprivations to the greatest number of people. That is, the main criterion driving the optimization solution is that the most cost-effective deprivations are lifted to the poorest poor households. Naturally, the next guiding principle in the optimization problem is, from the poorest households, choose the largest ones, to guarantee that, among the poorest poor, deprivations are lifted to the greatest number. That is why we call this the LNOB-MaxN criterion, as it simply inverts the order of the guiding optimization principles compared to the previous criterion.

In this case, the optimization criterion needs to work iteratively even when the union poverty cutoff is used because poor households with the maximum deprivation score need to be identified after each round of lifting deprivations. Thus, we directly detail the algorithm. Noteworthy, the algorithm is based on the same decision tool as the MaxN-LNOB criterion, the $\boldsymbol{CE^{IE}_{\_v}}$ matrix. The only difference is the order in which the elements of the matrix are selected.

Using the $\boldsymbol{CE^{IE}}$ matrix, the optimization LNOB-MaxN algorithm, developed in Mathematica, proceeds in this way:

1) It orders the rows of the $\boldsymbol{CE^{IE}_{\_v}}$ matrix by the censored deprivation score $c_i(k)$, from poorest to least poor.
2) It finds the maximum value(s) of the $\boldsymbol{CE^{IE}_{\_v}}$ matrix in the rows corresponding to the maximum deprivation score.
    a. Whenever there are two equal cost-effectiveness values for different households with the highest deprivation score, it selects the household which has the biggest size and removes that deprivation from that household.
3) It verifies that the accumulated cost of removing that deprivation j from household h is within the budget, ie.: $AccCost_{\_v} \leq B$. If this is not the case, it discards removing that *j* deprivation to that household.
4) The algorithm re-identifies the poor (according to the k value) and it censors the deprivations of the non-poor. It computes the $\boldsymbol{CE^{IE}_{(\_v+1)}}$ matrix (i.e. the cost-effectiveness matrix for the next iteration).
5) Steps 1-4 are repeated until the budget limit is reached, i.e. until

$$AccCost_{\_v} = B$$

or until the remaining budget is not enough to remove any other deprivation.

4.3.5 Defining the optimal allocation under the LNOB-MaxN criterion for the Spending Effectiveness Indicator

Analogously, the algorithm that implements the LNOB-MaxN criterion to find the optimal allocation for the SE^{MP} indicator, uses the same decision tool as the MaxN-LNOB criterion, the $\boldsymbol{CE}_{\_v}^{SE}$ matrix, with the only difference being the order in which the elements of the matrix are selected.

Using the $\boldsymbol{CE}^{SE}$ matrix, the optimization LNOB-MaxN algorithm, developed in Mathematica, proceeds in this way:

1) It orders the rows of the $\boldsymbol{CE}_{\_v}^{SE}$ matrix by the censored deprivation score $c_i(k)$, from poorest to least poor.
2) It finds the maximum value(s) of the $\boldsymbol{CE}_{\_v}^{SE}$ matrix in the rows corresponding to the maximum deprivation score.
    a. Whenever there are two equal cost-effectiveness values for different households with the highest deprivation score, it selects the household which has the biggest size and removes that deprivation from that household.
3) It verifies that the accumulated poverty reduction has not yet reached the poverty reduction target DM, i.e. that $AccDM_{(\_v)} < DM$.
4) It re-identifies the poor (according to the k value) and censors the deprivations of the non-poor. It computes the $\boldsymbol{CE}_{(\_v+1)}^{SE}$ matrix.
5) It repeats steps 1-4 until the accumulated decrease in M_0 is at least DM, that is: $AccDM_{(\_v)} \geq DM$

As a general note, we should remark that the four described algorithms (MaxN-LNOB and LNOB-MaxN, for impact and spending effectiveness), which are based in the $\boldsymbol{CE}_{\_v}^{IE}$ and the $\boldsymbol{CE}_{\_v}^{SE}$ matrices correspondingly could, in certain cases and due to the discrete character of the procedure (deprivations are lifted one household at a time), generate suboptimal allocations in the 'last mile' of the algorithm. That is, the last deprivations selected to be removed such that the constraint is satisfied, may result in falling short of using all the budget in IE, or in exceeding by too much the poverty reduction target in SE. In such cases, it may be possible to find an alternative combination of deprivations' removal which may perfect the original matrix solution, approximating the satisfaction of the corresponding constraint with finer tuning. However, this is not a matter for concern when implementing the algorithms with real data, which have population sizes in which the discrete effect is diluted. To avoid those possible cases in which the solution would be sub-optimal the algorithms could adopt a different approach, doing a step-by-step iterative optimization process but at the cost of a substantial longer computing time. We understand that such cost is not worth it given the marginal effective incidence of this issue in real data applications.

5. Examples illustrating the methodology

In this section we will exemplify the two alternative criteria to determine the optimal allocation for the IE^{MP} and the SE^{MP}indicators: MaxN-LNOB *vs.* LNOB-MaxN under a union poverty cutoff as well as under an intermediate poverty cutoff. In this example we assume equal weights. In Appendix 2 we provide an example with the same initial distribution using unequal weights.

Assume a society of 10 households adding up to a total of 40 people. For simplicity we assume there are no survey weights, but these can be incorporated as detailed above. The example proceeds as if one had a panel and there was no population growth. In Section 7 we explain how to work with cross-section data and deal with population growth. Assume that multidimensional poverty is measured using four indicators which have received fiscal investment: water, sewage, natural gas and electricity. Let's consider a baseline case in which all indicators weight the same: $w_j = \frac{1}{4}$ for $j = 1,2,3,4.$ Consider the following per household costs of removing each deprivation. As in this case weights are equal across indicators, the order of priority is simply given by the cost, as detailed in Table 2.

Table 2. Per Household Costs – Example with equal weights

Dimension	phc *of each* j *dimension*	*Weighting in the* M_0 *of each* j *dimension*	*Cost-Effectiveness Ratios*	*Priorities*
Water	800	0.25	3200	2
Sanitation	2300	0.25	9200	4
Gas	900	0.25	3600	3
Electricity	400	0.25	1600	1

5.1.1 Impact Effectiveness under different optimal criteria and different poverty cutoffs

In Table 3 we present deprivation matrix in t_0 and in t_1, with households ordered from poorest to richest, using a union poverty cutoff (k=0.25), and the optimal distributions that result from the MaxN-LNOB and the LNOB-MaxN criteria correspondingly. We also present a column on whether the household is identified as poor or not in each distribution, as well as the censored deprivation score of each distribution, which, in this case, coincide with the uncensored deprivation scores. In Table 4 we present the same distributions but using an intermediate (high) poverty cutoff of k=0.75. Zeroes in light blue denote censored deprivations.

According to the distributions, we are assuming that, between these two points in time, poverty was reduced removing deprivation in water to two households, deprivation in sanitation to one household, deprivation in natural gas to two households and deprivation in electricity to three households, with a spent budget of \$6,900 ($B = 2 * 800 + 1 * 2300 + 2 * 900 + 3 * 400 = 6{,}900$). Deprivations that are removed appear as a red zero in the deprivation matrix in t_1 as well as in the optimal distributions.

As detailed in the first rows of Table 7, from t_0 to t_1 there was a reduction of 0.163 in the MPI using a union poverty cutoff, which is the result of reducing the poverty headcount ratio in 0.05 (note that household #7 stopped being poor), and a reduction in poverty intensity of 0.149. If an intermediate poverty cutoff of k=0.75 is used, poverty was reduced in 0.244, also with a reduction both in H and A. Note that in this case, because of the higher poverty cutoff, three households (#2, #3 and #4) stopped being poor.

Now suppose one wants to assess the impact effectiveness of that fiscal effort. Implementing the algorithm under the MaxN-LNOB criterion, which maximizes the reduction in M_0, gives the distribution described in the third matrix of Table 3, with the red zeroes denoting the deprivations that the optimal allocation removes. This distribution results from implementing the described algorithm using the $\boldsymbol{CE^{IE}}$ matrix. To illustrate the methodology, we present the $\boldsymbol{CE^{IE}}$ matrix in Table 5, with red entries on the deprivations that is optimal to remove according to each criterion. As in this case a union poverty cutoff is used, the coefficients of the cost-effectiveness matrix can only change from their initial values to zero (when deprivations are removed) and thus we can simply present one matrix.[17]

Note that in the optimal MaxN-LNOB distribution, the budget is used to completely remove deprivation in the most cost-effective dimension (i.w. that one with the lowest phc_j/w_j ratio), electricity. It is also used to remove deprivation in the second most cost-effective dimension -water- to two of the five households deprived in that indicator, but not to the five of them. This is because three of the households deprived in water are small (households #2 and #3 are of three members, and household #7 is of two members), and it reduces poverty more to remove deprivation in the third most cost-effective dimension -gas- to households that are larger. Being sanitation the least cost-effective dimension, no household is removed deprivation in this dimension. The total budget used is \$6,300. The remaining \$600 are not enough for reducing any of the deprivations left. The total MPI reduction under this allocation is 0.269, much higher than the observed reduction of 0.163, and thus the impact effectiveness indicator in this case is 61%, as detailed in Table 7, indicating that the spent budget only achieved 61% of the potential MPI reduction.

17 The MaxN-LNOB optimal distribution can also be obtained stating the linear programming problem in Mathematica, as detailed in Section 4.3.2.1.

However, also note that in this solution, two of the poorest households, households #2 and #3, which were deprived in all dimensions, only have seen reduced their weighted deprivations in 0.25, whereas, for example, household #4, with a lower initial deprivation score of 0.75, has seen its deprivation score being reduced in 0.5. As explained, this is because this the MaxN-LNOB criterion gives priority to removing deprivations to larger households over removing deprivations to more intensely poor households.

