Summer School on Multidimensional Poverty

8–19 July 2013

Institute for International Economic Policy (IIEP)
George Washington University
Washington, DC
Properties of Multidimensional Poverty Measures

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Oxford Poverty & Human Development Initiative (OPHI)
Focus of This Lecture

Discuss the properties that are considered ‘desirable’ for the measurement and understanding of poverty in the multidimensional context
Main Sources of this Lecture

• Bourguignon and Chakravarty (2003): The Measurement of Multidimensional Poverty
• Please see the reading list for others

• Alkire et al. (2013): Multidimensional Poverty: Measurement and Analysis, in progress
Preliminaries

Multiple dimensions
- Standard of living, knowledge, quality of health (referred as ‘achievements’)

Achievements of a society or country can be represented by a matrix or joint distribution

Unit of analysis may be individual or household
Preliminaries

A typical dataset or achievement matrix (with 4 dimensions)

<table>
<thead>
<tr>
<th></th>
<th>Income</th>
<th>Years of Education</th>
<th>Sanitation (Improved?)</th>
<th>Access to Electricity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>x =</strong></td>
<td>700</td>
<td>14</td>
<td>Yes</td>
<td>Yes</td>
<td>Person 1</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>13</td>
<td>Yes</td>
<td>No</td>
<td>Person 2</td>
</tr>
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<td></td>
<td>400</td>
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<td>No</td>
<td>Person 3</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>11</td>
<td>Yes</td>
<td>Yes</td>
<td>Person 4</td>
</tr>
<tr>
<td><strong>z =</strong></td>
<td>500</td>
<td>12</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>

*z is the vector of poverty lines*
Preliminaries

Matrix $x = [x_{ij}]_{n \times d}$ summarizes the joint distribution of ‘d’ attributes across ‘n’ individuals.

Row vector $x_{i \cdot}$ denotes the achievements of person $i$ in all $d$ dimensions.

Column vector $x_{\cdot j}$ denotes the achievements in dimension $d$ of all $n$ persons.

Vector $z = [z_1, ..., z_d]$ be the cut-off vector containing the poverty line of each dimension.
Preliminaries

A general achievement matrix

\( x_{ij} \): the achievement of
individual \( i \) in dimension \( j \)

**Example:**

\( x_{1d} \): the achievement of the 1\(^{st} \) individual in dimension \( d \)

\( x_{n1} \): the achievement of the \( n^{th} \) individual in the first dimension

\[
\begin{pmatrix}
  x_{11} & \ldots & x_{1d} \\
  x_{21} & \ldots & x_{2d} \\
   \vdots & \ddots & \vdots \\
  x_{n1} & \ldots & x_{nd}
\end{pmatrix}
\]

= 

\[
\begin{pmatrix}
  x_{1*} \\
  x_{2*} \\
  \vdots \\
  x_{n*}
\end{pmatrix}
\]
Measurement

Measurement of multidimensional poverty involves two major steps like unidimensional measurement:

- Identification
- Aggregation
First Step: Identification

Identification: *Who is multidimensionally poor?*

An ‘identification function’, $r$, decides who should be multidimensionally poor

$$r(x_i,z) = 1 \text{ if person } i \text{ is multidimensionally poor}$$

$$r(x_i,z) = 0 \text{ if person } i \text{ is not multidimensionally poor}$$

There can be two types of identification Approaches

- Dimension-specific Deprivation Approach (Includes Counting)
- Aggregate Poverty Line Approach
First Step: Identification

Identification: Dimension-specific Deprivation Approach

**First stage**: Determine whether individuals are deprived in each dimension

**Second stage**: Identify if someone is poor based on an identification function (criterion)

**Examples**:
- Union criterion (if deprived in at least one dimension)
- Intersection criterion (if deprived in all dimensions)
- Intermediate criterion
First Step: Identification

*Example:* Constructing first stage ‘Deprivation Matrix’

