Q 1 Suppose a composite indicator ($C$) consists of two dimensional achievements $x_1$ and $x_2$, such that

$$C(x; w) = w_1 x_1 + w_2 x_2,$$

where $w_i$ is the weight attached to the $i^{th}$ dimensional achievement, for $i = 1, 2$.

There are two countries, A and B, that needs to be ranked by using the composite indicator $C$. The achievement vectors of country A and country B are denoted by $x^A = (x_1^A, x_2^A) = (0.5, 0.5)$ and $x^B = (x_1^B, x_2^B) = (0.8, 0.3)$, respectively. A committee of advisors determine the weight vector to be $w^0 = (0.6, 0.4)$, but they also realize that they are only 80% confident about it. Answer the following questions.

(i) Which country would perform better if the committee were 100% confident about determining $w^0$ correctly? Would this comparison be robust?

(ii) Draw the set of all weighting vectors in a two-dimensional diagram such that the weights sum up to one.

(iii) Determine the set of all weighting vectors for which A would perform better than B. Depict it in the diagram you have drawn in part (ii).

(iv) Determine the set of all weighting vectors for which B would perform better than A. Depict it in the diagram you have drawn in part (ii).

(v) Find out the set of all reasonable weighting vectors if the committee is only 80% confident about determining $w^0$ correctly. Would the comparison be robust?

(vi) Finally, suppose the committee is only 50% confident about determining $w^0$ correctly. Would the comparison be robust?

Q 2 The achievement vectors of three countries are $x^A = (0.7, 0.7, 0.7, 0.8)$, $x^B = (0.6, 0.65, 0.7, 0.75)$, and $x^C = (0.5, 0.75, 0.7, 0.8)$. Determine which comparisons are totally robust. Which comparisons are robust for $r = 0.25$, given the initial weighting vector is an equally weighted one?