Now, suppose we implement the LNOB-MaxN criterion, looking for the most cost-effective eradication of deprivations but prioritizing the poorest poor households in every round of deprivation removal. Under this optimal criterion, which distribution is depicted in the fourth matrix of Table 3, deprivation in the first and the second most cost-effective dimensions -electricity and water- are lifted to all poor and deprived households in those dimensions. Note that lifting water deprivation to all households differs from the MaxN-LNOB solution, in which households #7, #2 and #3 are not removed this deprivation. This is because the algorithm works iteratively looking in each iteration the highest coefficients of the $\boldsymbol{CE^{IE}}$ matrix among households with the highest deprivation score.[18] In this case, the full $6,900 budget is used and the achieved MPI reduction is of 0.263, a bit lower than with the MaxN-LNOB criterion, thus, the impact effectiveness indicator is a bit higher under this optimal allocation, 62%.

However, while the reduction of MPI with a union poverty cutoff is not necessarily maximized with the LNOB-MaxN criterion, note that with this distribution no household is left with a deprivation of 0.75 or higher (the distribution of deprivation scores of the MaxN-LNOB criterion and the LNOB-MaxN criterion can be compared in last two columns of Table 3). Thus, impact effectiveness under this criterion should also be evaluated with an MPI with a higher deprivation cutoff. Indeed, if an MPI with a k=0.75 is computed over the two optimal distributions, as detailed in the three last columns of Table 7, one can see that poverty is reduced to 0 under the LNOB-MaxN distribution, and thus impact effectiveness is reduced to 56%. In contrast, under the MaxN-LNOB criterion, MPI with k=0.75 is reduced only to 0.112, and so impact effectiveness of the observed distribution is higher, 75%.

One way to visualize the difference between the two optimal allocations is depicting the multidimensional dominance curves introduced in Alkire et al (2015, ch. 7). Figure 5.2 depicts the Complementary

18 The sequence is as follows. From the $\boldsymbol{CE^{IE}}$ matrix depicted in the last four columns of Table 5, first deprivation in electricity is removed to household #1, next to households #2 and #3. Next, by looking at the maximum coefficients of all households with a deprivation score of 0.75 (now the highest), deprivation in electricity is removed to household #5. Then, deprivation in water is removed to household #4, and then to households #1, #2 and #3. Next, by looking at the maximum coefficients of all households with a deprivation score of 0.5 (now the highest), deprivation in electricity is removed to household #6. Finally, there is budget left to remove deprivation in gas to household #4. Finally, deprivation in natural gas can be removed to the poorest-biggest household (household #7) left after the removal of the other deprivations.

Cumulative Distribution Functions for MaxN-LNOB and LNOB-MaxN, which indicate (in the y-axis) the proportion of people who have a c_i score equal or higher than each k value, i.e., they indicate the H for the different k values (in the x-axis). These are first order dominance curves. In turn, Figure5.3 depicts the Adjusted Headcount Ratio dominance curves, which indicate the M_0 value at each possible k value. It is a second order dominance curve. The data used to construct these two curves is detailed in Appendix 1, Table A1.1. In both cases, whenever a distribution A has a curve which lies somewhere below and nowhere above the curve of another distribution B, we can say that distribution A stochastically dominates distribution B, meaning, in the first case, that distribution A has an equal or lower H than distribution B at all possible k values and, in the second case, that distribution A has an equal or lower M_0 than distribution B at all possible k values.

In both figures the curves of the MaxN-LNOB and the LNOB-MaxN cross. In Figure 5.1, for a k value of up to 0.5, the MaxN-LNOB solution dominates the LNOB-MaxN, as it has lower H, but from then onwards, the LNOB-MaxN dominates. In the second order dominance curves presented in Figure 5.2, we also notice a similar pattern: the MaxN-LNOB solution dominates the LNOB-MaxN up to a k of 0.75, meaning that it has a lower M_0 value, but from then onwards, the LNOB-MaxN dominates. Therefore, it is reasonable that for MPI values with high poverty cutoffs the impact effectiveness assessment under the LNOB-MaxN optimal distribution is more demanding than the MaxN-LNOB, giving an effectiveness of only 56% vs. an effectiveness of 75%. This simply implies that with $6,900 multidimensional poverty of high intensity could have been reduced much more, and in this example -in fact- eradicated, had the poorest households been prioritized.

What happens if the optimal distributions under the two alternative criteria are computed over the censored distribution? This is detailed in Table 4 for the case of an intermediate poverty cutoff $k = 0.75$. With an intermediate cutoff one needs to ignore the deprivations of those who are not identified as poor ($c_i < 0.75$), marked in light blue. In this case, because of using a poverty cutoff higher than the union one, the coefficients of the $\boldsymbol{CE^{IE}}$ matrix will change in each iteration, as deprivations are lifted not only to zero (when a deprivation is removed), but also because removing a deprivation may become more cost-effective if, after the removal of some other deprivation: lifting some of the remaining ones would move the household out of poverty. This is exemplified in Table 6, which depicts how the $\boldsymbol{CE^{IE}}$ matrix changes in each iteration when implementing the MaxN-LNOB criterion.

In this case, because of the censoring of the deprivations of those with a lower deprivation score, the optimal distribution according to the MaxN-LNOB criterion coincides with that of the LNOB-MaxN criterion (although the sequence to arrive to the same result is different): both eradicate poverty, and thus, the evaluations of impact effectiveness also coincide in being 56%, suggesting that poverty reduction for the poorest poor stayed about halfway (see Table 7). The fact that the two distributions coincide for a high poverty cutoff is not unequivocal however, it depends on the distribution of household sizes alongside the distribution of deprivation scores, and the particular k value used. In the unequal weights example presented in the Appendix 2, the two optimal distributions do not coincide even for high k values.

Also note that because of the censoring, the available budget under the two optimal distributions is underutilized: poverty, as measured by an MPI with k=0.75, is eradicated with $4800, much less than the available budget of $6900, missing the opportunity to lift the censored deprivations. In particular, when the uncensored distribution of deprivations is used, the optimal distribution under the LNOB-MaxN criterion, once having eradicated deprivation scores of 1 and 0.75, uses the remaining budget to reduce deprivations among households with the next highest poverty intensity.

If the aim is to reduce multidimensional poverty prioritizing the poorest poor and making use of all the available budget, the recommendable evaluation metric seems to be implementing the LNOB-MaxN optimization algorithm over the uncensored distribution of deprivations and evaluating impact effectiveness using alternative poverty cutoffs, from highest to lowest. Such evaluation will elucidate which poverty-intensity groups have been privileged by the fiscal effort.

Table 3. Impact Effectiveness – Equal Weights – Union Criterion (k=0.25) – MaxN-LNOB vs. LNOB-MaxN

HH#	HH Size	Weighted deprivations t0				Weighted deprivations t1				Weighted deprivations Optimal MaxN-LNOB				Weighted deprivations Optimal LNOB- MaxN				P t0	P t1	P MN-LN	P LN-MN	ci(k) t0	ci(k) t1	ci(k) MaxN-LNOB	ci(k) LNOB-MaxN
		W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E								
1	4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	0.25	0	0	0	0.25	0.25	0	1	1	1	1	1	1	0.25	0.5
2	3	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0.25	0.25	0.25	0	0	0.25	0.25	0	1	1	1	1	1	0.5	0.75	0.5
3	3	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0.25	0.25	0.25	0	0	0.25	0.25	0	1	1	1	1	1	0.5	0.75	0.5
4	5	0.25	0.25	0.25	0	0	0.25	0.25	0	0	0.25	0	0	0	0.25	0	0	1	1	1	1	0.75	0.5	0.25	0.25
5	5	0	0.25	0.25	0.25	0	0.25	0.25	0.25	0	0.25	0	0	0	0.25	0.25	0	1	1	1	1	0.75	0.75	0.25	0.5
6	5	0	0.25	0	0.25	0	0.25	0	0	0	0.25	0	0	0	0.25	0	0	1	1	1	1	0.5	0.25	0.25	0.25
7	2	0.25	0.25	0	0	0	0	0	0	0.25	0.25	0	0	0	0.25	0	0	1	0	1	1	0.5	0	0.5	0.25
8	7	0	0.25	0	0	0	0.25	0	0	0	0.25	0	0	0	0.25	0	0	1	1	1	1	0.25	0.25	0.25	0.25
9	3	0	0	0.25	0	0	0	0.25	0	0	0	0.25	0	0	0	0.25	0	1	1	1	1	0.25	0.25	0.25	0.25
10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
No. of People	**40**	**17**	**34**	**23**	**20**	**10**	**32**	**17**	**9**	**8**	**34**	**9**	**0**	**0**	**34**	**18**	**0**	**37**	**35**	**37**	**37**				
CHs		**43%**	**85%**	**58%**	**50%**	**25%**	**80%**	**43%**	**23%**	**20%**	**85%**	**23%**	**0%**	**0%**	**85%**	**45%**	**0%**								

Table 4. Impact Effectiveness – Equal Weights – Intermediate Criterion (k=0.75) – MaxN-LNOB vs. LNOB-MaxN

HH#	HH Size	Censored Weighted deprivations t0				Censored Weighted deprivations t1				Censored Weighted deprivations Optimal MaxN-LNOB= =Optimal LNOB-MaxN				P t0	P t1	P MN-LN= LN-MN	ci(k) t0	ci(k) t1	ci(k) MaxN-LNOB= LNOB-MaxN
		W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E						
1	4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	0.25	0.25	0	1	1	0	1	1	0
2	3	0.25	0.25	0.25	0.25	0	0	0	0	0	0.25	0.25	0	1	0	0	1	0	0
3	3	0.25	0.25	0.25	0.25	0	0	0	0	0	0.25	0.25	0	1	0	0	1	0	0
4	5	0.25	0.25	0.25	0	0	0	0	0	0	0.25	0.25	0	1	0	0	0.75	0	0
5	5	0	0.25	0.25	0.25	0	0.25	0.25	0.25	0	0	0	0	1	1	0	0.75	0.75	0
6	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Number of People	**40**	**15**	**20**	**20**	**15**	**4**	**9**	**9**	**9**	**0**	**15**	**15**	**0**	**20**	**9**	**0**			
CHs		**38%**	**50%**	**50%**	**38%**	**10%**	**23%**	**23%**	**23%**	**0%**	**37.5%**	**37.5%**	**0%**						

