OXFORD POVERTY & HUMAN DEVELOPMENT INITIATIVE
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UNIVERSITY OF OXFORD

Summer School on Capability and Multidimensional Poverty

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[T]he job of a 'measure' or an 'index' is to distill what is particularly relevant for our purpose, and then to focus specifically on that. … The central issues in devising an index relate to systematic assessment of importance. Measurement has to be integrated with evaluation. This is not an easy task.

–Amartya Sen (1989)

Background: to *axiomatic* measures

Axiomatic approaches to multidimensional poverty began to gain momentum in the late 1990s

Brandolini, A., D'Alessio, G., 1998. Measuring Well-being in the Functioning Space. *Mimeo*. Rome. Banco d'Italia Research Department.

Chakravarty, S.R., Mukherjee, D., Renade, R.R., 1998. On the Family of Subgroup and Factor Decomposable Measures of Multidimensional Poverty. *Research on Economic Inequality*, 8, 175-194.

Key papers

- **Anand**, S., **Sen**, A.K., 1997. Concepts of Human Development and Poverty: A Multidimensional Perspective. New York, UNDP.
- **Tsui, K. 2002.,** Multidimensional Poverty Indices. *Social Choice and Welfare,* vol. 19, pp. 69-93.
- **Atkinson**, A.B., 2003. Multidimensional Deprivation. Contrasting Social Welfare and Counting Approaches. *Journal of Economic Inequality*. 1, 51-65
- **Bourguignon**, F., **Chakravarty**, S. R., 2003. The Measurement of Multidimensional Poverty. *Journal of Economic Inequality*. 1, 25-49.

Collections of articles (*axiomatic, information theory, fuzzy*)

- Kakwani, N., Silber, J., 2008a. *The Many Dimensions of Poverty*. Palgrave MacMillan
- Kakwani, N., Silber, J., 2008b. *Quantitative Approaches to Multidimensional Poverty Measurement*. Palgrave Macmillan.
- *World Development* June 2008

Background: to *counting* measures

- Much larger and longer history; far more empirical applications; wide policy use.
- From 1968: Scandinavian level of living.
- Mack, J., Lansley S., 1985. *Poor Britain*.
- Smeeding *et al*. 1993. *Review of Income & Wealth*
- Jayaraj & Subramanian~on Child Labor India
- *2005 UNICEF Child Poverty Report*.
- 2006: Chakravarty & D'Ambrosio

What is not covered:

- We will focus on *one* of several new multidimensional poverty measures (AF), teach it and do exercises on it so that you are confident using it.
- However **there are other axiomatic measures**. Some are summarised in Chakravarty & Silber 2008. "Measuring Multidimensional Poverty: The Axiomatic Approach," in Kakwani & Silber, Eds., *Quantitative Approaches*... p 192-209.
- **There are also interesting nonaxiomatic approaches** (Info theory, fuzzy set, Multiple Correspondence analysis). For a review of some of these see Deutsch, J., Silber, J., 2005. Measuring Multidimensional Poverty. An Empirical Comparison of Various Approaches. *The Review of Income and Wealth*. 51, 145-174 ; also Asselin on MCA 2008.

Focus of this class

- Alkire, S., Foster, J.E., 2011. "Counting and Multidimensional Poverty Measurement." *Journal of Public Economics*

- See also Alkire, S., Foster, J.E., 2011. *Understandings and Misunderstandings of Multidimensional Poverty Measurement*

Multidimensional Poverty- our challenge:

- A government would like to create an official multidimensional poverty indicator
- Desiderata
  - It must understandable and easy to describe
  - It must conform to "common sense" notions of poverty
  - It must be able to target the poor, track changes, and guide policy.
  - It must be technically solid
  - It must be operationally viable
  - It must be easily replicable

    - **What would you advise?**

Multidimensional Poverty Comparisons

- **There are many steps to creating index:**
  - Choice of purpose for the index (monitor, target, etc)
  - Choice of Unit of Analysis (indy, hh, cty)
  - Choice of Dimensions
  - Choice of Variables/Indicator(s) for dimensions
  - Choice of Poverty Lines for each indicator/dimension
  - Choice of Weights for indicators within dimensions
  - If more than one indicator per dimension, aggregation
  - Choice of Weights across dimensions
  - **Identification method**
  - **Aggregation method – within /across dimensions.**

This morning's focus:

- **Identification** – Dual cutoffs
- **Aggregation** – Adjusted FGT

- Purpose, Variables, Dimensional Cutoffs, Weights and all other steps – Assume given

Key methodological points:

Multidimensional poverty methodology comprises <u>identification</u> and <u>aggregation</u>, as well as the choice of <u>space</u>. (Sen 1976)

- **Identification** is critically important
- **Axioms** for MD poverty are joint restrictions on identification *and* aggregation.
- **Ordinal data** are common.
- **Decomposability** by sub-group, and (post identification) by factor, is key for policy.

