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UNIVERSITY OF  
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# HDCA Summer School on Capability and Multidimensional Poverty

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**Delft University of Technology, Netherlands**

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*Tabita, Kenya*



*Rabiya, India*



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*Agathe, Madagascar*



*Dalmo, Kenya*



*Ann-Saphia, Kenya*



*Valérie, Madagascar*



# **Factor analysis uses for index design and subjective scale validation**

**AF Measure Analysis Issues IV: redundancy, correlation, complementarity, subjective scales validation**

**José Manuel Roche**

# Main Goals

- Discuss the advantages and disadvantages of factor analysis to deal with redundancy, design synthetic indicators, select dimensions and indicators, and setting weights
- Review how these techniques are use for validation of subjective scales.

# Outline

- Brief introduction to main uses of Factor Analysis
- Differences between exploratory and confirmatory factor analysis
- Steps and recommendations to run an exploratory factor analysis (EFA)
- Advantages and disadvantages of EFA
- Brief overview of subjective scales validation

# Factor Analysis

These statistical techniques are appropriate when dealing with large amounts of data, as they have a high power of data reduction and facilitate the design of aggregated variables. They analyse the interrelations among a large list of indicators in order to understand their underlying structure, making it possible to reduce it to a small number of aggregated variables.

## **Wealth index** (*Rustein and Johnston 2004*)

A composite measure of the cumulative living standard of a household currently used in the DHS and MICS

### ***How is it measured?***

- Based on a set of assets and services assessed in the surveys (e.g. Type of flooring, Refrigerator, Water supply, Type of vehicle, Sanitation facilities, Persons per sleeping room, Electricity, Ownership of agricultural land, Radio, Domestic servant, Television, Telephone)
- Each household asset and service for which information is collected is assigned a weight or factor score generated through principal components analysis.
- The first component of a PCA is interpreted as a continuous scale of relative wealth. The standardized scores are then used to create the break points that define wealth quintiles as: Lowest, Second, Middle, Fourth, and Highest.
- The Wealth Index is used as a background characteristic when analysing health status, or child rights.

## *Not only an ad-hod solution to aggregate information!*

### *Other possible uses:*

- Gives information to assess the underlying structure of the data (e.g. explore the pattern of the dataset or the dimensions)
- Avoid redundancy (e.g. reduce a large number of correlated variables, aggregate them or select one that represents some of them)
- To validate and evaluate subjective scales (e.g. convergence, differentiate, internal consistence)
- To measure non observable variables or theoretical concepts (e.g. provides the measurement error, goodness of fit)
- To include in the complex models (e.g. Regression analysis or structural multiple equation models)

# Examples...

## Factor Analysis vs. Fuzzy Sets Theory

(Lelli 2008)

Social Interaction  
(Factor 2)

Economic conditions  
(Factor 3)

Health  
(Factor 6)

Psychological distress  
(Factor 1)

Shelter  
(Factor 7)

Cultural life  
(Factor 4)

Working conditions  
(Factor 5)

Table 4.1 - Rotated factor pattern

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7
Friends	-5	15	-18	5	-1	4	-12
Sport match	-2	45*	7	0	1	-5	-5
Café	1	57*	2	13	0	-5	4
Restaurant	0	34*	24	37*	1	-9	-6
Disco	5	54*	-10	3	-1	-8	3
Games	3	52*	0	0	1	-7	4
Going out	3	54*	8	21	2	-11	7
Sport practice	-3	38*	13	13	1	-8	-5
Meet ends	-13	5	73*	11	4	-5	-13
Savings	-3	12	61*	9	2	-10	1
Perceived sit.	-6	10	46*	11	8	-8	0
Fin. difficulties	9	0	-30*	-7	-1	0	13
Non affordab.	8	1	-46*	-10	-1	2	24
Economic satisf.	-19	-1	61*	10	20	-2	-23
Health status	-24	26	15	9	8	-61*	-1
Chronic illness	10	-12	-7	-2	-1	54*	-1
Recent illness	10	-4	-3	-2	-3	47*	2
Hospital	7	0	0	-5	2	45*	-1
Generalist	8	-12	-9	-9	1	57*	-7
Specialist	15	-2	-1	2	0	47*	5
Other physician	7	-2	2	11	0	10	1
Psychol. problems	24	-5	-4	0	0	29	3
Depressed	71*	-8	-18	-2	-9	19	4
No appetite	44*	6	-6	-5	-5	22	8
Insomnia	49*	-10	-8	1	-6	22	4
Unrested	63*	-3	-7	1	-10	20	9
Nervous	42*	4	-2	-5	-1	3	5
Feel guilty	70*	9	-2	7	-5	-1	9
No concentration	67*	2	2	4	-9	6	8
Weeping	57*	-4	-9	1	2	12	-2
Pessimistic	71*	-4	-9	1	-9	8	3
Irritable	63*	6	0	3	-5	-1	9
Need reassurance	70*	-7	-3	7	-2	0	3
Out of sorts	75*	-2	-6	0	-7	18	5
Housing satisf.	-17	-7	19	2	16	5	-53*
Area	13	-2	-15	4	-4	4	46*
Dwelling	11	3	-13	0	-3	2	55*
Heating	4	5	20	17	1	-11	-21
Crowding	-7	-13	-4	16	-3	12	-16
Theatre	2	13	6	63*	3	-6	-2
Cinema	12	45*	-2	34*	-3	-17	6
Concert	1	26	8	60*	3	-5	1
Museum	1	5	10	69*	0	-5	1
Conference	-2	1	8	57*	-1	-6	-2
Creative activity	3	6	4	28	3	0	0
Association	-6	22	12	27	1	1	-6
Work certitude	-6	1	9	1	43*	2	-1
Work type	-10	0	2	3	65*	-2	-3
Number of hours	-7	4	1	-2	61*	3	1
Work schedule	-5	4	3	6	65*	3	3
Working environm.	-9	4	1	2	65*	-2	-3
Work distance	-2	-1	0	1	36*	-2	-5
Job search	-4	16	-1	2	-14	2	2
Overqualified	3	14	-7	0	-23	4	6

- Belgian Section of the European Community Household Panel
- 54 indicators classified into 7 categories
- The FA confirms the underlying structure
- The first 7 factors are retained for further analyses

# Psychometric validity and reliability test

Type of Evidence	Fundamental Questions	Type of Analysis
<b>Reliability</b>		
Internal consistency	Do the indicators in the scale produce similar scores?	<b>Coefficient Alpha Cronbach</b>
Test-retest	Does the scale produce similar scores under similar conditions?	Multiple administration
<b>Validity</b>		
Face	Does the scale appear to measure what it claims to measure?	Scale Developer “expert” assessment
Content	Does item content reflect the construct definition? Do the respondents understand the questions/terms in the same way?	Assessment by a pool of experts <b>Cognitive interview, Focus Group</b>
Factorial	Does the scale measure the right number of constructs?  Defensible constructs discovered? → (Early development)  Theorised constructs confirmed? → (Hypothesis testing)  Patterns comparable across relevant groups? →	<b>Exploratory Factor Analysis (EFA)</b>  <b>Confirmatory Factor Analysis (CFA)</b>  SEM with covariate DIF (Item invariance)
Construct (Convergent and Discriminant)	Do variables that should correlate with scale score do so? Do variables that should not correlate with the scale score not do so?	<b>Correlation, ANOVA, t-test</b>
Concurrent Criterion (known-groups or known-instruments)	Do scale scores adequately categorise respondents with known characteristics? Do categorisations based on new scale scores adequately match those based on previously standardised measures?	<b>Correlation, ANOVA, t-test, external validity</b>
Predictive	Do scale scores accurately predict future behaviours or attitudes of respondents?	Correlation, ANOVA, t-test, external validity

# Exploratory factor analysis

(Abell et al. 2009, Brown 2006)

A typical function of a factor model with one factor:

$$x_{ij} = \lambda_i \xi_j + \delta_{ij}$$

Where  $x_{ij}$  is the standardized score of the  $i^{\text{th}}$  item for the person  $j^{\text{th}}$  ;  
 $\xi_j$  is the latent variable of the person  $j^{\text{th}}$  with mean = 0 and variance = 1;  $\lambda_i$  is the factor contribution of the person  $i$ ;  $\delta_{ij}$  is the remaining portion non explained by the model or measurement error.