Table 5. Cost-effectiveness Matrix for Impact Effectiveness- Equal Weights – Union Criterion (k=0.25) – MaxN-LNOB vs. LNOB-MaxN

HH#	HH Size	ci(k) t0	Cost-Effectiveness Matrix Selected values under MaxN-LNOB				Cost-Effectiveness Matrix Selected values under LNOB-MaxN			
			CE_W	CE_S	CE_G	CE_E	CE_W	CE_S	CE_G	CE_E
1	4	1	31.3	10.9	27.8	62.5	31.3	10.9	27.8	62.5
2	3	1	23.4	8.2	20.8	46.9	23.4	8.2	20.8	46.9
3	3	1	23.4	8.2	20.8	46.9	23.4	8.2	20.8	46.9
4	5	0.75	39.1	13.6	34.7	0.0	39.1	13.6	34.7	0.0
5	5	0.75	0.0	13.6	34.7	78.1	0.0	13.6	34.7	78.1
6	5	0.5	0.0	13.6	0.0	78.1	0.0	13.6	0.0	78.1
7	2	0.5	15.6	5.4	0.0	0.0	15.6	5.4	0.0	0.0
8	7	0.25	0.0	19.0	0.0	0.0	0.0	19.0	0.0	0.0
9	3	0.25	0.0	0.0	20.8	0.0	0.0	0.0	20.8	0.0
10	3	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Note: Values of the CE matrix have been multiplied by 1,000,000 to facilitate the visualization.

Table 6. Cost-effectiveness Matrix for Impact Effectiveness- Equal Weights – Intermediate Criterion (k=0.75) MaxN-LNOB

HH #	HH Size	ci(k) t0	MaxN-LNOB ITERATION 1				MaxN-LNOB ITERATION 2				MaxN-LNOB ITERATION 3			
			W_A	W_S	W_G	W_E	W_A	W_S	W_G	W_E	W_A	W_S	W_G	W_E
1	4	1	31.3	10.9	27.8	62.5	31.3	10.9	27.8	62.5	93.8	24.5	83.3	0
2	3	1	23.4	8.2	20.8	46.9	23.4	8.2	20.8	46.9	70.3	24.5	62.5	0
3	3	1	23.4	8.2	20.8	46.9	23.4	8.2	20.8	46.9	70.3	24.5	62.5	0
4	5	0.75	117.2	40.8	104.2	0.0	0	**0**	0	0	0	0	0	0
5	5	0.75	0	40.8	104.2	234.4	0	**0**	0	0	0	0	0	0
6	5	0.5	0	0	0	0	0	0	0	0	0	0	0	0
7	2	0.5	0	0	0	0	0	0	0	0	0	0	0	0
8	7	0.25	0	0	0	0	0	0	0	0	0	0	0	0
9	3	0.25	0	0	0	0	0	0	0	0	0	0	0	0
10	3	0	0	0	0	0	0	0	0	0	0	0	0	0
			Cost=400+800=$1200				Cost=1200+3*400=$2400				Cost=2400+3*800=$4800			

Figure 5.1. Complementary Cumulative Distribution Functions for MaxN-LNOB and LNOB-MaxN optimal distributions computed under the uncensored distributions – Equal Weights

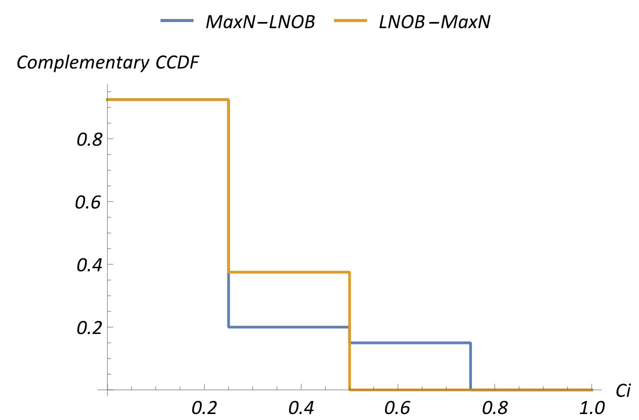

Figure 5.2. The Adjusted Headcount Ratio Dominance Curves for MaxN-LNOB and LNOB-MaxN optimal distributions computed under the uncensored distributions - Equal Weights

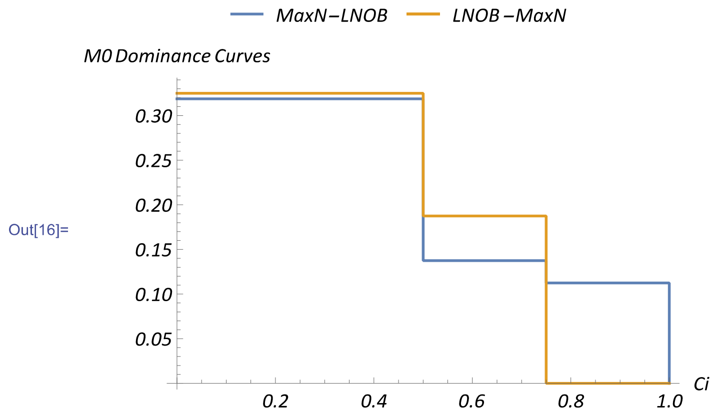

Table 7. Impact Effectiveness Indicators under different poverty cutoffs – Equal Weights- MaxN-LNOB vs. LNOB-MaxN

Distribution		Union criterion			Intermediate criterion		
		H (k=0.25)	A (k=0.25)	MPI (k=0.25)	H (k=0.75)	A (k=0.75)	MPI (k=0.75)
t_0	Poverty Measures	0.925	0.635	0.588	0.50	0.875	0.438
t_1	Poverty Measures	0.875	0.486	0.425	0.225	0.861	0.194
Observed poverty reduction		0.05	0.149	**0.163**	0.275	0.014	**0.244**
Observed spent budget		$6900			$6900		
Optimal MaxN-LNOB (over uncensored distribution)	Poverty Measures	0.925	0.345	0.319	0.15	0.75	0.112
	Poverty Reduction ($t_0 - Optimal$)	0	0.291	**0.269**	0.35	0.125	**0.326**
	Budget	**$6300**			**$6300**		
	IE^MPI	**0.163/0.269=61%**			**0.244/0.326=75%**		
Optimal LNOB-MaxN (over uncensored distribution)	Poverty Measures	0.925	0.351	0.325	0	0	0
	Poverty Reduction ($t_0 - Optimal$)	0	0.284	**0.263**	0.500	0.875	**0.438**
	Budget	**$6900**				**$6900**	
	IE^MPI	**0.163/0.263=62%**			**0.244/0.438=56%**		
Optimal MaxN-LNOB (over censored distribution)	Poverty Measures	NA			0	0	0
	Poverty Reduction ($t_0 - Optimal$)				0.500	0.875	**0.438**
	Budget				**$4800**		
	IE^MPI				**0.244/0.438=56%**		
Optimal LNOB-MaxN (over censored distribution)	Poverty Measures	NA			0	0	0
	Poverty Reduction ($t_0 - Optimal$)				0.500	0.875	**0.438**
	Budget				**$4800**		
	IE^MPI				**0.244/0.438=56%**		

NA: Non-applicable

5.1.2 Spending Effectiveness under different optimal criteria and different poverty cutoffs

Given the same initial and final deprivation matrices presented in Section 5.1.1, as well as the same weighting scheme and the same per household costs, we now want to know what would have been the minimum amount of fiscal effort B to achieve at least the same poverty reduction as the one observed. The matrices associated to this exercise are detailed in Table 8 for the case of the union poverty cutoff and in Table 9 for the case of a k=0.75.

With a union poverty cutoff, the poverty reduction target is the observed 0.163, which costed $6,900. The optimal distribution under the MaxN-LNOB criterion indicates that a slightly higher reduction, of 0.181, could have been achieved with a budget of $3600, if deprivations had been lifted in this way: remove deprivation in electricity (the most cost-effective dimension in this case) in five households in total, two of 5 members, one of 4 members and two of 3 members; remove deprivation in water (the second most cost-effective dimension) to one household of 5 members and one household of 4 members. As detailed in Table 10, with this benchmark, spending effectiveness is only 52%: the same and even higher MPI reduction could have been achieved with about half of the spent budget.[19] Naturally, in this example, this is because the budget in the optimal allocation is concentrated in the most cost-effective indicators, whereas in the observed one, some deprivations in sanitation -the most expensive dimension- were lifted. Note however, that if sanitation was a highly valued dimension, despite being the most expensive deprivation to lift, this could be accounted for in the weighting of the MPI indicators, and the 'cost-effectiveness' of removing this deprivation would change. This is exemplified with the case of unequal weights in the Appendix 2.