Review: Unidimensional Poverty

Variable – income

Identification – poverty line

Aggregation – Foster-Greer-Thorbecke '84

Example Incomes = (7,3,4,8) poverty line z = 5

Deprivation vector $g^0 = (0,1,1,0)$

Headcount ratio $P_0 = \mu(g^0) = 2/4$

Normalized gap vector $g^1 = (0, 2/5, 1/5, 0)$

Poverty gap = $P_1 = \mu(g^1) = 3/20$

Squared gap vector $g^2 = (0, 4/25, 1/25, 0)$

FGT Measure = $P_2 = \mu(g^2) = 5/100$

Unidimensional Methods: Challenges

- All components must be cardinally meaningful
- Aggregate reflects achievements and tradeoffs
- All components can be merged/freely traded.
- Empirical evidence for weights, functional form
- A shortfall in any component is not of concern

Poverty Measurement:

Examples

Welfare aggregation

Construct each person's welfare function

Set cutoff and apply unidimensional poverty index

Myriad assumptions needed

- Alkire and Foster (2010) "Designing the Inequality-Adjusted Human Development Index"
- Ordinal variables problematic
- Suggests dominance

Poverty Measurement:

Examples

Price aggregation

Construct each person's expenditure level

Set cutoff and apply unidimensional poverty index

Myriad assumptions needed

Alkire and Foster (2010) "Designing the Inequality-Adjusted Human Development Index"

Ordinal and nonmarket variables

Link to welfare (local, unidirectional)

Foster, Majumdar, Mitra (1990) "Inequality and Welfare in Market Economies" *JPubE*

Poverty Measurement:

Suppose

Many variables that *cannot* be meaningfully aggregated into some overall resource or achievement variable. How to measure poverty?

Multidimensional Data

Matrix of well-being scores for *n* persons in *d* domains

$$
\begin{array}{cc}
 & \text{Domains} \\
y = & \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix} \text{ Persons}
\end{array}
$$

Multidimensional Data

Matrix of well-being scores for n persons in d domains

$$\begin{array}{cc} & \text{Domains} \\ y = & \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix} \text{Persons} \end{array}$$

$$z \quad (13 \quad 12 \quad 3 \quad 1) \quad \text{Cutoffs}$$

z vector = Deprivation Cutoffs

- **Schooling:** "How many years of schooling have you completed?"
  - **6 or more (bold is non-poor)**
  - 1-5 years (non-bold is poor)
- **Drinking Water:** "What is the main water source for drinking for this household?"
  - **9. Piped Water**
  - **8. Well/Pump (electric, hand)**
  - 7. Well Water
  - 6. Spring Water / Rain Water / River/Creek Water / Pond/Fishpond
  - 5. Other
- **Sanitation:** "Where do the majority of householders go to the toilet?"
  - **11. Own toilet with septic tank**
  - **10. Own toilet without septic tank**
  - 9. Shared toilet
  - 8. Public toilet
  - 7. Creek/river/ditch (without toilet)
  - 6. Yard/field (without toilet)
  - 5. Sewer
  - 4. Pond/fishpond
  - 3. Animal stable
  - 2. Sea/lake
  - 1. Other

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

$$\begin{array}{cc} & \text{Domains} \\ y = & \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} \; \text{Persons} \end{array}$$

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

$$\begin{array}{cc} & \text{Domains} \\ g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{Persons} \end{array}$$

Normalized Gap Matrix

Normalized gap = $(z_j - y_{ji})/z_j$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
y = & \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

$$
z \quad (13 \quad 12 \quad 3 \quad 1) \quad \text{Cutoffs}
$$

<u>These entries fall below cutoffs</u>

Normalized Gap Matrix

Normalized gap = $(z_j - y_{ji})/z_j$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
g^1 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0.08 & 0 & 0 \end{bmatrix} \text{Persons}
\end{array}
$$

Squared Gap Matrix

Squared gap = $[(z_j - y_{ji})/z_j]^2$ if deprived, 0 if not deprived

$$\begin{array}{cc} & \text{Domains} \\ \boldsymbol{g}^1 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0.08 & 0 & 0 \end{bmatrix} \text{ Persons} \end{array}$$

Squared Gap Matrix

Squared gap = $[(z_j - y_{ji})/z_j]^2$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
g^2 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.176 & 0 & 1 \\ 0.002 & 0.029 & 0.449 & 1 \\ 0 & 0.006 & 0 & 0 \end{bmatrix} \text{Persons}
\end{array}
$$

Identification

$$
\begin{array}{cc}
 & \text{Domains} \\
\boldsymbol{g}^{0} = & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \text{Persons}
\end{array}
$$

Matrix of deprivations

Identification – Counting Deprivations

$$\begin{array}{cccc} & \text{Domains} & c & \\ g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix} & \text{Persons} \end{array}$$

Identification – Counting Deprivations

Q/ Who is poor?