A typical function for the factor analysis made up from three models:

$$x_{ij} = \lambda_{1i} \xi_{1j} + \lambda_{2i} \xi_{2j} + \lambda_{3i} \xi_{3j} + \delta_{ij}$$

The generalized function would be:

$$x_{ij} = \lambda_{1i} \xi_{1j} + \lambda_{2i} \xi_{2j} + \dots \lambda_{di} \xi_{dj} + \delta_{ij}$$

where  $x_{ij}$  is the standardized score of the  $i^{\text{th}}$  item for the person  $j^{\text{th}}$  ;  $\xi_{dj}$  is the latent variable for the person  $j^{\text{th}}$  in the factor  $d$  which normally has mean = 0 and variance = 1;  $\lambda_{id}$  is the factor contribution of the item  $i$  en el factor  $d$ ; and  $\delta_{ij}$  is the residual portion not explained by the model.

## **Measure of deprivation** (Klasen 2000)

Comparing a standard expenditure-based poverty measure with a specifically created composite measure of deprivation using the household survey data from South Africa.

### **Variables and weights according to the PCA**

Expenditure quintile	0.36
Fuel	0.35
Sanitation	0.34
Durable goods	0.34
Water	0.33
Education	0.28
Safety	0.01
Stunting	0.15
Satisfaction	0.16
Transport	0.20

### **Advantage**

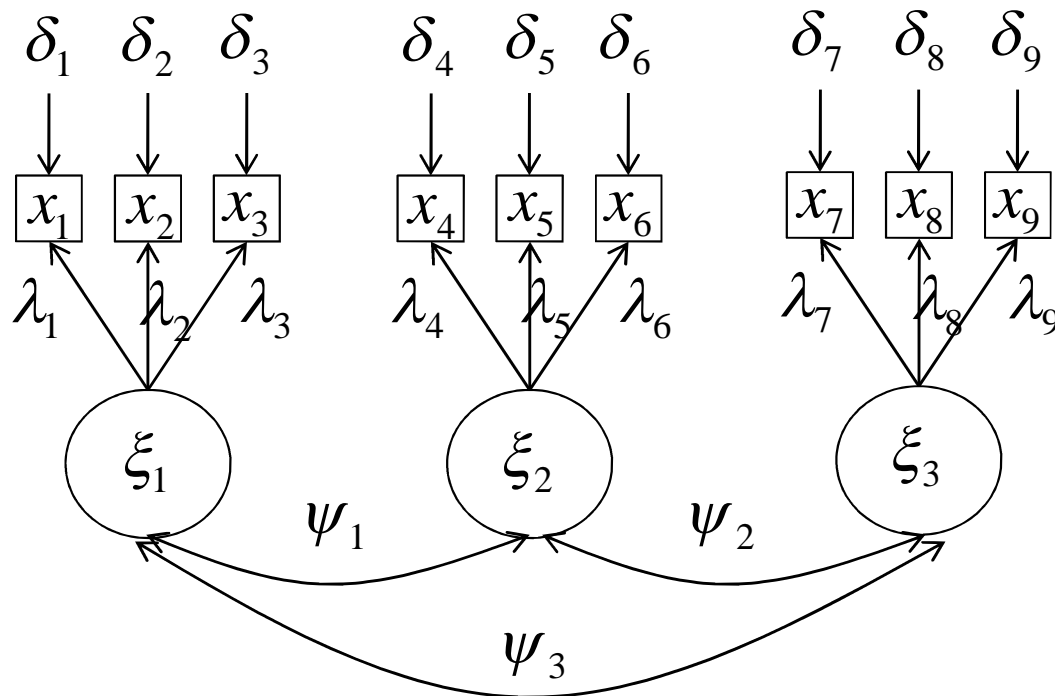
*'It uncovers empirically the commonalities between the individual components and bases the weights of these on the strength of the empirical relation between the deprivation measure and the individual capabilities' p39*

### **Disadvantage**

*'The disadvantage of such an approach is that it implicitly assumes that only components with strong correlations with each other are relevant for the deprivation measure which may be debatable in some cases' p39*

# Confirmatory factor analysis

(Abell et al. 2009, Brown 2006)



Name	Parameter	Type	Description
Lambda-Y	$\lambda_x$	Regression	Factor Loading
Delta	$\delta$	Variance-Covariance	Error variance and covariance
Psi	$\psi$	Variance – Covariance	Factor variance and covariance
Xi (Ksi)	$\xi$	Factor	Endogenous variable
$\mathcal{X}$ represents the item or exogenous (observed) variable			

Items 1-3:

$$x_{ij} = \lambda_{1i}\xi_{1j} + 0\xi_{2j} + 0\xi_{3j} + \delta_{ij}$$

$$x_{ij} = \lambda_{1i}\xi_{1j} + \delta_{ij}$$

Items 4-6:

$$x_{ij} = 0\xi_{1j} + \lambda_{2i}\xi_{2j} + 0\xi_{3j} + \delta_{ij}$$

$$x_{ij} = \lambda_{2i}\xi_{2j} + \delta_{ij}$$

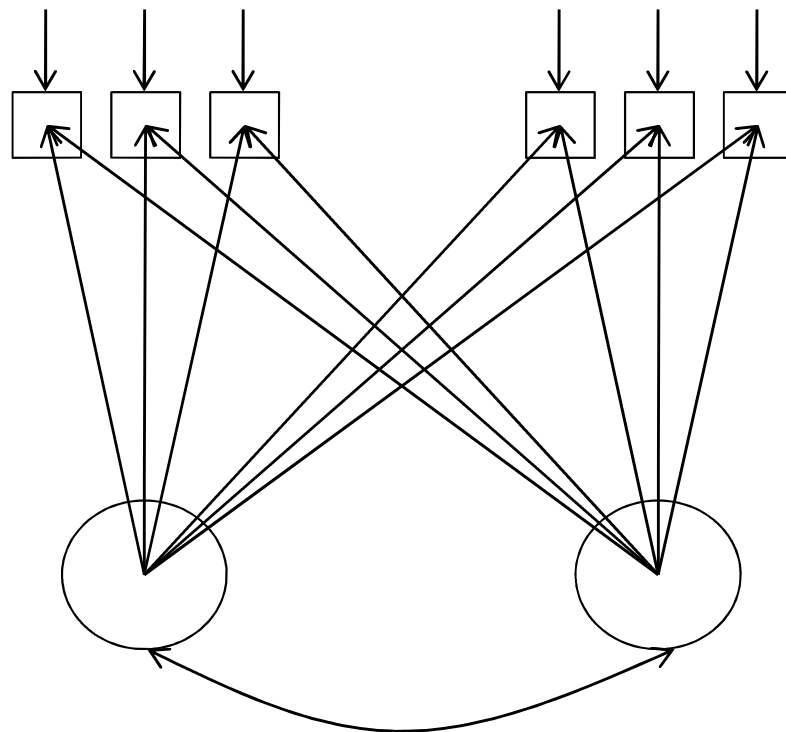
Items 7-9:

$$x_{ij} = 0\xi_{1j} + 0\xi_{2j} + \lambda_{3i}\xi_{3j} + \delta_{ij}$$

$$x_{ij} = \lambda_{3i}\xi_{3j} + \delta_{ij}$$

# Path diagram for the EFA?

*(Two factors with oblique rotation)*



# The most commonly used indices of goodness of fit

*(Abell et al. 2009, Bryne 2010, Brown 2006)*

## Chi-Square $\chi^2$

The most commonly used goodness of fit measure. Assesses the statistical significance of the difference across the variance - covariance matrix observed and estimated. Low values indicate well goodness of fit. For large samples the null hypothesis tends to be rejected.

## Root mean square residual (RMR)

It is also an absolute measure of goodness of fit. It reflects the difference between the observed and estimated covariance. It can be more reliable than the chi-square and behaves better with large samples. Takes values from zero to one, where 0.0 indicates perfect goodness of fit. A value of 0.05 or less indicates well goodness of fit.

## Root mean square error of approximation (RMSEA)

This index is made from a penalty function with low parsimonious of the model when takes into account the number of estimated parameters. A value of 0.05 or less suggests a reasonable goodness of fit.

## Comparative Fit Index (CIF)

Evaluates the goodness of fit from the model against the independence of the model. Set the covariance of the indicators as zero. A value less than 0.95 means excellent fit.

## Tucker-Lewis index (TLI)

Evaluates the value of the chi-square on the degrees of freedom of the proposed model for the same amount of the null hypothesis. A value less than 0.90 indicates an acceptable goodness of fit.