If we now consider the optimal allocation under the LNOB-MAxN criterion, we can see that the same poverty reduction of 0.169 is achieved with a budget of $4,000, higher than the one used with the MaxN-LNOB criterion. The distribution is as follows: the most cost-effective dimension (electricity) is lifted to the three poorest households, and then to household #5 with a deprivation score of 0.75 (the poorest after removing the first three deprivations). Next, deprivation in water is lifted to household#4 and household #1, with a score of 0.75 and of the biggest size among the poorest and deprived in water. Finally,

19 This is an example of a case in which the algorithm that operationalizes the MaxN-LNOB criterion in a reasonable computing time may produce a suboptimal allocation. In fact, poverty could be reduced in 0.169 if deprivation in electricity was lifted to households #1, #2, #4 and #5, and deprivation in water and gas was lifted to household #4, with a budget of $3300. This occurs because the algorithm proceeds one iteration at a time and decides in each iteration the optimal value. But as it approaches the end (i.e. as it is reaching the poverty reduction target in SE, and the budget in IE), better options may arise if the algorithm could evaluate two iterations forward, for example. Similar cases may arise for impact effectiveness and for the LNOB-MaxN criterion. However, as explained at the end of Section 4.3.5, the occurrence of suboptimal allocations is unlikely to have incidence in real data applications with big population sizes.

deprivation in water is lifted to household #2, also with a score of 0.75, and of the biggest size among the deprived in that dimension. Note that this poverty reduction is lower than the one achieved with the MaxN-LNOB criterion and yet more expensive. This is because LNOB-MaxN prioritizes reducing the deprivations in the households with the highest deprivation scores in the first place, and not necessarily the biggest ones. As the minimum budget under the LNOB-MaxN optimal distribution is higher than under the MaxN-LNOB one, spending effectiveness evaluated under the LNOB-MaxN criterion is also higher (58%) (Table 10).

As detailed in Table 9, the MaxN-LNOB and the LNOB-MaxN optimal distributions for spending effectiveness computed over the censored distribution with k=0.75 coincide in this case (but this need not always be the case for high poverty cutoffs). The poverty reduction target with this poverty cutoff is 0.244. The achieved reduction under the optimal distributions is 0.250, with a budget of $2400. In such case, spending effectiveness is evaluated to be of only 35% (see Table 10). That is, if focused on the poorest poor, an MPI reduction of 0.244 could have been achieved with only 35% of the spent budget.

Table 8. Spending Effectiveness – Equal Weights – Union Criterion (k=0.25) – MaxN-LNOB vs. LNOB-MaxN

HH#	HH Size	Weighted deprivations t0				Weighted deprivations t1				Weighted deprivations Optimal MaxN-LNOB				Weighted deprivations Optimal LNOB- MaxN				P t0	P t1	P MN-LN	P LN-MN	ci(k) t0	ci(k) t1	ci(k) MaxN-LNOB	ci(k) LNOB-MaxN
		W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E								
1	4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	0.25	0.25	0	0	0.25	0.25	0	1	1	1	1	1	1	0.75	0.5
2	3	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0.25	0.25	0.25	0	0	0.25	0.25	0	1	1	1	1	1	0.5	0.75	0.5
3	3	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0.25	0.25	0.25	0	0.25	0.25	0.25	0	1	1	1	1	1	0.5	1	0.75
4	5	0.25	0.25	0.25	0	0	0.25	0.25	0	0	0.25	0.25	0	0	0.25	0.25	0	1	1	1	1	0.75	0.5	0.25	0.5
5	5	0	0.25	0.25	0.25	0	0.25	0.25	0.25	0	0.25	0.25	0	0	0.25	0.25	0	1	1	1	1	0.75	0.75	0.5	0.5
6	5	0	0.25	0	0.25	0	0.25	0	0	0	0.25	0	0	0	0.25	0	0.25	1	1	1	1	0.5	0.25	0.25	0.5
7	2	0.25	0.25	0	0	0	0	0	0	0.25	0.25	0	0	0.25	0.25	0	0	1	0	1	1	0.5	0	0.5	0.5
8	7	0	0.25	0	0	0	0.25	0	0	0	0.25	0	0	0	0.25	0	0	1	1	1	1	0.25	0.25	0.25	0.25
9	3	0	0	0.25	0	0	0	0.25	0	0	0	0.25	0	0	0	0.25	0	1	1	1	1	0.25	0.25	0.25	0.25
10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Number of People	**40**	**17**	**34**	**23**	**20**	**10**	**32**	**17**	**9**	**12**	**34**	**18**	**3**	**8**	**34**	**20**	**5**	**37**	**35**	**37**	**37**				
Censored HRs		**43%**	**85%**	**58%**	**50%**	**25%**	**80%**	**43%**	**23%**	**30%**	**85%**	**45%**	**8%**	**20%**	**85%**	**50%**	**13%**								

Table 9. Spending Effectiveness – Equal Weights – Intermediate Criterion (k=0.75) – MaxN-LNOB vs. LNOB-MaxN

HH#	HH Size	Censored Weighted deprivations t0				Censored Weighted deprivations t1				Censored Weighted deprivations Optimal MaxN-LNOB= =Optimal LNOB-MaxN				P t0	P t1	P MN-LN= LN-MN	ci(k) t0	ci(k) t1	ci(k) MaxN-LNOB= LNOB-MaxN
		W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E						
1	4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	1	1	1	1	1	0.75
2	3	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0	1	0	1	1	0	0.75
3	3	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0	1	0	1	1	0	0.75
4	5	0.25	0.25	0.25	0	0	0	0	0	0	0.25	0.25	0	1	0	0	0.75	0	0
5	5	0	0.25	0.25	0.25	0	0.25	0.25	0.25	0	0.25	0.25	0	1	1	0	0.75	0.75	0
6	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Number of People	**40**	**15**	**20**	**20**	**15**	**4**	**9**	**9**	**9**	**10**	**20**	**20**	**0**	**20**	**9**	**10**			
Censored HRs		**38%**	**50%**	**50%**	**38%**	**10%**	**23%**	**23%**	**23%**	**25%**	**50%**	**50%**	**0%**						

Table 10. Spending Effectiveness Indicators under different poverty cutoffs – Equal Weights- MaxN-LNOB vs. LNOB-MaxN

		Union criterion			**Intermediate criterion**		
Distribution		**H (k=0.25)**	**A (k=0.25)**	**MPI (k=0.25)**	**H (k=0.75)**	**A (k=0.75)**	**MPI (k=0.75)**
t_0	Poverty Measures	0.925	0.635	0.588	0.50	0.875	0.438
t_1	Poverty Measures	0.875	0.486	0.425	0.225	0.861	0.194
Observed poverty reduction (TARGET)		0.05	0.149	**0.163**	0.275	0.014	**0.244**
Observed spent budget		***$6900***			***$6900***		
Optimal MaxN-LNOB (over uncensored distribution)	Poverty Measures	0.925	0.453	0.419	NA		
	Poverty Reduction $(t_0 - Optimal)$	0	0.182	**0.181**			
	Minimum Budget	**$3600**					
	SE^{MPI}	**3300/6900=52%**					
Optimal LNOB-MaxN (over uncensored distribution)	Poverty Measures	0.925	0.453	0.419	NA		
	Poverty Reduction $(t_0 - Optimal)$	0	0.182	**0.169**			
	Minimum Budget	***$4000***					
	SE^{MPI}	**4000/6900=58%**					
Optimal MaxN-LNOB (over censored distribution)	Poverty Measures	NA			0.250	0.750	0.188
	Poverty Reduction $(t_0 - Optimal)$				0.250	0.125	**0.250**
	Minimum Budget				**$2400**		
	SE^{MPI}				**2400/6900=35%**		
Optimal LNOB-MaxN (over censored distribution)	Poverty Measures	NA			0	0	0
	Poverty Reduction $(t_0 - Optimal)$				0.250	0.125	**0.250**
	Minimum Budget				**$2400**		
	SE^{MPI}				**2400/6900=35%**		

NA: Non-applicable.

5.2 Examples on the marginal contribution indicator with union and intermediate poverty cutoffs

As detailed in Section 4.2, the change in the censored headcount ratio of each j dimension can be interpreted as a marginal contribution indicator to poverty reduction. Continuing with the same example, Table 11 presents the censored headcount ratios with a union poverty cutoff $k = 0.25$, which coincide with the uncensored headcount ratios, and with an intermediate poverty cutoff of $k = 0.7$, at the initial and final moment. Neatly, the weighted sum of the change in the four censored headcount ratios equals the total change in M_0. Thus, the ratio of the weighted change in each censored headcount ratio to the total change in M_0 can be interpreted as the marginal contribution of dimension j to poverty reduction. In this example, with the union poverty cutoff we see that the dimension that contributed the most to multidimensional poverty reduction has been electricity (42.3%), followed by water (26.9%), gas (23.1%), and in fourth place, sanitation (7.7%).