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ 2 \\ 4 \\ 1 \end{array} \quad \text{Persons}
\end{array}
$$

Identification – Union Approach

Q/ Who is poor?

A1/ Poor if deprived in any dimension $c_i \geq 1$

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix} \quad \text{Persons}
\end{array}
$$

Identification – Union Approach

Q/ Who is poor?

A1/ Poor if deprived in any dimension $c_i \geq 1$

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c \\
\text{Persons} \quad & g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ \underline{1} \end{matrix}
\end{array}
$$

Observations

Union approach often predicts high numbers.

Charavarty et al '98, Tsui '02, Bourguignon & Chakravarty 2003 etc use the union approach

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ 2 \\ 4 \\ 1 \end{array} \quad \text{Persons}
\end{array}
$$

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ 2 \\ \underline{4} \\ 1 \end{array} \qquad \text{Persons}
\end{array}
$$

Observations

Demanding requirement (especially if d large)

Often identifies a very narrow slice of population

Atkinson 2003 first to apply these terms.

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$

$$
\begin{array}{cc}
\text{Domains} & c \\
g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ 2 \\ 4 \\ 1 \end{array}
\end{array}
\quad \text{Persons}
$$

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $c_i \geq k$ **(Ex: k = 2)**

$$
\begin{array}{cc}
 & \text{Domains} \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\end{array}
\quad
\begin{array}{c}
c \\ 0 \\ \underline{2} \\ \underline{4} \\ 1
\end{array}
\quad \text{Persons}
$$

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $c_i \geq k$ **(Ex: k = 2)**

$$
\begin{array}{cccc}
 & \text{Domains} & & c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{array} & \quad \text{Persons}
\end{array}
$$

Note

Includes both union ($k = 1$) and intersection ($k = d$)

Identification – The problem empirically

k =	H
Union 1	91.2%
2	75.5%
3	54.4%
4	33.3%
5	16.5%
6	6.3%
7	1.5%
8	0.2%
9	0.0%
Inters. 10	0.0%

Poverty in India for 10 dimensions:

91% of population would be targeted using union,

0% using intersection

Need something in the middle.

(Alkire and Seth 2009)

Aggregation

Censor data of nonpoor

$$
\begin{array}{cc}
 & \begin{array}{cc} \text{Domains} & \quad c \end{array} \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{array} \quad \text{Persons}
\end{array}
$$

Aggregation

Censor data of nonpoor

$$
\begin{array}{ccc}
 & \text{Domains} & c(k) & \\
g^{0}(k) = & \left[\begin{array}{cccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right] & \begin{array}{c} \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \mathbf{0} \end{array} & \text{Persons}
\end{array}
$$

Aggregation

Censor data of nonpoor

$$
\begin{array}{cc}
 & \text{Domains} \\
g^{0}(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
\quad
\begin{array}{c}
c(k) \\ 0 \\ \underline{2} \\ \underline{4} \\ 0
\end{array}
\quad \text{Persons}
$$

Similarly for $g^1(k)$, etc

Aggregation – Headcount Ratio

$$\begin{array}{ccc} & \text{Domains} & c(k) & \\ g^{0}(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \text{Persons} \end{array}$$

Aggregation – Headcount Ratio

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \text{Persons}
\end{array}
$$

Two poor persons out of four: **H = 1/2**

Critique

Suppose the number of deprivations rises for person 2

$$
\begin{array}{cccc}
 & \text{Domains} & \quad & c(k) & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons}
\end{array}
$$

Two poor persons out of four: **H = 1/2**

Critique

Suppose the number of deprivations rises for person 2

$$
\begin{array}{cc}
 & \text{Domains} \quad\quad \boldsymbol{c(k)} \\
g^{0}(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{matrix} \quad \text{Persons}
\end{array}
$$

Two poor persons out of four: **H = 1/2**

Critique

Suppose the number of deprivations rises for person 2

$$\begin{array}{cccc} & \text{Domains} & c(k) & \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons} \end{array}$$

Two poor persons out of four: **H = 1/2**

No change!

Critique

Suppose the number of deprivations rises for person 2

$$\begin{array}{cc} & \text{Domains} \quad\quad c(k) \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{array} \quad \text{Persons} \end{array}$$

Two poor persons out of four: **H = 1/2**

No change!