# The step of Exploratory Factor Analysis: procedures and recommendations

*(Ver: Brown 2006 The Common Factor Model and EFA)*

1. Select the indicators and choose the unit of analysis
2. Choose an extraction model and calculate initial factor loadings
3. Determine the appropriate number of factors
4. In multifactorial model, rotate the solution to obtain simple structure model
5. Interpret the factors and evaluate the quality of the solution
6. Re-run and (ideally) replicate the factor analysis

# Monitoring Inequality between social groups

(Roche 2008)

*Examples...*

**Focus:** Housing conditions  
(the capability of 'being well sheltered')

**Context:** Venezuela

**Data:** Household Survey (2001)  
Census ('71, '81, '90, 2001)



## Selected Indicators

Sewage system

Water

Electricity

Fuel

Floors

Roofs

Walls

Housing Overcrowding Index

## 2. Choice of the extraction method

- **Principal factor (pf):** The contributions (factor loading) are computed using the squared multiple correlations as estimates of the communality. It is one of the methods more used and is preferable when we want to avoid multivariable normality assumption.
- **Principal-component factor (pcf):** similar to principal component analysis where the communalities are assumed to be 1. It strictly does not correspond to a factorial analysis.
- **Iterated principal-factor (ipf):** This reestimates the communalities iteratively.
- **Maximum-likelihood factor (ml):** Allows statistical test to determine the goodness of fit of the factor analysis in terms of reproducing of the correlation of the original indicators. Assumes multivariable normality.

### 3. Determine the appropriate number of factors

✓ **Kaiser Criterion (Guttman, 1954):**

Factors with eigenvalue of 1.0 or higher. The rationale is that one factor should not explain less than the equivalent of any of the given variables included in the analysis.

Factor analysis/correlation  
Method: principal factors  
Rotation: (unrotated)

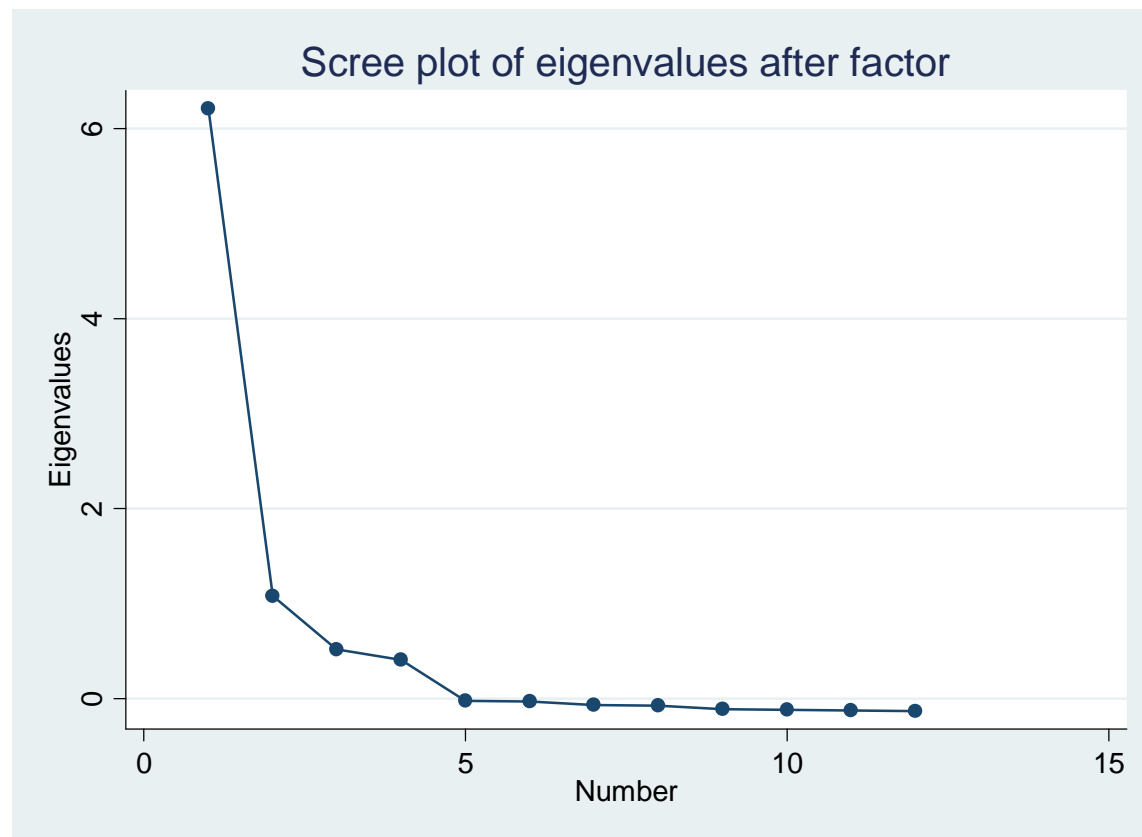
Number of obs = 1235  
Retained factors = 4  
Number of params = 74

Factor	Eigenvalue	Difference	Proportion	Cumulative
Factor1	6.40885	2.92466	0.4859	0.4859
Factor2	3.48419	1.36797	0.2642	0.7501
Factor3	2.11622	1.04221	0.1604	0.9105
Factor4	1.07401	0.54659	0.0814	0.9920
Factor5	0.52741	0.11662	0.0400	1.0319
Factor6	0.41079	0.25353	0.0311	1.0631
Factor7	0.15726	0.14523	0.0119	1.0750
Factor8	0.01202	0.02494	0.0009	1.0759
Factor9	-0.01292	0.01580	-0.0010	1.0749
Factor10	-0.02873	0.00545	-0.0022	1.0728
Factor11	-0.03418	0.01343	-0.0026	1.0702
Factor12	-0.04761	0.01739	-0.0036	1.0666
Factor13	-0.06500	0.01028	-0.0049	1.0616
Factor14	-0.07528	0.01756	-0.0057	1.0559
Factor15	-0.09283	0.02063	-0.0070	1.0489
Factor16	-0.11347	0.00738	-0.0086	1.0403
Factor17	-0.12085	0.00862	-0.0092	1.0311
Factor18	-0.12947	0.00287	-0.0098	1.0213
Factor19	-0.13234	0.01635	-0.0100	1.0113
Factor20	-0.14868	.	-0.0113	1.0000

LR test: independent vs. saturated:  $\chi^2(190) = 2.0e+04$  Prob> $\chi^2 = 0.0000$

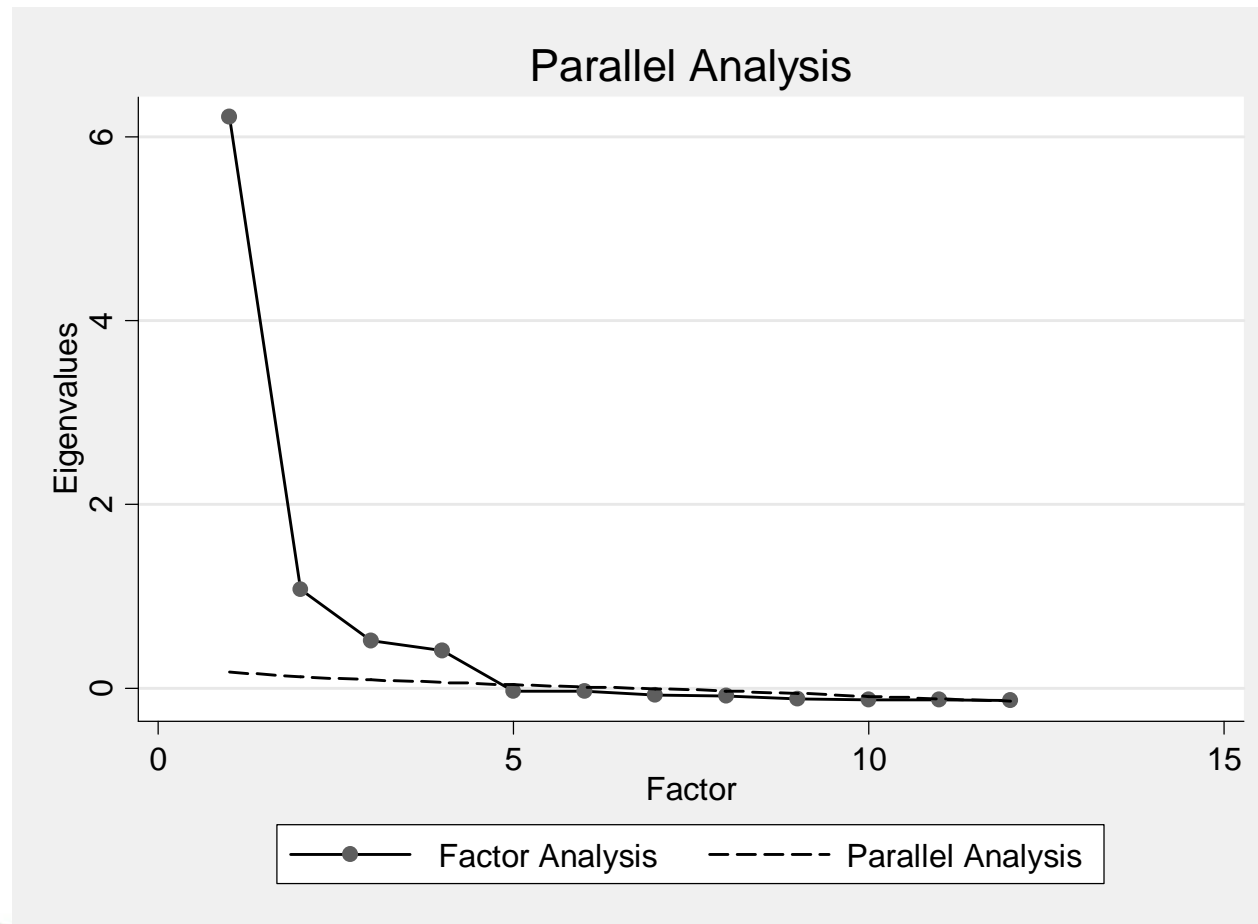
### 3. Determine the appropriate number of factors

✓ ***Analysis of the Scree Plot (Cattell, 1966):*** it identifies the inflexion point of the scree plot with the aim to select a small number of factors with eigenvalues significantly higher than the remaining one.



