Robustness analysis with the AF measures

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- In general robustness analysis seeks to assess the sensitivity of rankings generated by an indicator to changes in the indicator's key parameters.
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- Stochastic dominance conditions provide an extreme form of robustness: if they are fulfilled a comparison is robust to a broad range of parameter values.
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$$H^A(k) \leq H^B(k) \forall k \in [1, D] \iff F^A(c) \geq F^B(c) \forall c \in [0, D]$$
Robustness of rankings generated by the AF measures (including the MPI)

The rankings generated by the AF measures can be sensitive to changes in the measures’ key parameters, namely:

1. The dimension-specific poverty lines (i.e. the "first" cut-off): $z_d$
2. The weights attached to every variable/dimension: $w_d$
3. The value that the weighted sum of deprivations need to surpass in order to identify someone as poor (i.e. the "second" cut-off): $k$
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2. To derive conditions under which a ranking is robust regardless of lines, weights and multidimensional counting thresholds (this is harder but still doable)
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- Some basic (first-order) stochastic dominance conditions for the M0 and H involving multidimensional thresholds, weights and lines.
- Some basic robustness tests for weights.
The counting vector: key ingredient for dominance conditions for M0 and H

For individual $i$ define $D - c_i$, where:
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The counting vector: key ingredient for dominance conditions for M0 and H

For individual $i$ define $D - c_i$, where:

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c_i = \sum_{d=1}^{D} w_d I(x_{id} \leq z_d)
$$

Then consider a distribution of deprivations, $D - c$, in the population that can take values from 0 (poor in every dimension) to $D$ (non-poor in every dimension). A typical cumulative distribution is:
Robustness analysis with the AF measures
Stochastic dominance conditions for H and M0

A typical cumulative distribution of D-c

The non-poors in every dimension

The poor by the intersection approach

The poor by the union approach
The dominance condition over $k$

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\[ F^A(D - c) \leq F^B(D - c) \forall (D - c) \in [0, D] \iff H^A \leq H^B \forall (D - c) \in [0, D] \]

\[ H^A \leq H^B \forall (D - c) \in [0, D] \rightarrow M^A \leq M^B \forall (D - c) \in [0, D] \]
Robustness analysis with the AF measures

Stochastic dominance conditions for $H$ and $M_0$

The key dominance result in a pair of graphs: 1

$M_0(1) = M_0(0) = A(0)$

$= \text{sum of uncensored headcounts}$
The key dominance result in a pair of graphs: II
Proof: Alkire and Foster explained

Notice that $M_0$ can be expressed in terms of $H$ the following way:

$$M_0(k) = \frac{1}{D} [H(D)D + \sum_{j=k}^{D-1} j[H(j) - H(j + 1)]]$$
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Therefore $H^A(k) \leq H^B(k) \forall k \in [1, D] \rightarrow M^A(k) \leq M^B(k) \forall k \in [1, D]$
Robustness analysis with the AF measures

Further dominance results: now incorporating weights and poverty lines

We saw that for $M^A(k) \leq M^B(k) \forall k \in [1, D]$ to hold we need:
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Are there any conditions that ensure that the latter holds, in turn, for any weights and poverty lines?
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Are there any conditions that ensure that the latter holds, in turn, for any weights and poverty lines?

Yes, it is work in progress, but the condition seems to be the following:

$$F^A(x_1, \ldots, x_D) \geq F^B(x_1, \ldots, x_D) \forall (x_1, \ldots, x_D) \in [x_1, \text{min}, x_1, \text{max}] \times \ldots \times [x_D, \text{min}, x_D, \text{max}]$$

$$F^A(x_1, \ldots, x_D) \leq F^B(x_1, \ldots, x_D) \forall (x_1, \ldots, x_D) \in [x_1, \text{min}, x_1, \text{max}] \times \ldots \times [x_D, \text{min}, x_D, \text{max}]$$
Example: Test of dominance across six African countries by Batana
Robustness analysis with the AF measures