Violates 'dimensional monotonicity'

Aggregation

Return to the original matrix

$$\begin{array}{cccc} & \text{Domains} & c(k) & \\ g^{0}(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons} \end{array}$$

Aggregation

Return to the original matrix

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c(k) \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} \quad \text{Persons}
\end{array}
$$

Aggregation

Need to augment information — deprivation shares among poor

$$
\begin{array}{ccc}
 & \text{Domains} & \begin{array}{cc} c(k) & c(k)/d \end{array} \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{cc} 0 & \\ \underline{2} & 2/4 \\ \underline{4} & 4/4 \\ 0 & \end{array} \quad \text{Persons}
\end{array}
$$

Aggregation

Need to augment information

deprivation shares among poor

$$
\begin{array}{ccc}
 & \text{Domains} & c(k) \quad c(k)/d \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 & \\ \underline{2} & 2/4 \\ \underline{4} & 4/4 \\ 0 & \end{matrix} \quad \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \begin{array}{c} \\ 2/4 \\ 4/4 \\ \\ \end{array} & \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\mu(g^0(k))$

$$\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \begin{array}{c} \\ 2/4 \\ 4/4 \\ \\ \end{array}
\end{array} \quad \text{Persons}$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\boldsymbol{\mu(g^0(k))}$ = **6/16 = .375**

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \begin{matrix} \\ 2/4 \\ 4/4 \\ \\ \end{matrix} & \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\boldsymbol{\mu(g^0(k))}$ **= 7/16 = 0.44**

$$\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{l} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{array} & \begin{array}{l} \\ \ldots 3/4 \\ \ldots 4/4 \\ \\ \end{array} & \text{Persons}
\end{array}$$

A = average deprivation share among poor = 3/4

Note: if person 2 has an additional deprivation, M_0 rises

Satisfies dimensional monotonicity

Adjusted Headcount Ratio $\mathcal{M}_{k0}=(\rho_k,M_0)$

Valid for ordinal data (identification & aggregation) – robust to monotonic transformations of data.

Similar to traditional gap P_1 = HI ; this = HA

Easy to calculate, easy to interpret

Can be broken down by dimension – policy

Dominance Results (mentioned later)

Characterization via freedom – P&X 1990

Note: If cardinal variables, can go further

Pattanaik and Xu 1990 and M_0

- Freedom = the number of elements in a set.
- But does not consider the *value* of elements
- If dimensions are of intrinsic value and are usually valued, then *every deprivation* can be interpreted as a shortfall of intrinsic concern.
- the (weighted) sum of deprivations can be interpreted as the unfreedoms of each person
- Adjusted Headcount can be interpreted as a measure of unfreedoms across a population.

Aggregation: Adjusted Poverty Gap

Need to augment information of M_0 Use normalized gaps

$$
\begin{array}{cc}
 & \text{Domains} \\
g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Persons}
\end{array}
$$

Average **gap** across all deprived dimensions of the poor:

$G = (0.04+0.42+0.17+0.67+1+1)/6$

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = M_1 = M_0G = HAG

$$\begin{array}{cc} & \text{Domains} \\ g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \text{Persons} \end{array}$$

Average **gap** across all deprived dimensions of the poor:

$G = (0.04+0.42+0.17+0.67+1+1)/6$

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = M_1 = $M_0 G$ = HAG = $\mu(g^1(k))$

$$
\begin{array}{cc}
 & \text{Domains} \\
g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Persons}
\end{array}
$$

Average **gap** across all deprived dimensions of the poor:

G = (0.04+0.42+0.17+0.67+1+1)/6

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = $M_1 = M_0G$ = HAG = **$\mu(g^1(k))$**

$$
\begin{array}{ccc}
 & \text{Domains} & \\
g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \text{Persons}
\end{array}
$$

Obviously, if in a deprived dimension, a poor person becomes even more deprived, then M_1 will rise.

Satisfies monotonicity

Aggregation: Adjusted FGT

Consider the matrix of squared gaps

$$
\begin{array}{cc}
 & \text{Domains} \\
g^2(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

Aggregation: Adjusted FGT

Adjusted FGT is $M_2 = \mu(g^2(k))$

$$\begin{array}{cc} & \text{Domains} \\ g^2(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Persons} \end{array}$$

Aggregation: Adjusted FGT

Adjusted FGT is M_2 = **$\mu(g^2(k))$**

$$
\begin{array}{cc}
 & \text{Domains} \\
g^2(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

Satisfies transfer axiom

Aggregation: Adjusted FGT Family

Adjusted FGT is $M_\alpha = \mu(g^\alpha(\tau))$ for $\alpha \geq 0$

$$\begin{array}{cc} & \text{Domains} \\ g^\alpha(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^\alpha & 0 & 1^\alpha \\ 0.04^\alpha & 0.17^\alpha & 0.67^\alpha & 1^\alpha \\ 0 & 0 & 0 & 0 \end{bmatrix} & \text{Persons} \end{array}$$

Theorem 1 For any given weighting vector and cutoffs, the methodology $M_{k\alpha} = (\rho_k, M_\alpha)$ satisfies: decomposability, replication invariance, symmetry, poverty and deprivation focus, weak and dimensional monotonicity, nontriviality, normalisation, and weak rearrangement for $\alpha \geq 0$; monotonicity for $\alpha > 0$; and weak transfer for $\alpha \geq 1$.