### 3. Determine the appropriate number of factors

✓ **Parallel Analysis (Horn, 1965):** the factor to extract should account for more variance than the expected random variance



### 3. Determine the appropriate number of factors

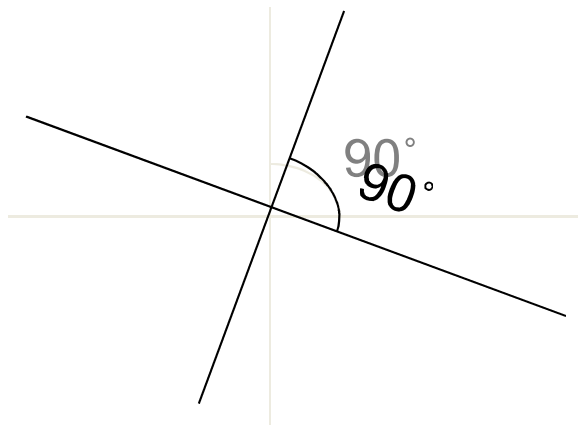
✓ ***Normative judgement:*** In practice, different methods can lead to conflicting conclusions so it is important to also consider the theoretical judgment of the analyst. Occasionally, previous theory might indicate the number of relevant factors to extract. The analyst might be interested in assessing if the variables converge in the factor they are expected to, and have a relatively low loading factors in factors associated to other constructs – this is the procedure that is followed in scale validation. In other occasions the analysis might be more interested in exploring the data, so will experiment with different extraction solutions based on the previous methods and will determine if the number of extracted variables is theoretically consistent.

## 4. Rotate the solution to obtain simple structure model

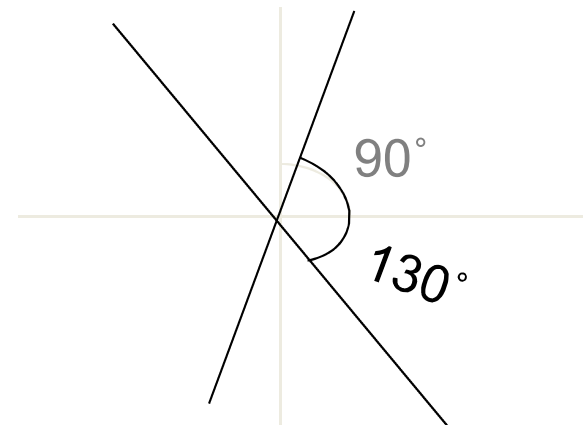
The Factors are orthogonal solutions which implies independence (no correlation).

The factors can be rotated in order to help interpretation. This is roughly to spread the variability among the factors.

As a result we increase the factor loading of some indicators in some factors, while decrease in others. THE TOTAL VARIANCE DESCRIBED BY THE FACTORS REMAIN UNCHANGED



Orthogonal Rotation  
(e.g. Varimax)



Oblique Rotation  
(promax or oblimin)

The decision is normally based on theory (should the dimensions be correlated?)

There is not a unique solution!

# Monitoring Inequality between social groups

(Roche, 2008)

*Example...*

## Unrotated, Varimax-rotated common components matrix

	Unrotated Component			VARIMAX-rotated Component		
	1	2	3	1	2	3
<b>Sewage</b>	0.734	0.120	-0.010	0.518	0.418	0.331
<b>Water</b>	0.565	0.435	0.144	0.695	0.100	0.190
<b>Electricity</b>	0.420	0.529	0.138	0.687	-0.014	0.061
<b>Fuel used for cooking</b>	0.401	0.495	-0.088	0.620	0.147	-0.087
<b>Floors</b>	0.752	-0.208	-0.310	0.226	0.752	0.297
<b>Roofs</b>	0.597	-0.312	-0.595	0.018	0.897	0.070
<b>Walls</b>	0.692	-0.228	0.345	0.258	0.250	0.721
<b>Housing Overcrowding Index</b>	0.495	-0.513	0.513	-0.064	0.101	0.870

Extraction Method: Principal Component Analysis. 3 components extracted.

VARIMAX: Rotation converged in 4 iterations.

Oblimin: Rotation converged in 9 iterations.

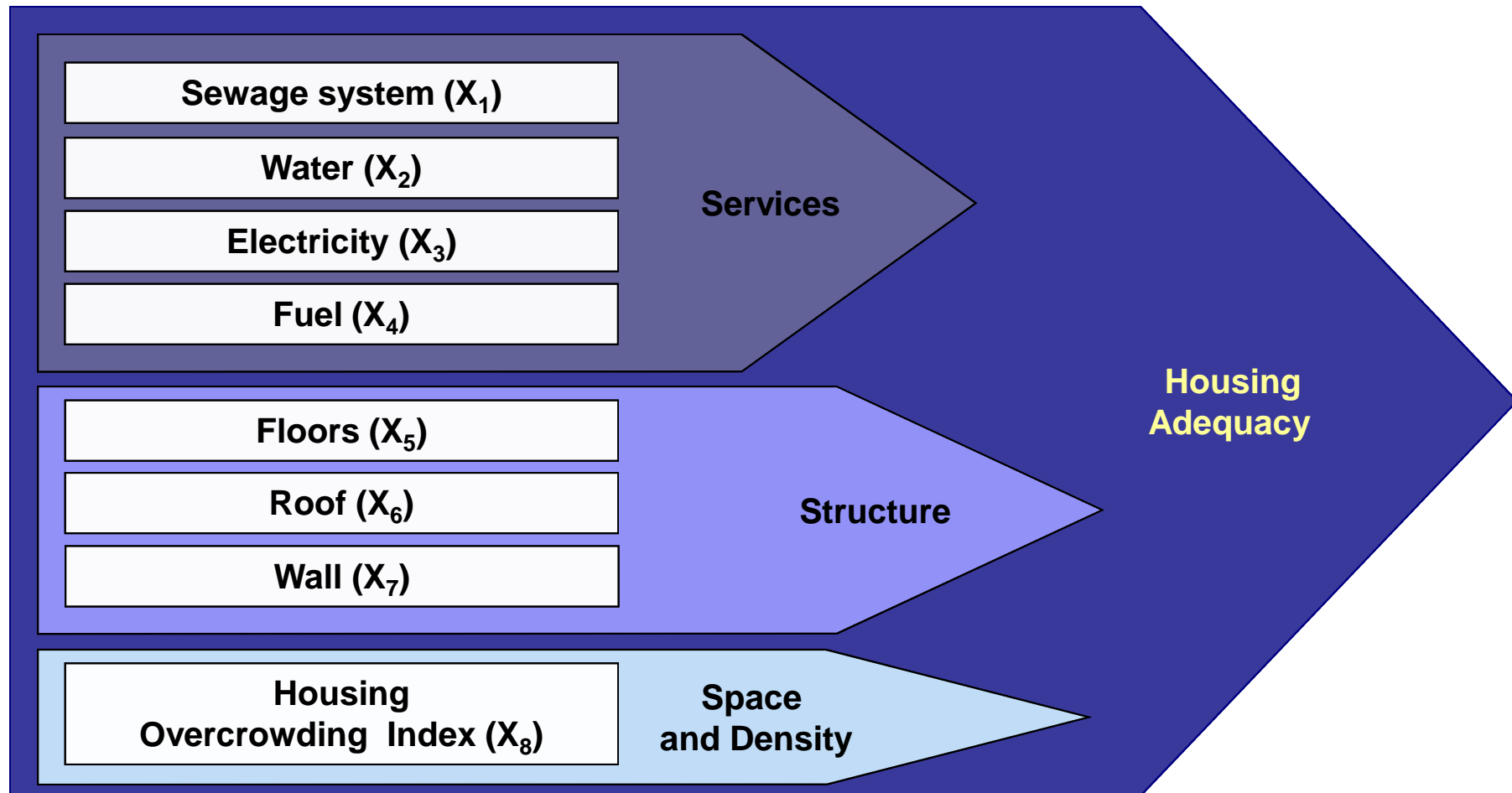
## 5. Interpretation and evaluation of the quality of the solution

- Consider the meaningfulness and interpretability of the solution
- Eliminate poorly defined factors.
- Eliminate poorly defined items (indicators) (items with higher loading in one or more factors, or with small loading in one factor)
- The process can be iterative, running new tests until reaching a satisfactory solution.
- If the purpose is to reach a theoretical conclusion, the analysis should be replicated using different datasets and performing a Confirmatory Factor Analysis.