Replace entries: 1 if deprived, 0 if not deprived

<table>
<thead>
<tr>
<th>Income</th>
<th>Years of Education</th>
<th>Sanitation (Improved?)</th>
<th>Access to Electricity</th>
<th>Person</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>14</td>
<td>Yes</td>
<td>Yes</td>
<td>Person 1</td>
</tr>
<tr>
<td>300</td>
<td>13</td>
<td>Yes</td>
<td>No</td>
<td>Person 2</td>
</tr>
<tr>
<td>400</td>
<td>10</td>
<td>No</td>
<td>No</td>
<td>Person 3</td>
</tr>
<tr>
<td>800</td>
<td>11</td>
<td>Yes</td>
<td>Yes</td>
<td>Person 4</td>
</tr>
</tbody>
</table>

\[ z = \begin{array}{c|c|c|c|c} 
500 & 12 & Yes & Yes & \\
\end{array} \]
First Step: Identification

*Example:* Constructing first stage ‘Deprivation Matrix’

Replace entries: 1 if deprived, 0 if not deprived

<table>
<thead>
<tr>
<th></th>
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<th>Access to Electricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Person 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Person 2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Person 3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Person 4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ g^0 = \]

\[ z = \begin{array}{cccc}
500 & 12 & \text{Yes} & \text{Yes}
\end{array} \]

These entries fall below cutoffs
First Step: Identification

Example: Equivalently ‘Censored Deprivation Matrix’

\[ x_{ij}^* = x_{ij} \text{ if } x_{ij}^* < z_j \text{ and } x_{ij}^* = z_j \text{ if } x_{ij}^* \geq z_j \]

<table>
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</table>

These entries fall below cutoffs

\[ z = \begin{array}{cccc}
500 & 12 & Yes & Yes \\
\end{array} \]
First Step: Identification

Example:

<table>
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<tr>
<th></th>
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<tr>
<td>$g^0 =$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>0</td>
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<td>1</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$z =$  

| 500 | 12 | Yes | Yes |

Union? Intersection?
First Step: Identification

*Example*: Constructing first stage ‘Deprivation Matrix’

Replace entries: 1 if deprived, 0 if not deprived

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<td>0</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>Person 4</td>
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</tbody>
</table>

$z = \begin{bmatrix} 500 \\ 12 \end{bmatrix}$

Union? Intersection?
First Step: Identification

Identification: Aggregate Poverty Line Approach

A person is identified as poor if her aggregate achievement falls below an aggregate poverty line.

Let the aggregation function be denoted by $f$.

Then,

\[ r(x_i, z) = 1 \quad \text{if} \quad f(x_i) < f(z) \]
\[ r(x_i, z) = 0 \quad \text{if} \quad f(x_i) \geq f(z) \]

Example consumer expenditure approach

*Note: No deprivation matrix was created in this situation*
Second Step: Aggregation

Aggregation: *How poor is the society?*

Based on the identification criterion, this step constructs an index of poverty $P(x;z)$ summarizing the information of the poor (*a censored matrix can be created just as in the unidimensional framework*)
Axioms
Axioms in Multidimensional Context

Two types

1. *Natural extensions* of the unidimensional framework.

2. Axioms specific to the multidimensional context
Natural Extensions

*Symmetry (Anonymity):*
Natural Extensions

**Symmetry (Anonymity):** If matrix $y$ is obtained from matrix $x$ by a *permutation* of achievements and the poverty lines remain unchanged, then $P(y;z) = P(x;z)$

$y$ is obtained from $x$ by a *permutation* of incomes if $x = Py$, where $P$ is a permutation matrix.