Robustness Results

Other ways of testing robustness of ranking

Why we need other ways

- Stochastic Dominance test are useful for pair-by-pair analysis
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- If one country stochastically dominates another country then the result holds for all parameters (all weights and cut-offs)
Robustness analysis with the AF measures

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  - Smaller sample size for extreme values of the cut-off
Framework

- There are $N$ countries
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- Weights for $D$ dimensions are denoted by the vector

$$w = (w_1, w_2, \ldots, w_D)$$
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- Set of first cut-offs for $D$ dimensions is denoted by the vector $z = (z_1, z_2, \ldots, z_D)$

- Second cut-off is denoted by $k \in \{1, 2, \ldots, D\}$
- Let us denote the rank of $N$ countries by the column vector $R = (R_1, R_2, \ldots, R_N)$

We assume $R_1 < R_2 < \ldots < R_N$
Framework

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If $R_n = R'_n$ for all $n = 1, \ldots, N$ then the ranking is completely robust with respect to this alternative specification.

However, if it is not then we need to find a method to check the robustness of ranking.
Checking rank correlation

- One useful method is to check the rank correlation between different sets of ranks
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    - Also called Kendall’s Tau
  - Spearman’s Rank Correlation Method ($\rho$)
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- The total number of concordance pairs \( (C) \)
- The total number of discordant pairs \( (D) \)

\begin{align*}
\text{Concordant pairs} &: \text{A pair } n \text{ and } \bar{n} \text{ is concordant if } R_n > R_{\bar{n}} \text{ and } R'_n > R'_{\bar{n}} \\
\text{Discordant pairs} &: \text{The pair is discordant if } R_n > R_{\bar{n}} \text{ but } R'_n < R'_{\bar{n}} \\
\end{align*}

Then

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\tau = \frac{C - D}{C + D}
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Kendall’s Rank Correlation Method

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Exploring Kendall’s Tau

▶ Kendall’s Tau is the normalized difference between the total concordant and discordant pairs
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- Note that $C + D$ is the total number of comparisons given that there is not tie of ranks
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Robustness analysis with the AF measures

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Other ways of testing robustness of ranking

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- The maximum value that $\tau$ may take is $+1$.
  - Recall the situation when $R_n = R'_n$ for all $n$.
- The minimum value that $\tau$ may take is $-1$.
- When the number of concordant pairs is equal to the number of discordant pairs, then $\tau = 0$. 
Spearman’s Rank Correlation Method

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\rho = 1 - \frac{6 \sum_{n=1}^{N} r_n^2}{n(n^2 - 1)}
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- when $R_n = R'_{N-n}$ for all $n$, then $\rho = -1$
Robustness analysis with the AF measures

Robustness Results

Other ways of testing robustness of ranking

Empirical Illustration

▶ Alkire and Seth (2008) - Spearman’s Rank Correlation Table
Empirical Illustration

- Alkire and Seth (2008) - Spearman’s Rank Correlation Table
- Application on 28 Indian states and nine dimensions using the dataset of National Family Health Survey (NFHS) 2005
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<th>Cut-off (k)</th>
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<th>5</th>
<th>6</th>
<th>7</th>
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### Robustness analysis with the AF measures

- **Robustness Results**
- **Other ways of testing robustness of ranking**

## Empirical Illustration: Alkire ans Santos 2010

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<th>MPI 1 Excluding Enrolment</th>
<th>MPI 2 Using weight-for-age Selected Measure</th>
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<tr>
<td><strong>MPI 2</strong></td>
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<tr>
<td>Using weight-for-age (Selected Measure)</td>
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<tr>
<td>Pearson</td>
<td>0.989</td>
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<tr>
<td>Spearman</td>
<td>0.988</td>
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<tr>
<td>Kendall (Taub)</td>
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<tr>
<td><strong>MPI 3</strong></td>
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<tr>
<td>Using weight-for-height</td>
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<td>0.998</td>
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<td>Using under 5 mortality (rather than age non-specific mortality)</td>
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<td>0.991</td>
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**Number of countries:** 51 (All DHS and three MICS countries which have Birth History)