# Monitoring Inequality between social groups

(Roche 2008)

*Examples...*

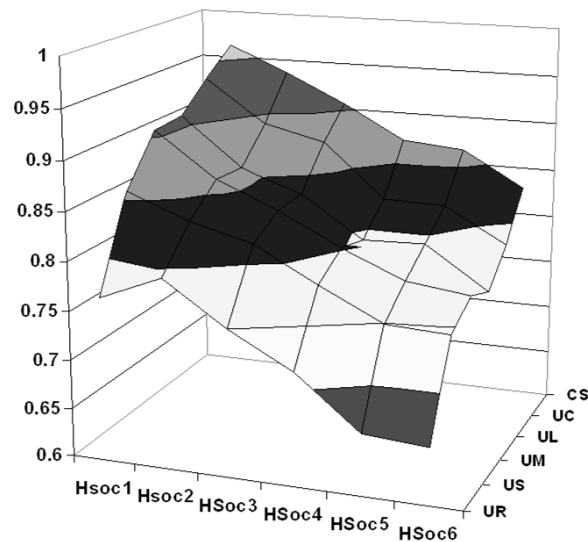


$$HAI = 1/3(X_1 + X_2 + X_3 + X_4) + 1/3(X_5 + X_6 + X_7) + 1/3(X_8)$$

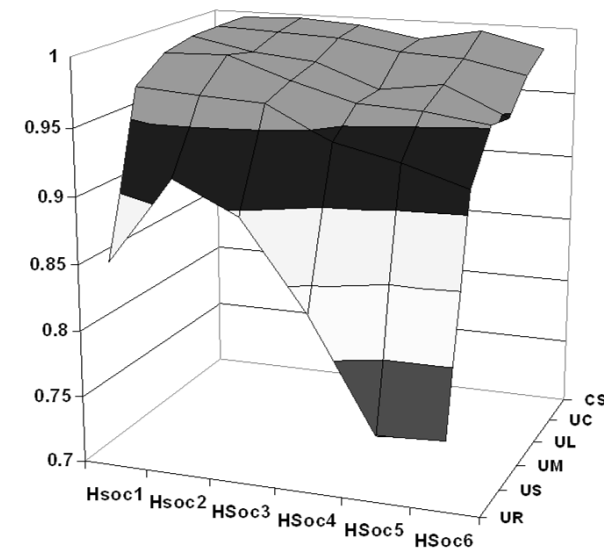
Perhaps an analysis on housing adequacy should **observe these different levels**, and not just focus on an overall housing adequacy.

# Capabilities and Groups Inequalities (Roche, 2009)

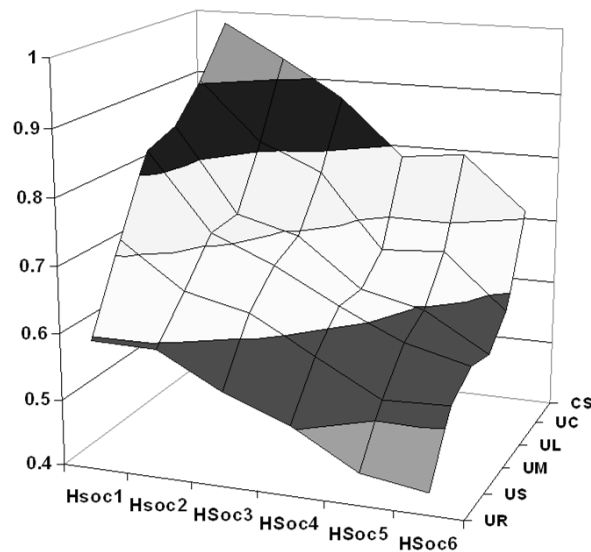
*Example...*



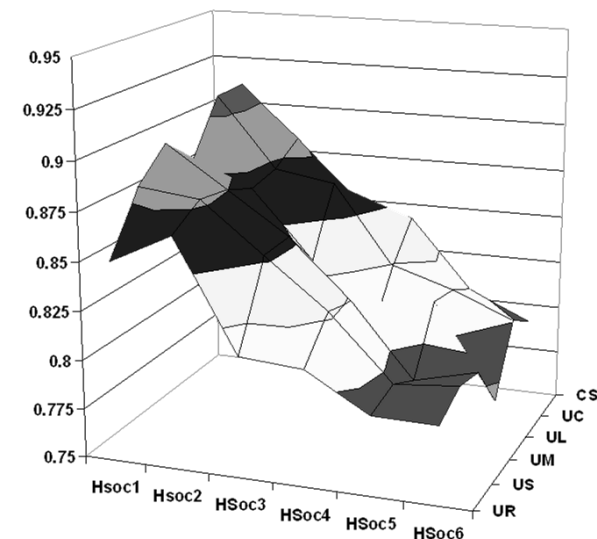
Overall housing adequacy



Housing Services



Housing structure



Space and density

# Capabilities and Groups Inequalities

(Roche 2009)

*Example...*

## Adj. R-Squared for different models

	Overall Adequacy (HAI)	Services (HSI)	Structure (HTI)	Space and Density (HDI)
<b>Model 1:</b> Income and constant only $Y = c + \lambda_1 X_1 + e$	15.1%	4.8%	15.1%	6.2%
<b>Model 2:</b> Income, demographic factors and constant $Y = c + \lambda_1 X_1 + \beta_3 Z_3 + e$	20.4%	8.5%	16.5%	19.9%
<b>Model 3:</b> Income, Hsoc, demographic factors and constant $Y = c + \lambda_1 X_1 + \beta_1 Z_1 + \beta_3 Z_3 + e$	25.0%	10.0%	21.7%	21.2%
<b>Model 4:</b> Income, Hsoc, ZXT, ZXR, demographic factors and constant $Y = c + \lambda_1 X_1 + \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + e$	32.1%	28.8%	28.6%	21.8%
<b>Model 5:</b> Income, Hsoc, ZXT, ZXR, other occupational variables (EcoAct, SecInf, SecPub), demographic factors and constant $Y = c + \lambda_1 X_1 + \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + e$	34.0%	33.6%	29.8%	22.2%

# Another example: MPI Ven

(Gallo & Roche 2011)

Option 1  
(5 dimensions)

Dimensions and Indicators		Weights
<b>Housing</b>		<b>1/5</b>
Overcrowding		1/10
Housing conditions (wall, floor, roof)		1/10
<b>Services</b>		<b>1/5</b>
Drinking water		1/15
Sanitation (tiolet)		1/15
Garbage Collection		1/15
<b>Living standards</b>		<b>1/5</b>
<u>Assests</u>		<u>1/5</u>
Electric or gas cooking fuel		
Laundry machine		
Fridge		
T.V.		
Air Conditionaire		
Boiler		
Tumble Dryer		
Car		
<b>Education</b>		<b>1/5</b>
School attendance		1/10
Years of schooling (9 years)		1/10
<b>Laboral</b>		<b>1/5</b>
Occupation		1/10
Minimum income		1/10

Option 2  
(4 dimensions)

Dimensions and Indicators		Weights
<b>Habitad and housing</b>		<b>1/4</b>
<u>Housing</u>		<u>1/8</u>
Overcrowding		1/16
Housing conditions (wall, floor, roof)		1/16
<u>Services</u>		<u>1/8</u>
Drinking water		1/16
Sanitation (tiolet)		1/16
Garbage Collection		1/16
<b>Living standards</b>		<b>1/4</b>
<u>Assests</u>		<u>1/4</u>
Electric or gas cooking fuel		
Laundry machine		
Fridge		
T.V.		
Air Conditionaire		
Boiler		
Tumble Dryer		
Car		
<b>Education</b>		<b>1/4</b>
School attendance		1/8
Years of schooling (9 years)		1/8
<b>Laboral</b>		<b>1/4</b>
Occupation		1/8
Minimum income		1/8

There is an implicit weight in how we cluster the indicators by dimension!!!