**Example:**

$$y = Px = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 4 \\ 4 & 4 & 2 \\ 8 & 6 & 3 \end{bmatrix}$$
Natural Extensions

*Replication Invariance (Population Principle):*
Natural Extensions

**Replication Invariance (Population Principle):** If matrix \( y \) is obtained from matrix \( x \) by a *replication* and the poverty lines remain unchanged, then \( P(y;z) = P(x;z) \)

\( y \) is obtained from \( x \) by a *replication* if each person’s achievement vector in \( x \) is simply repeated a finite number of times

**Example:**

\[
\begin{bmatrix}
4 & 4 & 2 \\
3 & 5 & 4 \\
8 & 6 & 3
\end{bmatrix}
= \begin{bmatrix}
4 & 4 & 2 \\
4 & 4 & 2 \\
3 & 5 & 4 \\
3 & 5 & 4 \\
8 & 6 & 3 \\
8 & 6 & 3
\end{bmatrix}
\]
Natural Extensions

*Scale Invariance (Homogeneity of Zero-Degree)*:
Natural Extensions

*Scale Invariance (Homogeneity of Zero-Degree)*: If all achievements in matrix $x$ and all poverty lines in $z$ are changed by the same proportion $a > 0$, then $P(ax;az) = P(x;z)$.

Example:

$$X = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} \quad z = \begin{bmatrix} 4 & 5 & 3 \end{bmatrix}$$

$$\alpha X = \begin{bmatrix} 2(4) & 2(4) & 2(2) \\ 2(3) & 2(5) & 2(4) \\ 2(8) & 2(6) & 2(3) \end{bmatrix} \quad \alpha z = \begin{bmatrix} 2(4) & 2(5) & 2(3) \end{bmatrix}$$

Useful but May be controversial!
Natural Extensions

*Focus:*
Natural Extensions

**Focus:** Unlike in the unidimensional framework, there are two types of focus axiom

*Type I* Focus on those identified as multidimensionally poor’ *(we are not interested in those who are not multidimensionally poor)*

*Type II* Focus on dimensions where multidimensionally poor are deprived *(we are not interested in dimensions in which they are not deprived)*
Natural Extensions

**Poverty Focus (Type I):** If $y$ is obtained from $x$ by an increment to a non-poor person’s achievements and the poverty lines remain unchanged, then $P(y;z) = P(x;z)$

**Example:** \[ x = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 4 \end{bmatrix}, \quad z = (5,6,4), \quad \text{and} \quad g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

Person 3 is not multidimensionally poor, does it matter if he/she experiences an increase in any of the dimensions?
Natural Extensions

Deprivation Focus (Type II): If y is obtained from x by an increment in achievements among the non-deprived, then $P(X;z)=P(Y;z)$. [Recall Deprived vs. Poor]

Example: $x = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 4 \end{bmatrix}$, $z = (5,6,4)$, and $g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Suppose person 2 is considered multidimensionally poor, does it matter if he/she experiences an increment in the third dimension in which he/she is not deprived?
Natural Extensions

Focus Axioms and Types of Identification

Each of the two focus axioms is attributed to a each identification technique introduced earlier

– Poverty focus is attributed to the Aggregated Poverty Line Approach

– Deprivation focus is attributed to the Dimension-specific Deprivation Approach
Natural Extensions

**Continuity:** For any sequence $x$, if $x'$ converges to $x$, then $P(x';z)$ converges to $P(x;z)$

A technical assumption. It prevents poverty measures from changing abruptly for changes in distribution of achievements.

Similar intuitive interpretation as the assumption in single dimensional framework.
Natural Extensions

Monotonicity:
Natural Extensions

**Monotonicity:** If $y$ is obtained from $x$ by a *deprived increment* among the poor and the poverty line remains unchanged, then $P(y,z) < P(x,z)$

$y$ is obtained from $x$ by a *deprived increment* if there is an increment in a deprived achievement of a multidimensionally poor

\[
\begin{bmatrix}
4 & 4 & 2 \\
3 & 5 & 4 \\
8 & 6 & 3
\end{bmatrix}
, z = (5, 6, 4),
\begin{bmatrix}
4 & 4 & 3 \\
3 & 5 & 4 \\
8 & 6 & 3
\end{bmatrix}
\]

*Example:* $x = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix}$, $z = (5, 6, 4)$, $y = \begin{bmatrix} 4 & 4 & 3 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix}$

Person 1 is multidimensionally poor, and experiences an improvement in the *third dimension.*
Suppose the population size of $x$ is denoted by $n(x)$. Matrix $x$ is divided into two population subgroups: $x'$ with population size $n(x')$ and $x''$ with population size $n(x'')$ such that $n(x) = n(x') + n(x'')$.

