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Summer School on Multidimensional Poverty Analysis

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A Review of the Unidimensional Poverty Measurement

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Main Sources of this Lecture

Papers/chapters/books	Author/s	Year
1. Multidimensional Poverty Measurement and Analysis	Alkire et al	2015
2. A Unified Approach to Measuring Poverty and Inequality: Theory and Practice	Foster et al	2013
3. On Economic Inequality (Annexe)	Foster and Sen	1997
4. Poverty Indices	Foster	2006

Note: Full references are available in the reading list

Preliminaries

- Reference population
  - We refer as '*Society*' (region/country/state/district)
- Unit of measurement
  - We refer as '*Person*' (child/women/households)
  - Suppose there are n persons in the society (n may vary)
- Variables or dimensions for assessing poverty
  - We refer as '*Space*' (e.g. income sources, commodities)
  - Suppose there are d such variables (fixed set)

Preliminaries

- <u>Achievement</u>
  - x_{ij}: Achievement of person i (=1,…,n) in dimension j (=1,…, d)
- <u>Achievement matrix</u>
  - Summary of achievements of all n persons in d dimensions
- <u>Achievement vector of a Person</u>
  - May contain incomes from d different sources or d different commodities consumed

Dimensions

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1d} \\ x_{21} & \cdots & x_{2d} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nd} \end{bmatrix}$$

Persons

Preliminaries

- Overall achievement of each person
  - x_i : Obtained by meaningfully combining *d* dimensions
    - Also referred as *resource variable* or *welfare indicator*
  - When the space is a set of income sources
    - Total income of person *i*: $x_i = \Sigma_j x_{ij} = x_{i1} + \ldots + x_{id}$
  - When the space is a set of commodities consumed
    - There is a set of *d* commodity prices: $(p_1,\ldots, p_d)$ and total consumption expenditure of person *i*:

$$x_i = \Sigma_j p_j x_{ij} = p_1 x_{i1} + \ldots + p_d x_{id}$$

Preliminaries

- Overall achievement vector: $x = (x_1,\ldots, x_n)$
  - Example: Suppose there are four persons in a society with overall incomes \$9, \$4, \$15 and \$8
  - Then $x = (9, 4, 15, 8)$ is a vector representing the incomes of the society

- Ordered overall achievement vector
  - An ordered vector ranks or orders individuals by their achievements
  - Example: $(9, 4, 15, 8) \rightarrow (4, 8, 9, 15)$

Preliminaries

- A policy maker is generally interested in the following three aspects of a distribution or a vector
  - Size (Welfare): e.g. per-capita income
  - Spread (Inequality): e.g. Gini coefficient
  - Base (Poverty)
    - Welfare of the population below a certain level of income

In this summer school, we focus on the third aspect

Poverty Measurement

Unidimensional poverty measurement involves two steps (Sen 1976): **Identification** and **Aggregation**

Identification: Who is poor?

- Dichotomises population into *poor* and *non-poor*
- Main tool: The Poverty Line (z)
- Person i is poor if $x_i < z$ and is non-poor if $x_i \geq z$

x_i is the i^{th} element of vector x

Significance of Poverty Line (z)

1. A *benchmark* that enables policy makers to identify the poor

   - Enabling '*targeting*', i.e.: who qualifies for social assistance

2. A *benchmark* for policy makers to bring the poor at z

   - For poverty depth analysis, achievement of the non-poor <u>above</u> the poverty line is ignored

Censoring at Poverty Line

Having z as benchmark, allows us to create a censored distribution of x, referred as x^*, where

