Qualitative Response

Examples:
- Poor or not
- Catastrophic health expenditures or not
- Employed or not

Model:
\[ y = \begin{cases} 
1 & \text{if poor} \\
0 & \text{otherwise} 
\end{cases} \]

Postulate existence of an underlying latent variable \( y^* \), say vulnerability, such that
\[ y^* = x'\beta + \varepsilon \]

Then, \( y = 1 \iff y^* > \text{threshold, say } c \)
\[ P(y = 1) = P(y^* > c) = P(x'\beta + \varepsilon > c) = P(\varepsilon > c - x'\beta) = F(c + x'\beta) \]
and
\[ P(y = 0) = 1 - F(c + x'\beta) \]

Estimation by Maximum Likelihood:
\[ \max_{\beta} \log \text{Likelihood} = \sum_{i \in \text{poor}} \log F(c + x_i'\beta) + \sum_{i \notin \text{poor}} \log [1 - F(c + x_i'\beta)] \]
Qualitative Response (cont'd.)

Marginal Effects:

\[ \frac{\partial P(y=1)}{\partial x_k} = \frac{\partial F(c+x'\beta)}{\partial x_k} = \frac{\partial F}{\partial (c+x'\beta)} \cdot \frac{\partial (c+x'\beta)}{\partial x_k} \]

\[ = f(-c+x'\beta) \cdot \beta_k \]

Choice of F:

- F = Normal Distribution \rightarrow \text{Probit Model}
- F = Logistic Distribution \rightarrow \text{Logit Model}

Remark

For identification purposes we set \( c = 0 \), \( V(u_i) = 1 \).

Extensions:

- Multinomial:
  More than 2 outcomes, say \( m \) outcomes
  No hierarchy in response/outcome

- Ordered Categorical:
  More than 2 outcomes, say \( m \) outcomes
  There is a hierarchy in the outcomes/responses.