Income  Education  Health

$x =$

\[
\begin{bmatrix}
4 & 4 & 2 \\
3 & 5 & 4 \\
8 & 6 & 3
\end{bmatrix}
\]

Person 1  Person 2  Person 3
Natural Extensions

Population Subgroup Consistency:

Population Subgroup Decomposability:
Natural Extensions

Population Subgroup Consistency: If \( P(y';z) > P(x';z) \) and \( P(y'';z) = P(x'';z) \), and \( n(x') = n(y') \), \( n(y'') = n(x'') \), then \( P(y;z) > P(x;z) \)

Population Subgroup Decomposability:
Natural Extensions

**Population Subgroup Consistency:** If \( P(y';z) > P(x';z) \) and \( P(y'';z) = P(x'';z) \), and \( n(x') = n(y') \), \( n(y'') = n(x'') \), then \( P(y;z) > P(x;z) \)

**Population Subgroup Decomposability:** A poverty measure is additive decomposable if:

\[
P(x) = \frac{n(x')}{n} P(x') + \frac{n(x'')}{n} P(x'')
\]

Recall: *decomposability implies subgroup consistency, but the converse does not hold*
Natural Extensions

Transfer in unidimensional context:
Natural Extensions

**Transfer in unidimensional context:** If y is obtained from x by a progressive transfer among the poor, then $P(y;z) < P(x;z)$

**Recall** if income is transferred from a person to another who is not richer than the former, keeping mean income same, the transfer is called a *progressive transfer*

This is also known as *Pigou-Dalton* transfer principle

**Example:** $z = 10$, $x = (9,4,15,8)$; $y = (9,5,15,7)$
Natural Extensions

Transfer in multidimensional context:

**Bistochastic matrix (B):** A matrix whose row elements and column element sum up to one

\[
\begin{bmatrix}
0.5 & 0.3 & 0.2 \\
0.4 & 0.3 & 0.3 \\
0.1 & 0.4 & 0.5
\end{bmatrix}
\]

*Example:* A general bistochastic matrix

Multiply a vector by a bistochastic matrix

\[
\begin{bmatrix}
0.5 & 0.3 & 0.2 \\
0.4 & 0.3 & 0.3 \\
0.1 & 0.4 & 0.5
\end{bmatrix}
\begin{bmatrix}
4 \\
8 \\
16
\end{bmatrix}
= 
\begin{bmatrix}
7.6 \\
8.8 \\
11.6
\end{bmatrix}
\]
Transfer in multidimensional context:

Bistochastic matrix (B): A matrix whose row elements and column elements sum up to one.

Example: What bistochastic matrix is used to obtain \( y = (9,5,15,7) \) from \( x = (9,4,15,8) \)?

\[
B = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0.75 & 0 & 0.25 \\
0 & 0 & 1 & 0 \\
0 & 0.25 & 0 & 0.75
\end{bmatrix}
\]
Uniform Majorization (UM): Matrix $y$ is obtained from $x$ by a Uniform Majorization among the poor (an averaging of achievements among the poor) if $y = Bx$, where $B$ is an $n \times n$ bistochastic matrix but not a permutation matrix, and $b_{ii}=1$ for every non-poor person $i$ in $Y$.

$$X = BY = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 3.5 & 4.5 & 3 \\ 3.5 & 4.5 & 3 \\ 8 & 6 & 3 \end{bmatrix}, \text{ and } z = [5 \ 6 \ 5]$$

Achievements of the first two persons (poor) were smoothed
Transfer Under UM: If $y$ is obtained from $x$ by a uniform majorization among the poor (an averaging of achievements among the poor), then $P(y;z) \leq P(x;z)$. 
Axioms Specific to the Multidimensional Context
Axioms Specific to the Multidimensional Context

\[
x = \begin{bmatrix}
4 & 4 & 2 \\
3 & 5 & 4 \\
8 & 6 & 3
\end{bmatrix}
\]

Income Education Health

Person 1
Person 2
Person 3

Dimensional Breakdown

It is a purely multidimensional concept, where the overall poverty can be expressed as an weighted average of dimensional deprivations of the poor.
Axioms Specific to the Multidimensional Context

**Dimensional Breakdown**

Formally, let $P_j(x_j; z)$ summarizes the post-identification (r) deprivation profile of all poor in dimension $j$.