Setting cutoff *k*: *n*ormative or policy

- Depends on: purpose of exercise, data, and weights
  - "In the final analysis, how reasonable the identification rule is depends, *inter alia*, on the attributes included and how imperative these attributes are to leading a meaningful life." (Tsui 2002 p. 74).
- E.g. a measure of Human Rights; data good = union
- Targeting: according to category (poorest 5%). Or budget (we can cover 18% - who are they?)
- Poor data, or people do not value all dimensions: k<d
- Some particular combination (e.g. the intersection of income deprived *and* deprived in any other dimension)

Robustness tests for *k*

- *Theorem 2* Where a and a' are the respective attainment vectors for y and y' in Y $(a_i = d - c_i)$, we have:
- (i) $y \, \boldsymbol{H} \, y' \Leftrightarrow a \, \boldsymbol{FD} \, a'$
- (ii) $a \, \boldsymbol{FD} \, a' \Rightarrow y \, \boldsymbol{M_0} \, y' \Rightarrow a \, \boldsymbol{SD} \, a'$, and the converse does not hold.

(i) akin to Foster Shorrocks: first order dominance over attainment vectors ensures that multidimensional headcount is lower (or no higher) for all possible values of k – and the converse is also true.

(ii) shows that $\boldsymbol{M_0}$ is implied by first order dominance, and implies second order, in turn

Properties for Multidimensional Poverty Methodologies

- axioms are ***joint restrictions*** on $\boldsymbol{\mathcal{M}} = (\varrho, M)$

- Identification is vital for some axioms (poverty focus).

- Previously defined axioms used union approach

- Our axioms are applicable to $0 < k \leq d$

Example:

- **Unidimensional Focus Axiom:** requires a poverty measure to be independent of the data of the non-poor (incomes at/above z)

- In a multidimensional setting:
  - a non-poor person might be deprived in several dimensions
  - a poor person might not be deprived in *all* dimensions.
- How do we adapt the focus axiom?

Example:

- **Poverty Focus**: If x is obtained from y by a simple increment among the non-poor, then $M(x;z)=M(y;z)$.
- **Deprivation Focus**: If x is obtained from y by a simple increment among the nondeprived, then $M(x;z)=M(y;z)$.

Union: deprivation focus implies poverty focus

Intersection: poverty focus implies deprivation

Bourguignon and Chakravarty (2003) assume the deprivation focus axiom (their 'strong focus axiom') along with union identification, so their methodology automatically satisfies the poverty focus axiom.

Another Example:

- *deprived increment* (still below cutoff, deprived)
- *dimensional increment* (now non-deprived)
- **Weak Monotonicity**: If x is obtained from y by a simple increment, then $M(x;z) \leq M(y;z)$.
- **Monotonicity**: M satisfies weak monotonicity and the following: if x is obtained from y by a deprived increment among the poor then $M(x;z) < M(y;z)$.
- **Dimensional Monotonicity**: If x is obtained from y by a dimensional increment among the poor, then $M(x;z) < M(y;z)$.

Properties

- Our methodology satisfies a number of typical properties of multidimensional poverty measures (suitably extended):
- *Symmetry,* *Scale invariance*
  Normalization *Replication invariance*
  Poverty Focus *Weak Monotonicity*
  Deprivation Focus *Weak Re-arrangement*

- M_0, M_1 and M_2 satisfy *Dimensional Monotonicity, Decomposability*

- M_1 and M_2 satisfy *Monotonicity* (for $\alpha > 0$) – that is, they are sensitive to changes in the depth of deprivation in all domains with cardinal data.

- M_2 satisfies *Weak Transfer* (for $\alpha > 1$).

Extension: General Weights

Modifying for weights at two points:

1) Identification (k is now a cutoff of the weighted sum of dimensions)
2) Aggregation (simply weight matrix prior to taking the mean)

Extension – General Weights

Modifying for weights: identification and aggregation (technically weights need not be the same, but conceptually probably should be)

- Use the g_0 or g_1 matrix
- Choose relative weights for each dimension w_d
- Important: *weights must add up to the number of dimensions*
- Apply the weights (sum = d) to the matrix
- c_k now reflects the *weighted sum* of the dimensions.
- Set cutoff k across the weighted sum.
- Censor data as before to create $g_0(k)$ or $g_1(k)$
- Measures are *still* the mean of the matrix.