The same analysis can be done if an intermediate poverty cutoff is used, such as $k = 0.75$. Note however, that in that case, electricity appears as the dimension contributing the least to poverty reduction, and the reduction in deprivation in water appears as contributing the same as the reduction in deprivations in sanitation and gas. However, as it can be seen from the exercise with the union poverty cutoff, i.e., the reductions of the uncensored headcount ratios, electricity and water were the ones with the biggest reductions. Moreover, comparing the uncensored initial and final distributions detailed in Table 4, one can see that the removal of deprivations in electricity and water led household #4, #2 and #3 to reduce their deprivation score below 0.75, that is, to being lifted out from multidimensional poverty. Because of this, their remaining deprivations are not counted any longer (i.e. are censored) and thus they do not 'appear' in the numerator of the other censored headcount ratios, magnifying the reduction in these other headcount ratios. For this reason, and especially considering that we aim at evaluating the effectiveness of the fiscal effort in reducing non-monetary deprivations, it seems recommendable to use a union criterion for the evaluation of the marginal contributions or, alternatively, to consider the changes in the censored headcount ratios alongside the changes in the uncensored headcount ratios.[20]

Table 11. Dimensional Marginal contributions – Observed and hypothetical ones- Equal Weights

	Poverty cutoff	M_0	CH_{water}	$CH_{sanitation}$	CH_{gas}	$CH_{electricity}$
t_0		0.588	0.43	0.85	0.58	0.50
t_1	k=0.25 (union)	0.425	0.25	0.80	0.43	0.23
$Change$		0.163	0.18	0.05	0.15	0.28
MC_j^{MPI}			26.9%	7.7%	23.1%	42.3%
t_0		0.438	0.38	0.50	0.50	0.38
t_1	k=0.75	0.194	0.10	0.23	0.23	0.23
$Change$	(intermediate)	0.244	0.28	0.28	0.28	0.15
MC_j^{MPI}			28.2%	28.2%	28.2%	15.4%
MaxN-LNOB		0.319	0.20	0.85	0.23	0
$Change$	k=0.25 (union)					
$(t_0 - MAxN - LNOB)$		0.269	0.22	0	0.35	0.50
MC_j^{MPI}			21.0%	0%	32.5%	46.4%
LNOB-MaxN		0.325	0	0.85	0.45	0
$Change$	k=0.25 (union)					
$(t_0 - LNOB - MaxN)$		0.263	0.43	0	0.13	0.50
MC_j^{MPI}			40.4%	0%	11.9%	47.5%

Note: Percent contributions may not add exactly to 100% due to rounding. CH: censored headcount ratio.

Interestingly, the observed dimensional marginal contributions can be compared to those that would have resulted if the optimal distributions had been achieved, which is exemplified for the case of the union approach in the last two blocks of rows of Table 11. For instance, both under the MaxN-LNOB and the LNOB-MaxN optimal distributions, the contribution of the reduction of deprivations in electricity would have been the greatest to overall poverty reduction (46.4% and 47.5% correspondingly), but in the MaxN-LNOB distribution this would have been followed by reducing deprivations in natural gas (contributing with 32.5% to poverty reduction), whereas in the LNOB-MaxN distribution this would have been followed by reductions in deprivations in water (contributing with 40.4% to poverty reduction).

20 See also Seth and Alkire (2015).

6. A brief note on reranking

Up to this point we have not addressed the fact that, as a result of the fiscal intervention, households may change their relative position in the distribution of deprivation scores, that is, there might be reranking. In the income space, "the definition of horizontal equity postulates that the pre-fiscal policy income ranking should be preserved (Duclos and Araar, 2006). In other words, if individual A was poorer than individual B before the fiscal interventions, individual A should continue to be poorer than individual B after the interventions." In fact, "…reranking is interpreted as a measure of fiscally induced horizontal inequality." (Lustig et al., 2018, p. 11-12).

Taken to the multidimensional context, this principle would imply that when constructing the optimal distributions -by any criterion- the original ranking of deprivation scores should be preserved. However, because of the indivisibilities detailed in Section 4.1.3, preserving the original ranking may not always be possible. The removal of deprivations produces indivisible reductions in deprivation scores that inevitably affect the ranking. This can be noticed in the examples of Section 5, comparing the columns of the censored deprivation scores. For instance, in the example of Table 3, households #2 and #3 have an initial deprivation score of 0.75, among the poorest, yet in the optimal LNOB-MaxN allocation, household #2 ends with a deprivation score of 0.25, in a better position than household #3, which ends with a deprivation score of 0.5. The example of unequal weights in Appendix 2 gives examples of even more dramatic re-rankings, simply by lifting one highly weighted deprivation. While it seems difficult to always comply with the horizontal equity principle in the multidimensional case, it must however be noted, that no one is made worse-off in absolute terms under any of the alternative optimal allocations.

7. Implementing the methodology with real data

The indicators proposed in this paper can be implemented with real data for two purposes. In the first place they can be implemented as an *ex-post* evaluation of the fiscal action. Second, they can be implemented *ex-ante*, as a government programmatic way to reduce multidimensional poverty. In this section we first explain how to deal with two technical issues when using real data, and then we detail the key pieces of information that are required for each kind of real-world implementations.

7.1 Dealing with population growth

In the proposed methodology we are considering the deprivation matrix in t_0 as the pre-fiscal matrix of the deprivation matrix in t_1. Until now we have assumed that the population of the deprivation matrices in t_0 and t_1 are the same, as well as the households' configurations, as if we had panel data. However, this methodology is intended to be implemented using repeated cross-sectional data. We have already detailed how to incorporate survey weights in the generation of the optimal distributions. Yet one further issue to consider is that, naturally, there will be population growth over time. Population growth affects the costing of poverty reduction. Reducing the MPI in "x" percent points is more costly if there is population growth than if there is not, as more households will have to be connected to services. Population growth can be easily incorporated through the survey weight variable.

To evaluate the fiscal effort done in reducing multidimensional poverty between t_0 and t_1, the relevant population size to consider is n_1, as that is the population over which MPI is computed in the final observation. One can compute the population growth between t_0 and t_1 and expand the survey weight variable in t_0, $p_{h_{t_0}}$, by the population growth between t_0 and t_1 ($n_g = (n_1 - n_0)/n_0$):

$$p'_{h_{t_0}} = p_{h_{t_0}}(1 + n_g)$$

By definition, $\sum_{h=1}^{T_0} s_h p'_{h_{t_0}} = n_1 = \sum_{h=1}^{T_1} s_h p_{h_{t_1}}$. The optimizing algorithms should then be implemented over the initial deprivation matrix $\boldsymbol{g}_{t_0}^0$, but using the expanded survey weights $p'_{h_{t_0}}$. By the replication invariance property, all the deprivation and poverty rates will remain unchanged using these expanded survey weights. In this way, the algorithm is implemented over the same deprivation and poverty metrics that a policy maker observes in t_0, but considering that the population will be that of t_1 by the time investments in removing deprivations are finalized.

There is, however, one limitation of proceeding in this way, which is that it assumes that all households, of different sizes, increase in the same proportion given by the population growth-rate. This may not always hold as households may tend to become smaller over time. That is, it is very likely that the number of households grows more than the population. Given that the costing of lifting deprivations is at the household level, the proposed procedure could overestimate the potential for poverty reduction that the observed budget could achieve if optimally allocated. At the same time, it is also worth noting that poor households, which in general tend to be bigger, may not register substantial reductions in their average size, at least in relatively short periods of time. In such case, applying the homogeneous population growth rate across households' sizes may not be that problematic in practice.

7.2 Data requirements for implementing the methodology as an ex-post evaluation of the fiscal action

If the methodology is implemented as an ex-post evaluation of the fiscal action one needs to define several issues simultaneously and interconnectedly: the indicators over which the fiscal action will be evaluated, the period for the evaluation, and check the availability of microdata for that period and the indicators that will be considered. The selection of indicators is not trivial. As argued earlier, the selected indicators need to be such that reductions in their deprivation rates can be reasonably attributed to the fiscal action.

For the effectiveness analysis two cross-section household survey data, at the initial and final point in time -the t_0 and t_1 moment, are required. For certain analysis, microdata from censuses could be convenient, as it covers all areas in a country and offers disaggregated level data, which household surveys do not. The drawback is that it is only collected every ten years.

Once the period, indicators and data sources have been decided, it is fundamental to have two additional pieces of information for the fiscal analysis: a) estimates of the cost of removing each deprivation under consideration, b) information on the public spending on those items over the period under study. While this kind of information should be available, it may be not so straightforward to find or obtain.

The provision of public services varies greatly across countries, from regulated private companies to public ones, with mixed ownership in between, and -very frequently- with different companies supplying different areas of a country. Thus, obtaining information on the cost of removing deprivation in services such as water, sewage sanitation, gas or electricity, may require investing some considerable time and resources. In most countries, there is a government department that collects that information, although it is not always readily available.[21] As mentioned in Section 4.3.1, most likely, there will be geographical variation of such costs.

The information on the public spending done in each area under analysis over the study period is relatively easier to obtain. Note, however, that for a proper assessment of effectiveness, this information should have the greatest level of disaggregation as possible, both in terms of the spending items (capital investments, operation or maintenance) as well as in terms of the geographical areas where this spending was allocated.

Finally, an important normative decision is the weighting scheme to implement in the MPI, as it directly determines the cost-effectiveness of removing each deprivation. Clearly, the weighting scheme needs to be properly justified. In any case, it is advisable that a robustness analysis is performed within a certain reasonable range of weights.

With this information, alongside the proposed procedure to deal with population growth, the optimal distributions under the alternative optimal criteria MaxN-LNOB and LNOB-MaxN can be computed, and

21 Despite data limitations and diverse technical complexities, Hutton and Varughese (2016) offer global costing estimates of extending water, sanitation and hygiene (WASH) services to meet the 6^{th} SDG.

thus the IE^{MP} and SE^{MP} indicators can be calculated. The dimensional marginal contributions to poverty reduction can also be computed and contrasted with those that would emerge from the optimal allocations.

7.3 Data requirements for implementing the methodology as an ex-ante poverty reduction government programme

Alternatively, a government may want to intervene in reducing multidimensional poverty and seek for the most cost-effective way to accomplish that. In such case, the optimal distributions under the MaxN-LNOB or the LNOB-MaxN criteria can be a guide. In particular, if the aim is to help the poorest poor, the LNOB-MaxN criterion should be implemented.

When implementing this analysis ex-ante, one needs microdata on the indicators on which the government plans to take action. In this case data at the starting point t_0 will serve both as a diagnosis of the initial state of deprivations, as well as the basis for planning the intervention. To cost and compute the optimal distribution under any of the two criteria, the government will have to consider the population projection for the target year, such that population growth can be factored in the budget as detailed in Section 7.1.