Then, $P(x; z) = w_1P_1(x_1; z) + \cdots + w_dP_d(x_d; z)$

where $w_j$ is the weight (normalized) assigned to dimension $j$.

For *union criterion*, it is referred as factor decomposability by Chakravarty, Mukherjee, and Ranade (1998).

$$P_j(x_j; z) = P_j(x_j; z_j)$$
**Axioms Specific to the Multidimensional Context**

**Rearrangements**

<table>
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<th>Income</th>
<th>Education</th>
<th>Health</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

Person 1

<table>
<thead>
<tr>
<th>Income</th>
<th>Education</th>
<th>Health</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

Person 2

<table>
<thead>
<tr>
<th>Income</th>
<th>Education</th>
<th>Health</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Person 3

Is the pattern of poverty same in both societies? If not, what is the difference?
Axioms Specific to the Multidimensional Context

Both matrices have the same distribution for each dimension (*marginal distribution*)

The correlation between dimensions are not same

Require an axiom based on *correlation/association* between dimension when marginals are same (Atkinson & Bourguignon, 1982; Boland & Proschan, 1988).

This axiom is intrinsic to the multivariate case
Axioms Specific to the Multidimensional Context

\[
x = \begin{bmatrix} 7 & 7 & 2 \\ 3 & 3 & 8 \\ 10 & 10 & 12 \end{bmatrix} \quad y = \begin{bmatrix} 7 & 7 & 8 \\ 3 & 3 & 2 \\ 10 & 10 & 12 \end{bmatrix}
\]

Ways to call the data transformation:

From x to y:
- association increasing rearrangement
- correlation-increasing transfer
- correlation increasing increasing switch

From y to x:
- association decreasing rearrangement
Axioms Specific to the Multidimensional Context

Question…

How do you think poverty should change under an association decreasing rearrangement?
Axioms Specific to the Multidimensional Context

If you think that good health can substitute (compensate) for bad income or bad education, then poverty should decrease.

If you think that good health is necessary (complementary) to achieve good income and good education, then poverty should increase.

If you think that health is not necessary to achieve good income and good education, and can not either substitute for any of these, (i.e., you think they are independent), then poverty should not change.

Bourguignon and Chakravarty (2003)
Axioms Specific to the Multidimensional Context

**Decreasing in Association Decreasing Rearrangement**: If an achievement matrix $x'$ is obtained from another achievement matrix $x$ by an association decreasing rearrangement among the poor, then $P(x';z) < P(x;z)$. 

[Achievements are assumed to be substitutes]

**Increasing in Association Decreasing Rearrangement**: If an achievement matrix $x'$ is obtained from another achievement matrix $x$ by an association decreasing rearrangement among the poor, then $P(x';z) > P(x;z)$. 

[Achievements are assumed to be complements]

There are weak versions as well
Axioms Specific to the Multidimensional Context

**Dimensional Monotonicity:** If y is obtained from x by a dimensional increment among the poor, then \(P(y,z) < P(x,z)\).

y is obtained from x by a dimensional increment among the poor if due to an increment in a deprived achievement of a poor, he or she becomes non-deprived in that dimension.

**Example:**

\[
\begin{bmatrix}
4 & 4 & 2 \\
3 & 5 & 4 \\
8 & 6 & 3 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
4 & 4 & 2 \\
3 & 6 & 4 \\
8 & 6 & 3 \\
\end{bmatrix}
\]

Suppose person 2 is considered multidimensionally poor, and experiences an increment in the second dimension and is no longer deprived in it.
Axioms Specific to the Multidimensional Context

*Dimensional Monotonicity:*

Why important?

Ordinality vs. cardinality