<b>Housing</b>		<b>1/6</b>
Overcrowding		1/12
Housing conditions (wall, floor, roof)		1/12
<b>Services</b>		<b>1/6</b>
Drinking water		1/24
Sanitation (tiolet)		1/24
Garbage Collection		1/24
Electric or gas cooking fuel		1/24
<b>Living standards</b>		<b>1/3</b>
<u>Assests</u>		<u>1/9</u>
Laundry machine		
Fridge		
T.V.		
Air Conditionaire		
Boiler		
Tumble Dryer		
Car		
<b>Occupation</b>		<b>1/9</b>
<b>Minimum income</b>		<b>1/9</b>
<b>Education</b>		<b>1/3</b>
School attendance		1/6
Years of schooling (9 years)		1/6

# Tetrachoric correlations

(Gallo & Roche 2011)

	overcrow	housing	water	toilet	garbage	fuel	attendance	years of sch	occupatio	assets	dependen	income
overcrow	1.0000											
housing	0.6943	1.0000										
water	0.3479	0.5969	1.0000									
toilet	0.5003	0.7422	0.7112	1.0000								
garbage	0.2528	0.4503	0.5615	0.5645	1.0000							
fuel	0.2309	0.4851	0.4792	0.5726	0.3984	1.0000						
attendance	0.4220	0.3387	0.2957	0.3730	0.2277	0.3001	1.0000					
years of sch	0.3360	0.5239	0.4736	0.5813	0.3804	0.4375	0.4072	1.0000				
occupatio	0.0979	0.1408	0.1397	0.1675	0.0674	0.1442	0.1091	0.2543	1.0000			
assets	0.4073	0.5652	0.4700	0.6282	0.3402	0.4698	0.3424	0.5477	0.1977	1.0000		
dependenc	0.4218	0.1935	0.1084	0.1764	0.0844	0.0392	0.1718	0.1552	0.1496	0.1779	1.0000	
income	0.3877	0.3542	0.3058	0.4110	0.2365	0.2790	0.3076	0.4232	0.3159	0.4440	0.5804	1.0000

Correlations are sufficiently high to look for underlying variables

# Factor Analysis Results

## (Gallo & Roche 2011)

All 12 indicators

Variable	Factor1	Factor2	Factor3	Uniqueness
overcrow	0.037	0.047	0.759	0.353
housing	0.510	-0.045	0.543	0.227
water	0.776	-0.005	0.020	0.388
toilet	0.738	0.099	0.160	0.204
garbage	0.697	-0.060	-0.030	0.577
fuel	0.564	0.191	-0.083	0.576
attendance	0.076	0.304	0.244	0.720
years of sch	0.334	0.483	-0.005	0.473
occupatio	-0.077	0.496	-0.112	0.830
assets	0.340	0.407	0.099	0.474
income	-0.006	0.581	0.100	0.597

Excluding education (to assess changes)

Variable	Factor1	Factor2	Factor3	Uniqueness
overcrow	0.0114	0.7302	0.0652	0.4052
housing	0.4337	0.6022	-0.0333	0.213
water	0.7707	0.0486	-0.0245	0.3856
toilet	0.7212	0.2028	0.0815	0.2012
garbage	0.6987	-0.0074	-0.0886	0.5747
fuel	0.5903	-0.058	0.1434	0.581
occupatio	-0.043	-0.0912	0.4747	0.8256
assets	0.3553	0.1462	0.3536	0.4875
income	0.0353	0.1142	0.5458	0.6006

Only housing and services

Variable	Factor1	Factor2	Uniqueness
overcrow	-0.0125	0.7563	0.4381
housing	0.3464	0.6552	0.2054
water	0.7376	0.0863	0.3797
toilet	0.6946	0.2857	0.2215
garbage	0.6568	-0.0045	0.5719
fuel	0.6063	0.0359	0.6076

There seems to be two dimensions of housing, and one dimension on living standards with education

There is enough reasons to separate education – we still get occupation with assets and income

Housing alone still distinguishes two dimensions: services and housing structure/space

# Another example: MPI Venezuela (Gallo & Roche 2011)

The decision on clustering the dimensions and setting weights is still normatively driven but the analysis helps to support the decision

## Option 3 (3 dimensions)

Dimensions and Indicators		Weights
<b>Habitad and housing</b>		<b>1/3</b>
	<b><u>Housing</u></b>	<b><u>1/6</u></b>
	Overcrowding	1/12
	Housing conditions (wall, floor, roof)	1/12
	<b><u>Services</u></b>	<b><u>1/6</u></b>
	Drinking water	1/24
	Sanitation (tiolet)	1/24
	Garbage Collection	1/24
	Electric or gas cooking fuel	1/24
<b>Living standards</b>		<b>1/3</b>
	<b><u>Assests</u></b>	<b><u>1/9</u></b>
	Laundry machine	
	Fridge	
	T.V.	
	Air Conditionaire	
	Boiler	
	Tumble Dryer	
	Car	
	<b><u>Occupation</u></b>	<b><u>1/9</u></b>
	<b><u>Minimum income</u></b>	<b><u>1/9</u></b>
<b>Education</b>		<b>1/3</b>
	School attendance	1/6
	Years of schooling (9 years)	1/6

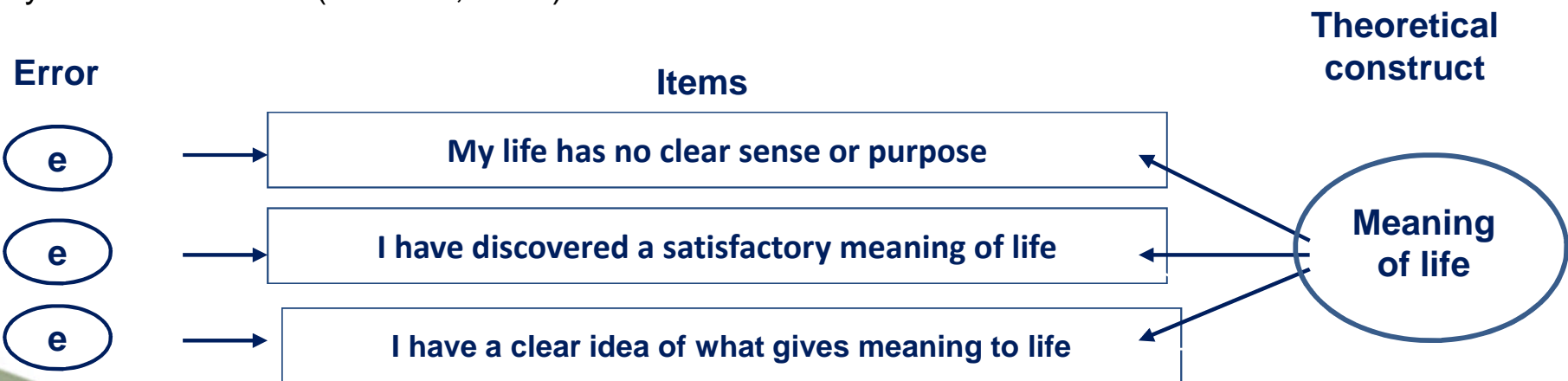
# Brief overview of subjective scales validation