Example: Weights

$$
\begin{array}{ccc}
 & \text{Domains} & \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \text{Persons}
\end{array}
$$

Matrix of deprivations

Weighting vector $\omega = \quad (.5 \quad 2 \quad 1 \quad .5)$

Example: Weights

$$\begin{array}{cc} & \text{Domains} \\ g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array} \quad \text{Persons}$$

Matrix of deprivations

Weighting vector ω = **(.5 2 1 .5)**

Example: Weights - Identification

$$g^0 = \begin{array}{c} \text{Domains} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ 0 \\ \mathbf{2.5} \\ \mathbf{4} \\ \mathbf{2} \end{array} \quad \text{Persons}$$

Matrix of deprivations

Weighting vector $\omega = \quad \mathbf{(.5 \quad 2 \quad 1 \quad .5)} \qquad \boldsymbol{k = 2}$

Identification changed!

Example: Weights - Identification

$$
\begin{array}{ccc}
 & \text{Domains} & \\
g^{0} = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{\mathbf{2.5}} \\ \underline{\mathbf{4}} \\ \mathbf{2} \end{matrix} \quad \text{Persons}
\end{array}
$$

Weighting vector $\omega =$ $\mathbf{(.5 \quad 2 \quad 1 \quad .5)}$ $\boldsymbol{k = 2.5}$

Original Identification for k=2.5

Example: Weights – Aggregation
k = 2.5

$$\begin{array}{cc} & \text{Domains} & & \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{\mathbf{2.5}} \\ \underline{\mathbf{4}} \\ 0 \end{array} & \text{Persons} \end{array}$$

M_0 still HA = mean of matrix = 6.5/16

H = 2/4

A = weighted = 6.5/8

etc.

Illustration: USA

- **Data Source:** National Health Interview Survey, 2004, *United States Department of Health and Human Services. National Center for Health Statistics* - ICPSR 4349.
- **Tables Generated By:** Suman Seth.
- **Unit of Analysis:** Individual.
- **Number of Observations:** 46009.
- **Variables:**
  - (1) income measured in poverty line increments and grouped into 15 categories
  - (2) self-reported health
  - (3) health insurance
  - (4) years of schooling.