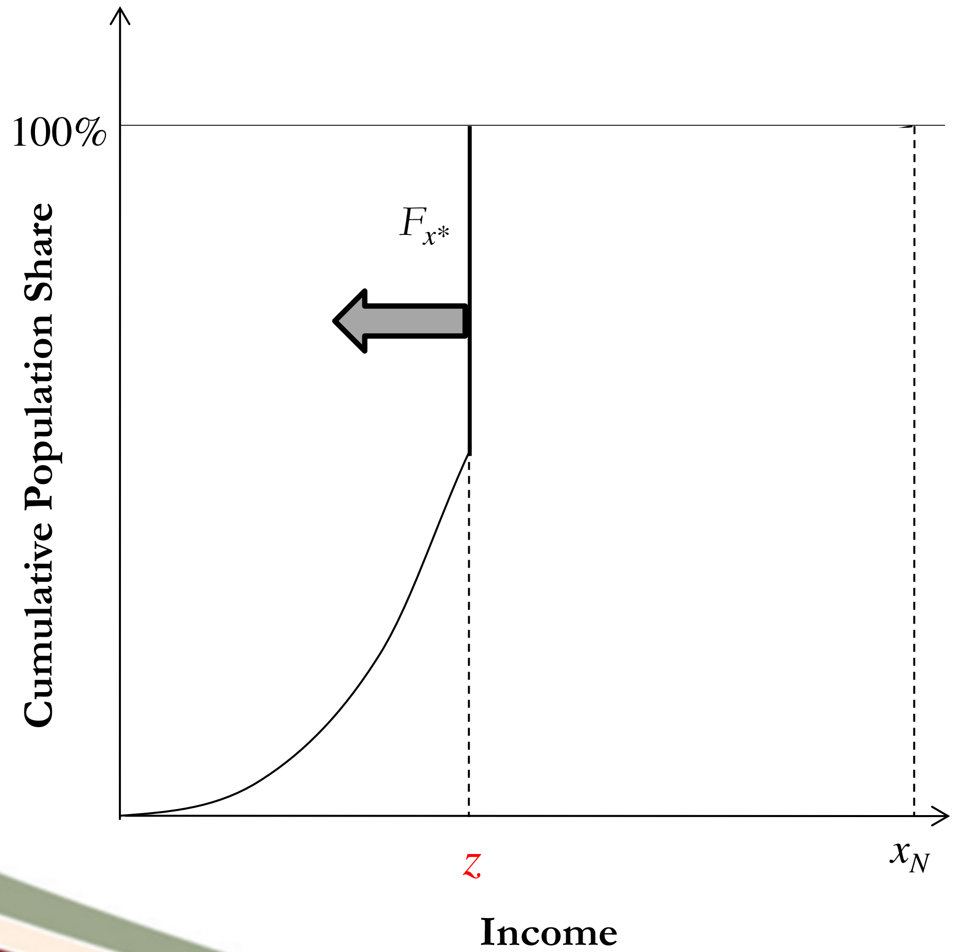
$$x_i^* = x_i \text{ if } x_i < z$$

and $$x_i^* = z \text{ if } x_i \geq z$$

Example: If $z = 10$ and $x = (9, 4, 15, 8)$, then

$$x^* = (9, 4, 10, 8)$$

Censoring at Poverty Line



Obtain the censored distribution x^*

$$x_i^* = x_i \text{ if } x_i < z$$
$$x_i^* = z \text{ if } x_i \geq z$$

Aggregation

- How poor is the society?
  - This step constructs an index of poverty summarizing the information in the censored achievement vector x^*
  - For each distribution x and poverty line z, $P(x;z)$ or $P(x^*)$ measures the level of poverty in the distribution

Behaviour of Poverty Measures

- How should a poverty measure change due to different data transformations?

- Consider certain examples

Example 1

- Which of the following distributions has more poverty?

  $x = (5,4,4,5)$;

  $y = (4,4,5,5)$; and

  $z = 10$

  – And why?

- <u>Symmetry</u>
  – If y is obtained from x by a <u>permutation of incomes</u> and z remains unchanged, then $P(y;z) = P(x;z)$

Example 2

- Which of the following distributions has more poverty?

  $x = (9,9,15)$;

  $y = (9,9,9,15,15,15,15,15)$; and

  $z = 10$

  – And why?

- <u>Replication invariance</u>
  – If y is obtained from x by <u>a replication</u> and z remains unchanged, then $P(y;z) = P(x;z)$
  – <u>Example</u>: $z = 10$, $x = (9,4,15,8)$, $y = (9,9,4,4,15,15,8,8)$

Example 3

- Which of the following distributions has more poverty?

  $x = (\$2, \$4, \$6, \$12)$ and $z_x = \$10$

  $y = (£3, £6, £9, £18)$ and $z_y = £15$ $\quad\quad$ $£1 = \$1.5$

  - And why?

- <u>Scale invariance</u>
  - If all incomes in *y* and *z* are changed by the <u>same proportion</u> $a > 0$, then $P(\alpha y; \alpha z) = P(y; z)$
  - Related properties: *Unit Consistency*, *Translation invariance*

Example 4

- Which of the following distributions has more poverty?

  $x = (5,4,4,15)$;
  $y = (5,4,4,25)$; and
  $z = 10$

  – And why? Hint: Censor

- Focus
  - If y is obtained from x by an increment to a non-poor person's income and z remains unchanged, then $P(y;z) = P(x;z)$
  - Two types of focus properties in multidimensional context

Example 5

- Which of the following distributions has more poverty?

  $x = (4,4,4,4)$;

  $y = (4,4,4,3)$; and

  $z = 10$

  – And why?