The information on the costing of the services on which the government wants to expand access will naturally also be needed, as in the ex-post case, as well as the intended budget for this poverty reduction programme. Note that depending on the results of the effectiveness analysis, it may happen that the relative allocation of funds across different ministries changes, which will require political negotiation. One more time, the weighting of the indicators will have to be transparently decided upfront.[22]

7.4 Assessing by geographical areas and decomposing by population subgroups

The proposed methodology assesses whether a certain public spending for expanding households' access in different dimensions was allocated in the best way across dimensions and households. Throughout the paper we have referred to the case of expanding access to public services as natural candidates for the multidimensional measure to consider. In consequence, to seek plausible optimal allocations, it is advisable that the proposed algorithms are implemented by areas within a country, which can be regions, provinces, or municipalities, depending on their extension. The prioritization of them within a country can follow different criteria, with the MPI value being an obvious strong candidate.

22 See the discussion in Barbieri and Higgins (2015) about the setting of the MPI's indicators weights from a political economy point of view.

Additionally, both for an ex-post or an ex-ante implementation of the methodology, decompositions across population subgroups, such as mono-parental-female headed households vs. biparental households, vs. households with no children can be incorporated, as the MPI is fully decomposable.

7.5 Considering a more comprehensive MPI

The assessment of impact and spending effectiveness on a few indicators, such as access to public services, on which there has been a fiscal effort to reduce deprivation can be implemented within a broader MPI, one that considers other key poverty dimensions. In fact, many countries now have a national MPI and may want to assess the fiscal effort done on a subgroup of indicators but keeping the complete national MPI as the metric (which may include nutrition, education or employment, for example). In such case, the methodology can be implemented with the following considerations. The observed change in the MPI, which is the denominator of the IE^{MP} indicator and the target for the SE^{MP} indicator, would only need to consider the change in the indicators that are under scrutiny, i.e. those reduced by the fiscal effort. However, note that both optimal criteria, MaxN-LNOB as well as LNOB-MaxN, would rank the households by their initial full MPI value.

8. Concluding remarks

In this paper we proposed analogue indicators of CEQ's fiscal incidence indicators for the case of multidimensional poverty under the AF measurement framework using the M_0 measure, with which a Multidimensional Poverty Index (MPI) can be defined. We have proposed an impact and a spending effectiveness indicator which can be implemented using cross-sectional household survey (or census) data at two points in time, alongside information on the cost of removing each deprivation at the household level, and information on the public spending the government has allocated or plans to allocate to the dimensions under analysis. We have also noted that changes in the censored headcount ratios (associated to the M_0 measure) expressed as a proportion of total change in poverty can be interpreted as an observed dimensional marginal contribution indicator, which in turn can be compared to the ones that emerge from the optimal allocations.

In the methodology presented here, poverty is identified at the household level and deprivations are also lifted at the household level, with per household costs associated to removing each considered deprivation. However, poverty is computed in population terms. This brings one tension: whether the optimal distribution to be considered for the impact and spending effectiveness indicators should prioritize reducing poverty to the biggest number, what we have named the MaxN-LNOB criterion, or rather to the poorest poor, what we have named the LNOB-MaxN criterion (LNOB for "Leave No One Behind"), a *prioritarianism* criterion. The first optimal criterion will produce a reduction of the MPI always equal or greater than the second criterion, but it may leave the poorest poor just as they were at the beginning. We consider that the LNOB-MaxN criterion truly embodies the 2030 Development Agenda as well as a more sensible ethical principle and should thus be preferred over the MaxN-LNOB one. We recommend implementing the LNOB-MaxN optimization algorithm over the uncensored distribution of deprivations and evaluating impact effectiveness using alternative poverty cutoffs, from highest to lowest, to elucidate which poverty-intensity groups have been privileged by the fiscal effort. Interestingly however, if poverty is identified at the individual level or if household sizes are ignored, the two criteria coincide.

Throughout the paper we have referred to the case of expanding access to public services as natural candidates for the multidimensional measure to consider. In such case, it is advisable that the proposed algorithms are implemented by areas within a country, which can be regions, provinces, or municipalities, depending on their extension. Their prioritization within a country can follow different criteria. While the MPI value is an obvious strong candidate, other options, such as the number of multidimensionally poor people may be justifiable.

The proposed indicators can be implemented *ex-post*, as an assessment of the effectiveness of certain areas of public spending over a certain period, but also *ex-ante*, to guide a poverty reduction programme. Decompositions across relevant population subgroups can be incorporated, as the MPI is fully decomposable. While the methodological requirement of information on the costing of removing deprivation in certain fundamental dimensions of wellbeing such as access to basic services can be challenging, it is not impossible, and the payoff of a more effective allocation of the fiscal budget for poverty reduction surely outweighs the difficulties of assembling such data. We hope this methodology can be useful for a better targeting of the policy aimed at reducing poverty in its many dimensions, contributing in this way to the achievement of the first SDG and related ones.

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Appendix 1. Distributional data to construct multidimensional poverty curves

Table A1.1. Distributional Data to construct multidimensional poverty dominance curves for MaxN-LNOB and LNOB-MaxN distributions. Example 1

	Frequency		Cumulative Frequency		Complementary Cumulative Frequency (1-Cumul Freq.)		Complementary Cumulative M_0	
ci	**MaxN-LNOB**	**LNOB-MaxN**	**MaxN-LNOB**	**LNOB-MaxN**	**MaxN-LNOB**	**LNOB-MaxN**	**MaxN-LNOB**	**LNOB-MaxN**
0	0.075	0.075	0.075	0.075	0.925	0.925	0.319	0.325
0.25	0.725	0.55	0.8	0.625	0.20	0.375	0.319	0.325
0.5	0.05	0.375	0.85	1	0.15	0	0.137	0.187
0.75	0.15	0	1	1	0	0	0.112	0
1	0	0	1	1	0	0	0	0

Example: Columns 4 and 5: In the MaxN-LNOB distribution there is one household of 3 members with a deprivation score of 0. That represents 3/40=0.075 of the population. Then, there are 6 households with a deprivation score of 0.25. These households add up to a total of 29 people, out of a total of 40, which is 29/40=0.725 of the population. Adding 0.075 with 0.725, we get 0.8, as detailed in second row, third column. And so on, with the other values. Columns 6 and 7 are the complementary of columns 4 and 5. Columns 8 and 9 are obtained computing the average of censored deprivation scores up to each ci value. In simpler terms, it is computing the M0 measure for the different k values. For example, for k=0.25, for distribution MaxN-LNOB, M_0=0*0.075+0.25*0.725+0.5*0.05+0.75*0.15+1*0=0.319. For k=0.5, for distribution MaxN-LNOB, M0=0.5*0.05+0.75*0.15+1*0=0.137, and so on for higher k values.

Appendix 2. Examples illustrating the methodology with unequal weights

In this Appendix we illustrate the methodology using unequal weights. We assume the same initial and final distributions and the same costs, but we now assume that each indicator has a different weight, which may emerge from participatory studies or normative reasons. In this case we suppose that overcoming deprivation in sanitation is highly valued, for example, because of its strong links to improvements in health outcomes.

Consider the per household costs of removing each deprivation and the weighting structure of the MPI indicators presented in Table A2.1. Now, sanitation, despite being the most expensive deprivation to remove, it is the most cost-effective because of its high weight in the MPI. We are assuming – on purpose – a very disbalanced weighting scheme with the aim of exemplifying the flexibility of the methodology. In real world exercises such imbalances should be very rare.

Table A2.1. Per Household Costs – Example with unequal weights

Dimension	phc *of each* j *dimension*	*Weighting in the* M_0 *of each* j *dimension*	*Cost-Effectiveness Ratios*	*Priorities*
Water	800	0.075	10,666.66	4
Sanitation	2300	0.75	3066.66	1
Gas	900	0.125	7200	2
Electricity	400	0.05	8000	3

Impact Effectiveness under different optimal criteria and different poverty cutoffs

In Table A2.2 we present deprivation matrix in t_0 and in t_1, with households ordered from poorest to richest, using a union poverty cutoff (k=0.05), and the optimal distributions that result from the MaxN-

LNOB criterion and the LNOB-MaxN criteria correspondingly. In Table A2.3 we present the same distributions but using an intermediate (high) poverty cutoff of k=0.80. Once again, zeroes in light blue denote censored deprivations. We are assuming that poverty was reduced lifting the same deprivations as in the example with equal weights, with a spent budget of $6,900. Deprivations that were removed appear as a red zeroes. For brevity of the tables, we are not including the columns on the poverty status of each household, but this can be easily inferred from the censored deprivation scores columns.

As detailed in the first rows of Table A2.4, from t_0 to t_1 there was a reduction in MPI using a union poverty cutoff and the unequal weighting scheme of 0.083. If an intermediate poverty cutoff of k=0.80 is used, poverty was reduced in 0.177. In both cases there were reductions of both H and A.

Now suppose one wants to assess the impact effectiveness of that fiscal effort. Implementing the algorithm under the MaxN-LNOB criterion, which maximizes the reduction in M_0 gives a distribution that allocates the $6,900 budget exclusively to the reduction of the most cost-effective deprivation -sanitation- to the three poor biggest households. While this deprivation was removed only to three households, because it is so highly valued, it reduces poverty in 0.319, much more than the observed reduction. Thus, impact effectiveness is only 26% (Table A2.4), indicating that, under this weighting scheme and optimal distribution, the spent budget achieved only 26% poverty reduction of what it could have achieved. However, note that in this optimal distribution the three most intensely deprived households, husehold #1, #2 and #3, which have a deprivation score of $c_{ht_0} = 1$, are not lifted any deprivation.