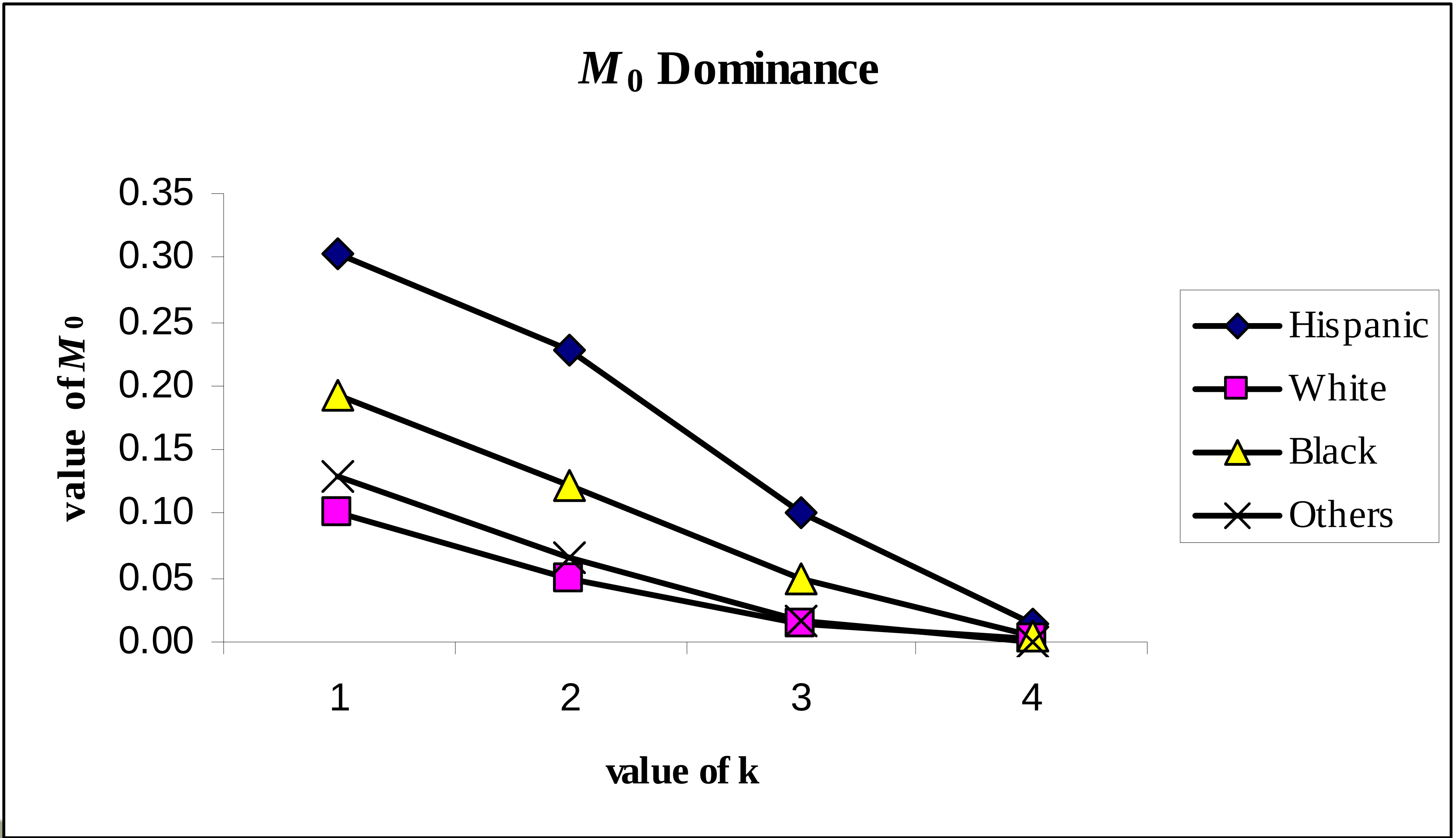
Illustration: USA

1	2	3	4	5	6	7	8	9
Ethnicity	**Population**	**Percentage Contributn**	**Income Poverty Headcount Ratio**	**Percentage Contributn**	***H***	**Percentage Contributn**	**M_0**	**Percentage Contributn**
Hispanic	9100	19.8%	0.23	37.5%	0.39	46.6%	0.229	47.8%
White	29184	63.6%	0.07	39.1%	0.09	34.4%	0.050	33.3%
Black	5742	12.5%	0.19	20.0%	0.21	16.0%	0.122	16.1%
Others	1858	4.1%	0.10	3.5%	0.12	3.0%	0.067	2.8%
Total	**45884**	**100.0%**	**0.12**	**100.0%**	**0.16**	**100.0%**	**0.09**	**100.0%**

Illustration: USA

1	2	3	4	5	6
Ethnicity	***H_1 Income***	***H_2 Health***	***H_3 H. Insurance***	***H_4 Schooling***	***M_0***
Hispanic	0.200	0.116	0.274	0.324	0.229
Percentage Contribution	21.8%	12.7%	30.0%	35.5%	100%
White	0.045	0.053	0.043	0.057	0.050
Percentage Contribution	22.9%	26.9%	21.5%	28.7%	100%
Black	0.142	0.112	0.095	0.138	0.122
Percentage Contribution	29.1%	23.0%	19.5%	28.4%	100%
Others	0.065	0.053	0.071	0.078	0.067
Percentage Contribution	24.2%	20.0%	26.5%	29.3%	100%

Illustration: USA – all values of *k*



Indonesia: Deprivation by dimension

Deprivation	Percentage of Population
Expenditure	30.1%
Health (*BMI*)	17.5%
Schooling	36.4%
Drinking Water	43.9%
Sanitation	33.8%

Indonesia: Breadth of Deprivation

Number of Deprivations	Percentage of Population
One	26%
Two	23%
Three	17%
Four	8%
Five	2%

Identification as *k* varies

Cutoff *k*	Percentage of Population
1	74.9%
2	49.2%
3	26.4%
4	9.7%
5	1.7%

And interpretation?

Equal Weights			
Measure	k=1 (Union)	$\boldsymbol{k}$=2	k=3 (Intersection)
H	0.577	0.225	0.039
M_0	0.280	0.163	0.039
M_1	0.123	0.071	0.016
M_2	0.088	0.051	0.011

General Weights				
Measure	k = 0.75 (Union)	k = 1.5	k = 2.25	k = 3 (Intersection)
H	0.577	0.346	0.180	0.039
M_0	0.285	0.228	0.145	0.039
M_1	0.114	0.084	0.058	0.015
M_2	0.075	0.051	0.036	0.010

And interpretation?

Equal Weights			
Measure	k=1 (Union)	k=2	k=3 (Intersection)
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M_0 = H for intersection

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M_1	0.114	0.084	0.058	0.015
M_2	0.075	0.051	0.036	0.010

And interpretation?

If all persons have *maximal* deprivation, then G=1, so $M_0 = M_1$. **Low gap** if M_0 is **higher than** M_1.

…eights

	k=1 (Union)	k=2	k=3 (Intersect…
	0.577	0.225	0.039
M_0	0.280	0.163	0.039
M_1	0.123	0.071	0.016
M_2	0.088	0.051	0.011

M_0 = H for intersection

General Weights

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M_2	0.075	0.051	0.036	0.010

And interpretation?

If all persons have *maximal* deprivation, then G=1, so M_0 = M_1. **Good** if M_0 is **different** from M_1.