# Psychometric evaluation of subjective scales

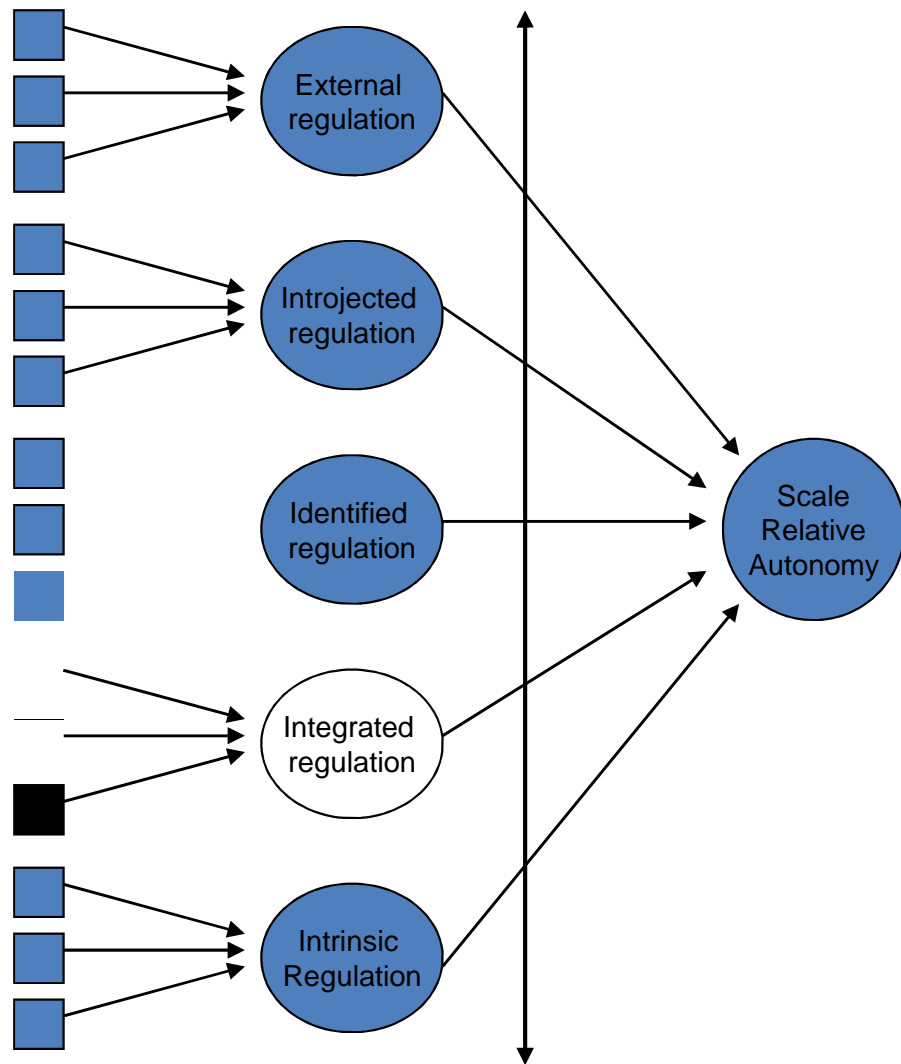
Psychometric scales attempt to measure a theoretical construct (i.e. meaning of life, perceived autonomy) using multiple items. Multi-item scales are generally more reliable than single-item scales. The underlying measurement theory indicates that items contain a “true” component and some “noise” (measurement error). Multiple items make it possible to reduce the error measurement and to identify consistency among items – in occasions, items might be capturing something else than what the analyst is interested in (Treiman, 2009).

**Scale dimensions  
(Abell et al., 2009)**

		Dimensions	
		Uni	Multi
Items	Uni	Simplest	Worst
	Multi	Most common	Most complex



**Eg. Gagne et al., 2009: The Motivation at Work Scale**



**Continuum**

External:

1. Because this job affords me a certain standard of living
2. Because it allows me to make a lot of money
3. I do this job for the paycheck

Introjected:

1. Because I have to be the best in my job, I have to be a “winner”
2. Because my work is my life and I don’t want to fail
3. Because my reputation depends on it

Identified:

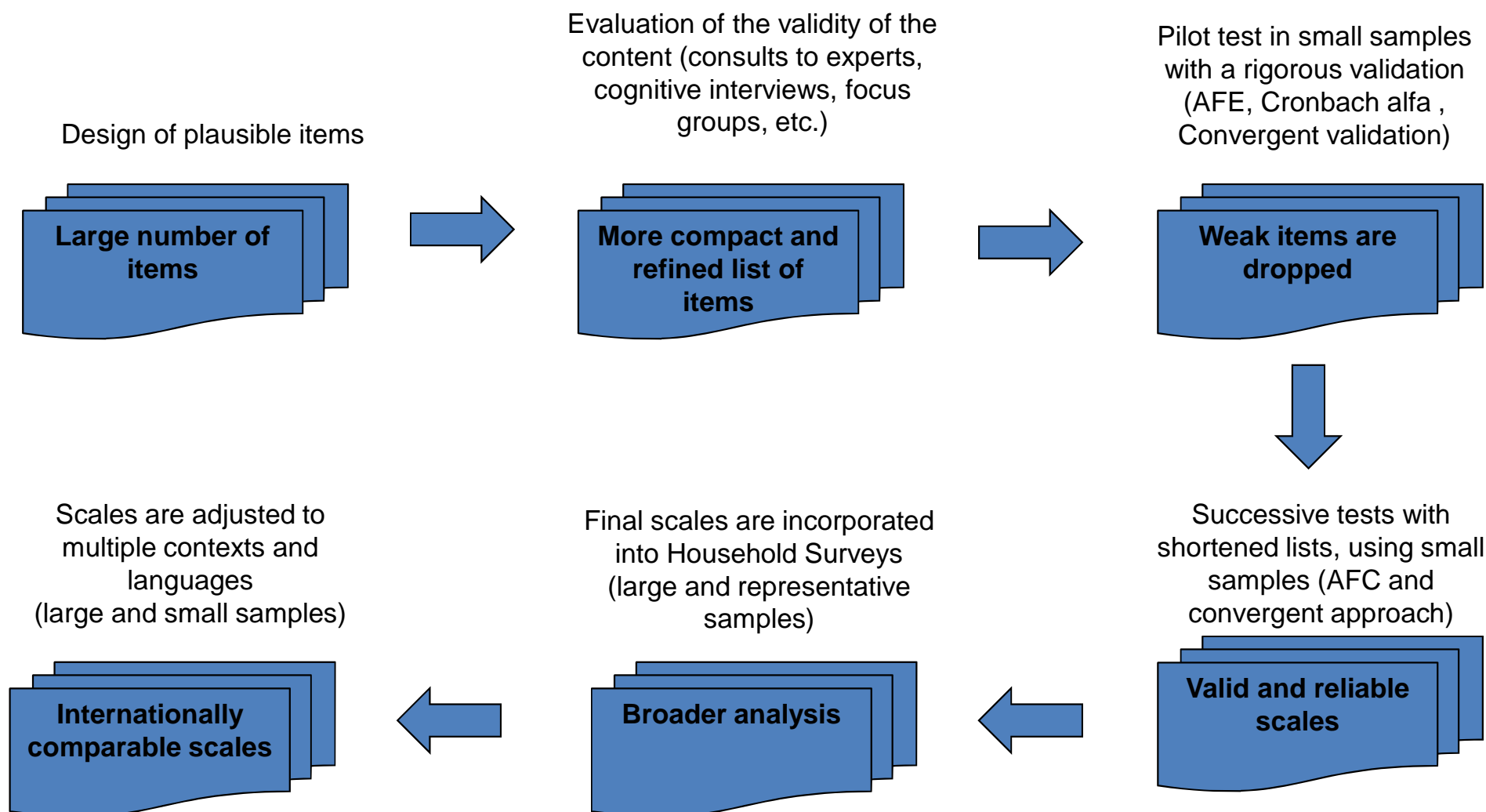
1. I chose this job because it allows me to reach my life goals
2. Because this job fulfills my career plans
3. Because this job fits my personal values

Intrinsic

1. Because I enjoy this work very much
2. Because I have fun doing my job
3. For the moments of pleasure that this job brings me

The stem is “Using the scale below, please indicate for each of the following statements to what degree they presently correspond to one of the reasons for which you are doing this specific job” and is accompanied by the scale 1= not at all; 2= very little; 3 = a little; 4 = moderately; 5 = strongly; 6 = very strongly; 7= exactly.

# Typical process to develop subjective scales



# Psychometric evaluation of subjective scales

Type of evidence	Main questions	Type of analysis
<b>Pruebas de Confiabilidad</b>		
Internal consistency	Do scale indicators measure similar levels?	Alfa Cronbach coefficient
Successive measurements	Does the scale produce similar measures under equivalent conditions?	Multiple administration
<b>Pruebas de Validez</b>		
Apearance	Does the scale seem to be measuring what is intended?	Evaluation by experts in developing scales
Content	Does the content of the items reflect the definition od the theoretical construct? Do the interviewees understand the questions/terminology in the same way?	Evaluation by a group of experts / cognitive interview / Focus Group
Factori	Does the scale measure the number of theoretical constructs? Is it possible to support the found constructs? (initial analysis) → Are the theoretical constructs confirmed? → (hypothesis test )  Is the structure comparable among relevant groups? →	Exploratory Factor Analysis Confirmatory Factor Analysis  with covariate DIF (Item invariance)
Construct (Convergence and discriminant)	Do the variables that should correlate with the scale actually do it? Do the variables that should not correlate with the scale actually do it?	Correlation, ANOVA, t-test
Concurrent Approach (known-groups or known-instruments)	Do the scale scores adequately represent interviewees with observable characteristics? Do the categorizations based in new scales correctly relate with those based in standarized previuos measures?	Correlation, ANOVA, t-test
Prediction	Do the scale scores adequately predict the future behaviuor or attitudes of interviewees?	Correlation, ANOVA, t-test

**Source:** Adapted from Abell et al. (2009) Developing and validating rapid assessment instruments, OUP.

## Convergent validation of the item: Kendall Tau b correlations

Are the scales correlated as expected with items, regarding sign and intensity?