Under the LNOB-MaxN criterion, the budget is also completely spent in removing deprivation in sanitation to three households, but the selected households are the poorest ones, such that no household remains with a deprivation score of 1. As these households are not the biggest ones, MPI is reduced in 0.187, over twice the observed reduction but less than with the MaxN-LNOB criterion. In consequence, impact effectiveness is evaluated to be 44%. (Table A2.4). However, as in the equal weights case, given that the LNOB-MaxN criterion prioritizes the poorest poor, it is convenient to also evaluate effectiveness using an MPI with a higher deprivation cutoff. Indeed, when an MPI with k=0.80 is used, poverty reduction with the LNOB-MaxN criterion (implemented over the uncensored distribution) is 0.249, higher than the 0.234 reduction achieved with the MaxN-LNOB criterion also implemented over the uncensored distribution (see Table A2.4). Therefore, with this MPI, impact effectiveness is lower with the LNOB-MaxN benchmark (71%) than with the MaxN-LNOB criterion (75%).

As with the case of equal weights, FigureA2.1 depicts the Complementary Cumulative Distribution Functions for MaxN-LNOB and LNOB-MaxN and Figure A2.2 depicts the Adjusted Headcount Ratio dominance curves. Once again, the curves cross in both cases, indicating that while the MaxN-LNOB distribution dominates the LNOB-MaxN distribution, both in terms of H (first order dominance) as in terms of M_0 (second order dominance) for the first part of the possible k values, for high k values, the converse holds.

Let's now consider the two criteria being implemented over the censored distribution. It is interesting to note that with k=0.80, the deprivation in sanitation of household #8, of 7 people, is censored. Thus, in contrast with the uncensored case, this deprivation is not lifted in the MaxN-LNOB optimal distribution. Household #6, of 5 people, is chosen instead. With this distribution, the MPI is reduced in 0.334 (Table A2.4), almost twice the observed reduction of 0.177, such that impact effectiveness is only 53%. Yet, note that even using this high poverty cutoff, the poorest poor households are left unchanged. In fact, the MPI reduction is achieved through a significant reduction in H but a simultaneous (small) increase in A (as the number of the poor was reduced). In turn, the LNOB-MaxN optimal distribution, focused on lifting the poorest poor, is the same as with the uncensored distribution: it lifts deprivation in sanitation to households #1, #2 and #3. As these are not the biggest households, the MPI with k=0.8 is reduced in 0.249, higher than the observed 0.177 reduction but lower than the reduction achieved with the MaxN-LNOB criterion, and thus, impact effectiveness is higher (71%) (Table A2.4). However, if the MPI was computed over these two distributions using an even higher poverty cutoff, for example with k=0.81, it could be appreciated that the LNOB-MaxN distribution removed the most intense poverty, and effectiveness would be evaluated to be lower as compared to the MaxN-LNOB criterion.

As argued with the equal weights example, it is recommendable to implement the LNOB-MaxN optimization algorithm over the uncensored distribution of deprivations and evaluate impact effectiveness using alternative poverty cutoffs, from highest to lowest, for an assessment of which poverty-intensity groups have been privileged by the fiscal effort.

Table A2.2. Impact Effectiveness – Unequal Weights – Union Criterion (k=0.05) – MaxN-LNOB vs. LNOB-MaxN

HH#	HH Size	Weighted deprivations t0				Weighted deprivations t1				Weighted deprivations Optimal MaxN-LNOB				Weighted deprivations Optimal LNOB- MaxN				ci(k) t0	ci(k) t1	ci(k) MaxN-LNOB	ci(k) LNOB-MaxN
		W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E				
1	4	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	1	1	0.25
2	3	0.075	0.75	0.125	0.05	0.075	0.75	0	0	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	0.825	1	0.25
3	3	0.075	0.75	0.125	0.05	0.075	0.75	0	0	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	0.825	1	0.25
4	5	0.075	0.75	0.125	0	0	0.75	0.125	0	0.075	0	0.125	0	0.075	0.75	0.125	0	0.95	0.875	0.2	0.95
5	5	0	0.75	0.125	0.05	0	0.75	0.125	0.05	0	0	0.125	0.05	0	0.75	0.125	0.05	0.925	0.925	0.175	0.925
7	2	0.075	0.75	0	0	0	0	0	0	0.075	0.75	0	0	0.075	0.75	0	0	0.825	0	0.825	0.825
6	5	0	0.75	0	0.05	0	0.75	0	0	0	0.75	0	0.05	0	0.75	0	0.05	0.8	0.75	0.8	0.8
8	7	0	0.75	0	0	0	0.75	0	0	0	0	0	0	0	0.75	0	0	0.75	0.75	0	0.75
9	3	0	0	0.125	0	0	0	0.125	0	0	0	0.125	0	0	0	0.125	0	0.125	0.125	0.125	0.125
10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
No. of people	**40**	**17**	**34**	**23**	**20**	**10**	**32**	**17**	**9**	**17**	**17**	**23**	**20**	**17**	**24**	**23**	**20**				
Cens. HRs		**43%**	**85%**	**58%**	**50%**	**25%**	**80%**	**43%**	**23%**	**43%**	**43%**	**58%**	**50%**	**43%**	**60%**	**58%**	**60%**				

Table A2.3. Impact Effectiveness – Unequal Weights – Intermediate Criterion (k=0.80) – MaxN-LNOB vs. LNOB-MaxN

HH#	HH Size	Censored Weighted deprivations t0				Censored Weighted deprivations t1				Censored Weighted deprivations Optimal MaxN-LNOB				Censored Weighted deprivations Optimal LNOB-MaxN				ci(k) t0	ci(k) t1	ci(k) MaxN-LNOB	ci(k) LNOB-MaxN
		W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E				
1	4	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	1	1	0
2	3	0.075	0.75	0.125	0.05	0.075	0.75	0	0	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	0.825	1	0
3	3	0.075	0.75	0.125	0.05	0.075	0.75	0	0	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	0.825	1	0
4	5	0.075	0.75	0.125	0	0	0.75	0.125	0	0.075	0	0.125	0	0.075	0.75	0.125	0	0.95	0.875	0	0.95
5	5	0	0.75	0.125	0.05	0	0.75	0.125	0.05	0	0	0.125	0.05	0	0.75	0.125	0.05	0.925	0.925	0.175	0.925
7	2	0.075	0.75	0	0	0	0	0	0	0.075	0.75	0	0	0.075	0.75	0	0	0.825	0	0.825	0.825
6	5	0	0.75	0	0.05	0	0	0	0	0	0	0	0	0	0.75	0	0.05	0.8	0	0	0.8
8	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
No. of people	**40**	**17**	**34**	**33**	**20**	**10**	**20**	**14**	**9**	**17**	**12**	**20**	**20**	**17**	**17**	**20**	**20**				
Cen. HRs		**43%**	**85%**	**58%**	**50%**	**25%**	**50%**	**35%**	**23%**	**43%**	**30%**	**50%**	**50%**	**43%**	**43%**	**50%**	**50%**				

Figure A2.1. Complementary Cumulative Distribution Functions for MaxN-LNOB and LNOB-MaxN optimal distributions computed under the uncensored distributions – Unequal Weights

Out[12]=

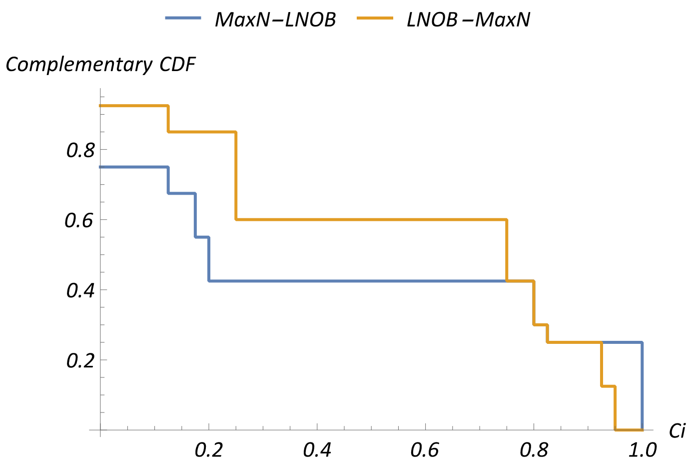

Figure A2.2. The Adjusted Headcount Ratio Dominance Curves for MaxN-LNOB and LNOB-MaxN optimal distributions computed under the uncensored distributions – Unequal Weights

Out[13]=

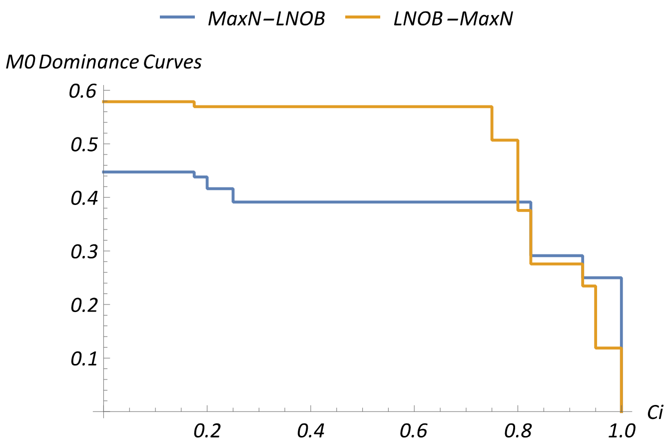

Table A2.4. Impact Effectiveness Indicators under different poverty cutoffs – Unequal Weights- MaxN-LNOB vs. LNOB-MaxN

		Union criterion			Intermediate criterion		
Distribution		**H (k=0.05)**	**A (k=0.05)**	**MPI (k=0.05)**	**H (k=0.80)**	**A (k=0.80)**	**MPI (k=0.80)**
t_0		0.925	0.828	0.766	0.675	0.927	0.626
t_1		0.875	0.781	0.683	0.5	0.898	0.449
Observed poverty reduction		**0.05**	**0.047**	**0.083**	**0.175**	**0.029**	**0.177**
Observed spent budget			$6900			$6900	
Optimal MaxN-LNOB (over uncensored distribution)	Poverty Measures	0.75	0.597	0.447	0.425	0.921	0.391
	Poverty Reduction $(t_0 - Optimal)$	0.175	0.231	0.319	0.25	0.006	0.234
	Budget		$6900			$6900	
	IE^{MPI}		**0.083/0.319=26%**			**0.177/0.234=75%**	
Optimal LNOB-MaxN (over uncensored distribution)	Poverty Measures	0.925	0.626	0.579	0.425	0.884	0.376
	Poverty Reduction $(t_0 - Optimal)$	0	0.202	0.187	0.25	0.043	0.249
	Budget		$6900			$6900	
	IE^{MPI}		**0.083/0.187=44%**			**0.177/0.249=71%**	
Optimal MaxN-LNOB (over censored distribution)	Poverty Measures				0.30	0.971	0.291
	Poverty Reduction $(t_0 - Optimal)$		NA		0.375	-0.044	0.334
	Budget					$6900	
	IE^{MPI}					**0.177/0.334=53%**	
Optimal LNOB-MaxN (over censored distribution)	Poverty Measures				0.425	0.884	0.376
	Poverty Reduction $(t_0 - Optimal)$		NA		0.25	0.043	0.249
	Budget					$6900	
	IE^{MPI}					**0.177/0.249=71%**	

NA: Non-applicable.