- <u>Monotonicity</u>
  - If y is obtained from x by a <u>decrement of incomes among the poor</u> and z remains unchanged, then $P(y,z) > P(x,z)$
- <u>Weak monotonicity</u>
  - If y is obtained from x by a decrement of incomes among the poor and z remains unchanged, then $P(y,z) \geq P(x,z)$

Example 6

- Which of the following distributions has more poverty?

$$x = (1,1,9,9);$$

$$y = (4,4,6,6); \text{ and}$$

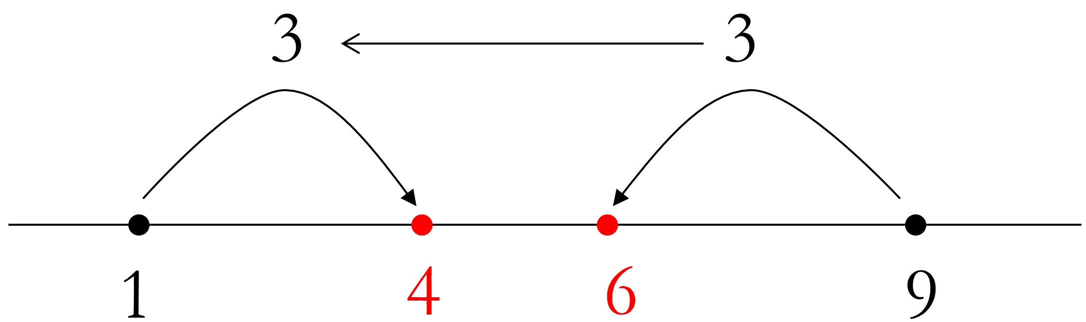
$$z = 10$$

  – And why?

- Transfer principle
  – If y is obtained from x by a progressive transfer among the poor and z remains unchanged then $P(y;z) < P(x;z)$
- Weak transfer principle
  – If y is obtained from x by a progressive transfer among the poor and z remains unchanged then $P(y;z) \leq P(x;z)$

Donaldson and Weymark (1986)

Progressive Transfer



Transfer: Is there a limit on the amount of transfer in this property?

- Yes. Post-transfer income cannot fall below the lower pre-transfer income
- What is the implication of this axiom for non-transferable dimensions?

Example 7

- Let z=10, x=(9,4,15,8), x^1=(9,4), and x^2=(15,8)
  - y is obtained from x such that y^1=(**6**,4), while $y^2 = x^2$
  - Which distribution has more poverty: y or x?
  - Note $P(y^1;z) > P(x^1;z)$ by monotonicity, but $P(y^2;z) = P(x^2;z)$, and so it is consistent to have $P(y;z) > P(x;z)$

- <u>Subgroup Consistency</u>
  - If y is obtained from x, such that

  (i) $P(y^1;z) > P(x^1;z)$,

  (ii) $P(y^2;z) = P(x^2;z)$, and

  (iii) the population size of each group and z remains unchanged, then $P(y;z) > P(x;z)$

Subgroup Consistency

- For which reason is the subgroup consistency property important?
  - Evaluation of poverty reduction programs!
  - It can be seen as an extension of monotonicity
    - Monotonicity: poverty must fall when a poor person experiences increment in their income.
    - Subgroup consistency: aggregate poverty must fall when a subgroup experiences increment in their income.
    - However, a reduction in subgroup poverty may be accompanied by both increase and fall in individual incomes

    (where Subgroup consistency differs from Monotonicity)

Additive Decomposability

- A poverty measure is additive decomposable if

$$P(x) = \frac{n^1}{n} P(x^1) + \frac{n^2}{n} P(x^2)$$

- One may compute the contribution of each group to overall poverty as

$$\frac{n^l}{n} \frac{P(x^l)}{P(x)}; l = 1, 2$$

  - Note: Additive decomposability implies subgroup consistency, but the converse does not hold

Some Technical Properties

- Normalization
  - A poverty measure should be bounded between 0 and 1
    - 0: non-poor and 1: poor
  - Example 1: $z = 2$ and $x = (9,4,15,8)$, then $P(x;z) = 0$
  - Example 2: $z = 16$ and $x = (9,4,15,8)$, then $P(x;z) = 1$
- Continuity
  - A poverty measure should be continuous on the achievements
  - This property prevents any sudden change in a poverty measure

Classification of Properties

- Invariance Properties
  - Ensure that poverty measures should not change under certain transformations of the achievement matrix
- Dominance Properties
  - Ensure that poverty measures should increase or decrease due to certain transformations in the achievement matrix
- Subgroup Properties
  - Relate overall poverty to either groups of people or groups of dimensions
- Technical Properties
  - Guarantee that measures behave within certain usual, convenient parameters

Poverty Measures

- Basic Measures
  - Headcount Ratio
  - Income Gap Ratio
  - Poverty Gap Ratio

- Advanced Measures
  - Squared Poverty Gap (Foster-Greer-Thorbecke)
  - Sen-Shorrocks-Thon Measure
  - Watts Measure
  - Clark-Hemming-Ulph-Chakravarty Class of Measures