## Internal consistency of the scale:

Cronbach Alpha coefficient (*Cronbach 1955*)

Psychologists pay attention to reliability:

*Do scale indicators produce similar scores?*

Economists concentrate on robustness:

*Does the scale generate similar rankings?*

$$\alpha = \frac{N \bar{r}}{1 + r(N - 1)}$$

N: number of items

r: average correlation  
among items

**TABLE 11.1.** Values of Cronbach's Alpha for Multiple-Item Scales with Various Combinations of the Number of Items and the Average Correlation Among Items.

<i>N</i>	.09	.25	.49
2	.17	.40	.66
3	.23	.50	.74
4	.28	.57	.79
5	.33	.62	.83
6	.37	.67	.85
7	.41	.70	.87
8	.44	.73	.88
9	.47	.75	.90
10	.50	.77	.91

(Treiman, 2009)

# The meaning of life

*Meaning in Life questionnaire* (Steger et al., 2006)

Meaning: The sense we have, and the meaning we feel in relation to the nature of our being and existence

**My life has a clear meaning or purpose**  
**I have discovered a satisfactory meaning of life**  
**I have a clear idea of what gives meaning to my life**

Reduced version of the scale 'presence of meaning', which measures if a person perceives that (s)he gives meaning to (her)his life and if this is translated into a satisfactory and clear purpose of life

# Basic Psychological Needs

3 sub-scales (Deci and Ryan, 2000)

Autonomy: Autonomous determination, freedom of speech, authenticity

**I feel that I am free to decide how I want to live my life**  
**In general, I feel that I can freely express my ideas and opinions**  
**I feel that I am honest with myself in every diary situation**

Competence: External appreciation, acknowledgement sense, self-effectiveness

**People who know me say I am capable/good in what I do**  
**Most of the time, I feel that I meet expectation in what I do**  
**In general, I feel very able/capable/effective**

Relationships with others: Social interaction, friendship, relationship with others

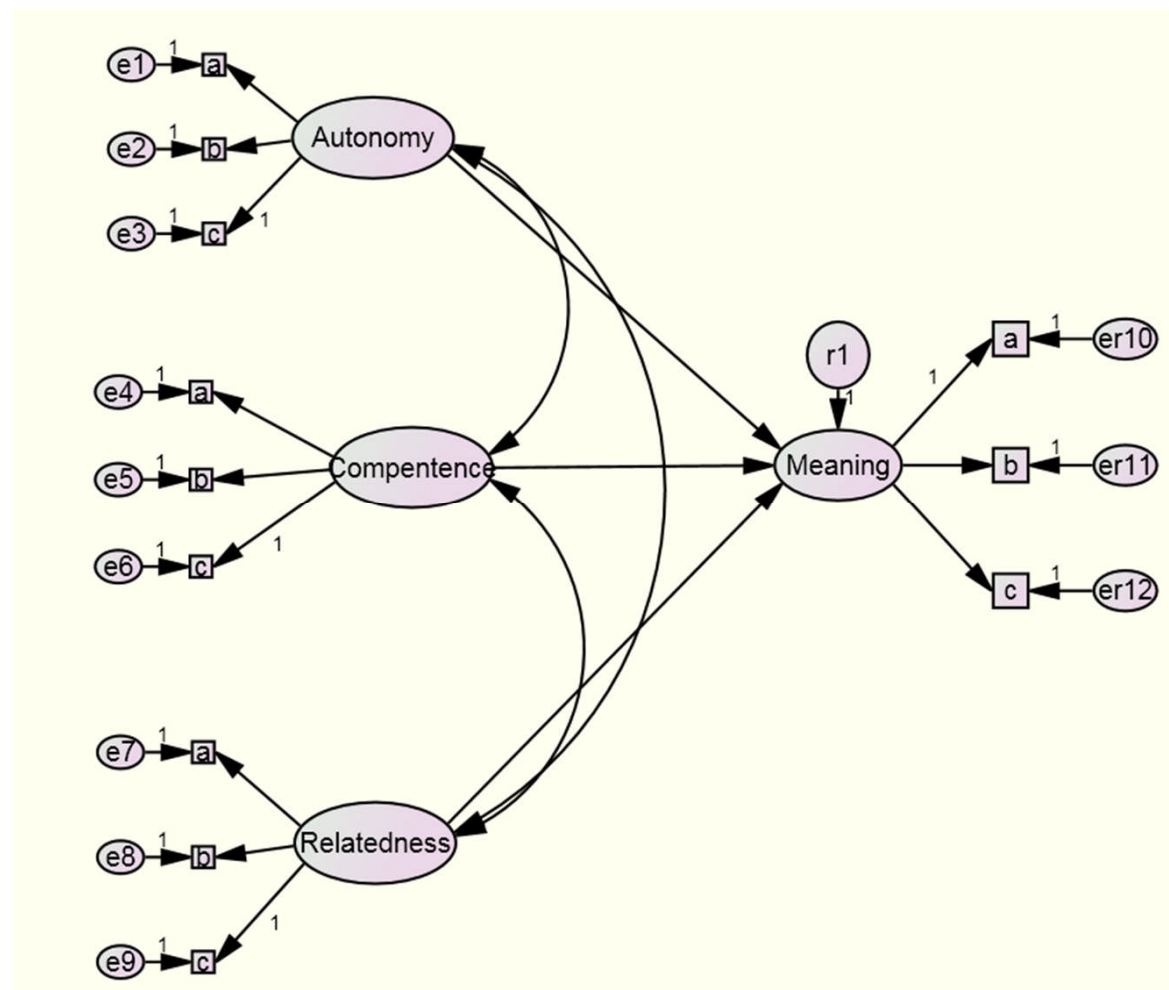
**I get along with people I have contact with**  
**I considered people I contact with to be close to me**  
**People around me cares about my wellbeing**

# Exploratory Factor Analysis

	Factor			
	1	2	3	4
mv3_a My life has a clear meaning or purpose	.759			
mv3_b I have found a satisfactory meaning in life	.920			
mv3_c I have a clear sense of what gives meaning to my life	.780			
mv4_a I feel free to decide for myself how to lead my life				.659
mv4_b I generally feel free to express my ideas and opinions				.974
mv4_c I feel like I can pretty much be honest with myself in daily situations				.632
mv5_a People I know tell me I am competent/capable at what I do		.740		
mv5_b Most of the time I feel a sense of accomplishment from what I do		.843		
mv5_c I generally feel very capable		.820		
mv6_a I get along well with people I come into contact with			.638	
mv6_b I consider myself close to the people I regularly interact with			.928	
mv6_c People in my life care about me			.641	
<b>Chronbach's Alpha</b>	<b>.878</b>	<b>.845</b>	<b>.859</b>	<b>.809</b>

**Note:** Only items with a loading higher than .300

# Confirmatory Factor Analysis



$\chi^2(48)=231.41$ ,  $p=.000$ ,  $RMR=.013$ ,  $RMSEA=.045$ ,  $CFI=.986$ ,  $TLI=.981$

# Evaluating validity of subjective and psychological wellbeing scales, using Chilean data

Factor: Exploratory factor analysis indicates that items converge and discriminate among them according to the four evaluated constructs (meaning of life, autonomy, competence and social relationships with others).

Reliability: High internal consistency (Cronbach's  $\alpha$ ), internal correlation among items in each scale and correlation across groups.

Structure: Confirmatory factor analysis – goodness of fit confirms the structure of the theoretical constructs.

Comparability across groups: factor invariance to genre and age groups.

Concurrence of criteria: expected correlation with other instruments

## Strengths

- ✓ Aggregation solution with high power of data reduction
- ✓ Deals well with measurement errors
- ✓ Suitable for exploratory analysis or confirmatory analysis in the identification of relevant underlying dimensions
- ✓ Reduces the chance of double-counting highly similar attributes and deals with issues concerning measurement error
- ✓ The factor loadings or component score can be saved and used in further analysis for inferences and model-testing (alternatively, incorporated directly into the model as in structural equation modelling)

## Weaknesses

- ✓ The final factors scores tend to be difficult to interpret
- ✓ Aggregation and weights would vary every time new data is considered, making comparisons more difficult (e.g. comparisons between years or countries)
- ✓ Not a single aggregation solution (depending in the choice of extraction and rotation method)
- ✓ In confirmatory analysis, the construct validity of the final factors depends on the theoretical relevance of the chosen initial indicators
- ✓ In most techniques, ordinal scale variables need to be interpreted in a cardinal sense (alternatively, nominal variables in multiple correspondence analysis, or latent continuous variables in structural equation modelling)