M_0 = H for intersection

...hts

	k=1 (Union)	k=2	k=3 (Intersect...)
	0.577	0.225	0.039
	0.280	0.163	0.039
M_1	0.123	0.071	0.016
M_2	0.088	0.051	0.011

General Weights

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M_0	0.285	0.228	0.145	0.039
M_1	0.114	0.084	0.058	0.015
M_2	0.075	0.051	0.036	0.010

Weights affect relevant *k* values.

Empirical Examples

Sub-Saharan Africa (14): Assets, Education, BMI, Empowerment

Latin America (6) Income, Child in School, hhh Education, Water, Sanitation, Housing

China Income, Education, BMI, Water, Sanitation, Electricity

India Assets, Education, BMI, Water, Sanitation, Housing, Electricity, Cooking Fuel, Livelihood, Child status, Empowerment.

Pakistan Expenditure, Assets, Education, Water, Sanitation, Electricity, Housing, Land, Empowerment

Bhutan I Income, Education, Rooms, Electricity, Water (land, roads used in rural areas only)

MPI – for 104 countries (10 indicators; 3 dimensions)

Empirical Applications

We can also choose a **unit of analysis** other than the individual (Bhutan) or Household (other), and use the same methodology with indicators of institutions, and z **cutoffs** representing quality, standards, or benchmarks.

Gross National Happiness (Bhutan)

Quality of Education (Mexico, Argentina)

Governance (Ibrahim Index)

Targeting (India BPL, Mexico Oportunidades)

Child Poverty (Afghanistan, Bangladesh)

Social Responsibility/Fair Trade (Altereco)

Human Rights (Benetech)

Joint Distribution vs Marginal

- Use a deprivation cutoff for each dimension {Bourguignon and Chakravarty (2003)}
- Hence each shortfall can be seen and may contribute independently to poverty.
- Ordinal data can be used.

These can be divided broadly into two types:

Marginal Measures

Measures that reflect Joint Distribution

Multidimensional Methods:

Marginal Measures:

- Apply a deprivation cutoff for each vector of achievements.
- Construct an aggregate
- Inadequate idenfication (if at all, union)
- Ignores joint distribution
- Examples:
    - HPI

Multidimensional Methods:

Our Proposal: Joint Measures

- Apply a deprivation cutoff for each vector of achievements.
- Identify *who is poor* – e.g. with dual-cutoff
- Aggregate across poor people
- Examples:
    - MPI
    - Counting
    - Basic Needs

Why do Joint Distribution methods add value?

Matrix 1

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} .25 & .25 & .25 & .25 \end{bmatrix}$$

Matrix 2

$$g^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} .25 & .25 & .25 & .25 \end{bmatrix}$$

Why do Joint Distribution methods add value?

Matrix 1

Marginal Measures ONLY use this vector to create their measures. So according to ANY marginal measure, the poverty of Matrix 1 = the poverty of Matrix 2.

$g^0 =$

$$\begin{bmatrix} .25 & .25 & .25 & .25 \end{bmatrix} = \begin{bmatrix} .25 & .25 & .25 & .25 \end{bmatrix}$$

$\mathbf{M_0}$ if k=1: 0.25;

H=.25; A=1

H=1; A=.25

Matrix 1

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} .25 & .25 & .25 & .25 \end{bmatrix}$$

Matrix 2

$$g^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} .25 & .25 & .25 & .25 \end{bmatrix}$$

M_0 if k=1: 0.25 0.25

M_0 if k=2: 0.25 0

Matrix 1 Matrix 2

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

$$[.25 \quad .25 \quad .25 \quad .25]$$

$$g^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$[.25 \quad .25 \quad .25 \quad .25]$$

Informal Note: order of operations

	Unidim.	Marginal	MD (Joint)
Identify Deprivations	n/a	1	1
Aggregate Across Dimensions ('count')	1	3	2
Identify Who is Poor	2	n/a	3
Aggregate across People	3	2	4

Key point: Deprivation and Censored Matrix

Deprivation Matrix

Censored Deprivation Matrix, k=2

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 4 \\ 1 \end{bmatrix} \quad \Longrightarrow \quad g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 4 \\ 0 \end{bmatrix}$$

AF Method: Decompositions

By Population Subgroup

M_α	Poverty
H	Headcount
A	Intensity

Post-identification: By Dimension

Censored Headcount

Percentage Contribution

All draw on <u>censored</u> matrix

misunderstood

Informal Glossary of Terms

Deprivation: if $y_{id} < z$ person *i* is **deprived** in y_d

Poverty: if $c_i \leq k$ person *i* is poor.

Deprivation cutoffs: the z cutoffs for each dimension

Poverty cutoff: the overall cutoff *k*

Dimension: for AF – a column in the matrix having its own deprivation cutoff (sometimes called an 'indicator')

Joint distribution: showing the simultaneous or coupled deprivations a person/hh has