Poverty Measures

- <u>The Headcount Ratio (H)</u>
  - The most commonly and widely used measure of poverty
  - It reports the proportion of the population that is poor
  - It ranges between 0 and 1
  - If q is the number of poor in vector x with population size n, then $H = q/n$
  - <u>Example</u>: Let $z = 10$ and $x = (9,4,15,8)$, then $H = 3/4$

Poverty Measures

- The Headcount Ratio (H)
  - What properties does this measure satisfy?
  - H satisfies symmetry, replication invariance, scale invariance, focus, normalization, and subgroup consistency
  - H does not satisfy monotonicity, transfer, continuity
  - Policy Implication?
    - Encourages a policy maker, with limited budget, to assist the marginally poor only instead of the severely poor

Poverty Measures

- Poverty Gap Ratio (PG)
  - This measure repairs some of the problems of the headcount ratio
  - It reports the average normalized income shortfall from the poverty line using the *censored* distribution x^*
  - The normalized shortfall of the i^{th} person is

$$g_i^* = (z - x_i^*)/z.$$

  - The average income normalized shortfall is

$$PG = (1/n)\,\Sigma_i\, g_i^* = (z - m^*)/z = \mathbf{H \times I}$$

$$I = (1/q)\Sigma_i (z - x_i)/z$$

Poverty Measures

- Poverty Gap Ratio (PG)
  - Example: $x=(9,4,15,8)$; $z=10$
  - Then $x^*=(9,4,\mathbf{10},8)$ and $g^* = (0.1,0.6,0,0.2)$
  - So, PG = 0.9/4 = 0.225

  - Alternatively, m^* = average of elements in x^*.
  - So, $m^* = 7.75$. Thus, PG = (10 – 7.75)/10 = 0.225

  - PG ranges between 0 and 1

Poverty Measures

- Poverty Gap Ratio (PG)
  - What axioms does this measure satisfy?
  - It satisfies - symmetry, replication invariance, scale invariance, focus, normalization, monotonicity, continuity and subgroup consistency
  - It does not satisfy – transfer

  - **Policy Implication**?
  - It does not encourage a policy maker to distinguish between a marginally poor and severely poor while assisting

Advance Poverty Measures

- Squared Poverty Gap (SG)
  - It reports the average of squared normalized income shortfalls from the poverty line using the *censored* distribution x^*. Also, known as *Foster-Greer-Thorbecke* (FGT) measure
  - The average normalized shortfall of the i^{th} person is
    $$g_i^* = (z - x_i^*)/z$$
  - The average of squared normalized income shortfalls
    $$SG = (1/n)\ \Sigma_i\ (g_i^*)^2$$

Advance Poverty Measures

- Squared Poverty Gap (SG)
  - Example: $x=(9,4,15,8)$; $z=10$.
  - Then $x^*=(9,4,\mathbf{10},8)$ and $g^* = (0.1,0.6,0,0.2)$
  - Squares of poverty gap are $sg^* = (0.1^2,0.6^2,0^2,0.2^2) = (0.01,0.36,0,0.04)$
  - Thus, SG = 0.41/4 = 0.102
  - SG ranges between 0 and 1.

Advance Poverty Measures

- <u>Squared Poverty Gap (SG)</u>
  - What axioms does this measure satisfy?
  - It satisfies symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, transfer, and subgroup consistency
  - This measure can be presented as: $SG = H[I^2 + (1 - I)^2 \times C_p^2]$
    - C_p is the coefficient of variation of income across the poor.
    - <u>Policy implication</u>: care for the severe poor first

Foster-Greer-Thorbecke (FGT) Class

The FGT class of measures is defined as

$$\text{FGT}_a = (1/n)\, \Sigma_i\, (g_i^{\ast})^a$$

where a is a parameter and $g_i^{\ast}$ is the normalized income gap of the i^{th} person in $x^{\ast}$

For $a = 0$, FGT is the Headcount Ratio

For $a = 1$, FGT is the Poverty Gap Ratio

For $a = 2$, FGT is the Squared Poverty Gap

Properties of the Alkire Foster (AF) method

- The AF methodology satisfies a number of typical properties of multidimensional poverty measures:

  Symmetry
  Scale invariance
  Normalization
  Replication invariance
  Ordinality
  Poverty Focus
  Deprivation Focus
  Weak Monotonicity
  Weak Deprivation Re-arrangement
  Dimensional Monotonicity
  Decomposability
  Dimensional breakdown.

Intuition of Key Properties

- **Poverty Focus:** Deprivations of non-poor do not matter
- **Deprivation Focus:** Improvements in non-deprived dimensions do not matter (whether poor or non-poor)
- **Monotonicity**: Improvements in deprived dimensions reduce poverty
- **Dimensional Monotonicity:** An additional deprivation for a poor person increases poverty

Thank you ☺

Questions?