Spending Effectiveness under different optimal criteria and different poverty cutoffs

We now turn to the dual indicator of spending effectiveness for the unequal weights case. The matrices associated to this exercise are detailed in Table A2.5 for the case of the union poverty cutoff and in Table A2.6 for the case of a k=0.80.

With these unequal weights and a union poverty cutoff the poverty reduction target is the observed 0.083, which costed $6,900. The optimal distribution under the MaxN-LNOB criterion indicates that a higher reduction of 0.131 could have been achieved with a budget of $2300, by simply lifting deprivation in sanitation to the biggest poor household (household #8). Note that because of removing this deprivation, this household stops being poor (and this is also why A increases). Then, as detailed in Table A2.7, with this benchmark, spending effectiveness is only 33%: an MPI reduction even higher than the observed one could have been achieved with a third of the spent budget, had the budget been allocated to the most cost-effective dimension.

If we now consider the optimal allocation under the LNOB-MAxN criterion, we can see that the same poverty reduction of 0.131 requires twice the budget, $4600. This is because deprivation in sanitation is removed to the poorest households, which in this case are not the biggest ones, and thus two households need to be removed this deprivation to achieve the target poverty reduction. As the minimum budget under the LNOB-MaxN optimal distribution is twice the minimum budget under the MaxN-LNOB one, spending effectiveness evaluated under the LNOB-MaxN criterion is also twice spending effectiveness under MaxN-LNOB (66%) (Table A2.7).

If we now look for the optimal distributions computed over the censored distribution with k=0.80, the target poverty reduction is 0.177. We can see in Table A2.6, that the MaxN-LNOB distribution now requires lifting two deprivations in sanitation to achieve the target poverty reduction, because the deprivation in sanitation of household #10 is now censored. Thus, the minimum budget is $4600, and now spending effectiveness is 66%. In turn, with the LNOB-MaxN criterion, deprivation in sanitation is lifted to the two poorest households (#1 and #2), and to achieve the target poverty reduction, deprivation in electricity is lifted to household #3. This is an example in which because the remaining poverty reduction to achieve the target is very small, it is optimal to lift a less costly deprivation as electricity, rather than sanitation, as captured in expression (26) of the CE^{SE} coefficients. In such way poverty is reduced in 0.179, just above the required 0.177, with a budget of $5000. Thus, spending effectiveness in this case is 72%.

Table A2.5. Spending Effectiveness – Unequal Weights – Union Criterion (k=0.05) – MaxN-LNOB vs. LNOB-MaxN

HH#	HH Size	Weighted deprivations t0				Weighted deprivations t1				Weighted deprivations Optimal MaxN-LNOB				Weighted deprivations Optimal LNOB- MaxN				ci(k) t0	ci(k) t1	ci(k) MaxN-LNOB	ci(k) LNOB-MaxN
		W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E				
1	4	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	1	1	0
2	3	0.075	0.75	0.125	0.05	0.075	0.75	0	0	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	0.825	1	0
3	3	0.075	0.75	0.125	0.05	0.075	0.75	0	0	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0.05	1	0.825	1	1
4	5	0.075	0.75	0.125	0	0	0.75	0.125	0	0.075	0.75	0.125	0	0.075	0.75	0.125	0	0.95	0.875	0.95	0.95
5	5	0	0.75	0.125	0.05	0	0.75	0.125	0.05	0	0.75	0.125	0.05	0	0.75	0.125	0.05	0.925	0.925	0.925	0.925
7	2	0.075	0.75	0	0	0	0	0	0	0.075	0.75	0	0	0.075	0.75	0	0	0.825	0	0.825	0.825
6	5	0	0.75	0	0.05	0	0.75	0	0	0	0.75	0	0.05	0	0.75	0	0.05	0.8	0.75	0.8	0.8
8	7	0	0.75	0	0	0	0.75	0	0	0	0	0	0	0	0.75	0	0	0.75	0.75	0	0.75
9	3	0	0	0.125	0	0	0	0.125	0	0	0	0.125	0	0	0	0.125	0	0.125	0.125	0.125	0.125
10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
No. of people	**40**	**17**	**34**	**23**	**20**	**10**	**32**	**17**	**9**	**17**	**27**	**23**	**20**	**17**	**27**	**23**	**20**				
Cens. HRs		**43%**	**85%**	**58%**	**50%**	**25%**	**80%**	**43%**	**23%**	**43%**	**68%**	**58%**	**50%**	**43%**	**68%**	**58%**	**50%**				

Table A2.6. Spending Effectiveness – Unequal Weights – Intermediate Criterion (k=0.80) – MaxN-LNOB vs. LNOB-MaxN

HH#	HH Size	Censored Weighted deprivations t0				Censored Weighted deprivations t1				Censored Weighted deprivations Optimal MaxN-LNOB				Censored Weighted deprivations Optimal LNOB-MaxN(*)				ci(k) t0	ci(k) t1	ci(k) MaxN-LNOB	ci(k) LNOB-MaxN
		W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E	W_W	W_S	W_G	W_E				
1	4	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	1	1	0
2	3	0.075	0.75	0.125	0.05	0.075	0.75	0	0	0.075	0.75	0.125	0.05	0.075	0	0.125	0.05	1	0.825	1	0
3	3	0.075	0.75	0.125	0.05	0.075	0.75	0	0	0.075	0.75	0.125	0.05	0.075	0.75	0.125	0	1	0.825	1	0.95
4	5	0.075	0.75	0.125	0	0	0.75	0.125	0	0.075	0	0	0	0.075	0.75	0.125	0	0.95	0.875	0	0.95
5	5	0	0.75	0.125	0.05	0	0.75	0.125	0.05	0	0	0	0	0	0.75	0.125	0.05	0.925	0.925	0	0.925
7	5	0.075	0.75	0	0	0	0	0	0	0.075	0.75	0	0	0.075	0.75	0	0	0.825	0	0.825	0.825
6	2	0	0.75	0	0.05	0	0	0	0	0	0.75	0	0.05	0	0.75	0	0.05	0.8	0	0.8	0.8
8	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
No of people	**40**	**17**	**34**	**33**	**20**	**10**	**20**	**14**	**9**	**17**	**17**	**20**	**20**	**17**	**20**	**20**	**20**				
Cen. HRs		**43%**	**85%**	**58%**	**50%**	**25%**	**50%**	**35%**	**23%**	**43%**	**43%**	**50%**	**50%**	**43%**	**50%**	**50%**	**50%**				

Table A2.7. Spending Effectiveness Indicators under different poverty cutoffs – Unequal Weights-MaxN-LNOB vs. LNOB-MaxN

Distribution		**Union criterion**			**Intermediate criterion**		
		H (k=0.05)	**A (k=0.20)**	**MPI (k=0.05)**	**H (k=0.80)**	**A (k=0.80)**	**MPI (k=0.80)**
t_0	Poverty Measures	0.925	0.828	0.766	0.675	0.927	0.626
t_1	Poverty Measures	0.875	0.781	0.683	0.5	0.898	0.449
Observed poverty reduction (TARGET)		0.05	0.048	**0.083**	0.175	0.029	**0.177**
Observed spent budget			**$6900**			**$6900**	
Optimal MaxN-LNOB (over entire distribution)	Poverty Measures	0.75	0.847	0.635		NA	
	Poverty Reduction ($t_0 - Optimal$)	0.175	-0.019	0.131			
	Minimum Budget		***$2300***				
	SE^{MPI}		**2300/6900=33%**				
Optimal LNOB-MaxN (over entire distribution)	Poverty Measures	0.925	0.686	0.635		NA	
	Poverty Reduction ($t_0 - Optimal$)	0	0.142	0.131			
	Minimum Budget		***$4600***				
	SE^{MPI}		**4160/6900=66%**				
Optimal MaxN-LNOB (over censored distribution)	Poverty Measures		NA		0.425	0.921	0.391
	Poverty Reduction ($t_0 - Optimal$)				0.25	0.006	0.235
	Minimum Budget					***$4600***	
	SE^{MPI}					**4600/6900=66%**	
Optimal LNOB-MaxN (over censored distribution)	Poverty Measures		NA		0.5	0.894	0.447
	Poverty Reduction ($t_0 - Optimal$)				0.175	0.033	**0.179**
	Minimum Budget					***$5000***	
	SE^{MPI}					**5000/6900=72%**	

NA: Non-applicable.