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## Measuring Human Development and Human Deprivations

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### Abstract

This paper is devoted to the discussion of the measurement of human development and poverty, especially in United Nations Development Program's global Human Development Reports. We first outline the methodological evolution of different indices over the last two decades, focusing on the well-known Human Development Index (HDI) and the poverty indices. We then critically evaluate these measures and discuss possible improvements that could be made.

**Keywords:** Human Development Report, Measurement of Human Development, Inequality-adjusted Human Development Index, Measurement of Multidimensional Poverty

**JEL classification:** O15, D63, I3

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## Contents

1. Introduction.....	2
2. Indices in the Human Development Reports 1990–2009 .....	4
2.1. The index for measuring human development .....	4
2.2. Indices for measuring human deprivations .....	5
2.3. A critical evaluation of the pre-2010 indices .....	7
3. Indices in the Human Development Report 2010 and Onwards .....	10
3.1. The Human Development Index .....	10
3.2. Adjusting inequality in human development .....	13
3.3. A new index for measuring human deprivations .....	14
4. A Critical Evaluation of the 2010 Indices .....	17
4.1. The Human Development Index .....	17
4.1.1. Choice of weights .....	18
4.1.2. Choice of variables .....	19
4.1.3. Modelling flaws.....	22
4.2. The Inequality-adjusted Human Development Index.....	23
4.3. The Multidimensional Poverty Index.....	25
5. Concluding Remarks.....	26

Developing better measures is not an end in itself but a means to enhance policies that improve people's lives.

– Ángel Gurría, OECD Secretary-General

## 1. Introduction

Gross Domestic Product (GDP) has become the yardstick to evaluate the overall economic performance or even the social wellbeing of a country or a region. The underlying assumption is that the wellbeing of an individual depends on the expenditure capacity of that person so that disposable income can be interpreted as a summary measure of her consumption opportunities. As the GDP is the market value of all new goods and services produced and provided in a given region during a year – or, equivalently, the total income of all individuals in the region in that year – it can be regarded as the macro counterpart of individual incomes or a measure of social wellbeing.

There are many problems associated with the use of GDP as a measure of social wellbeing and the use of consumption expenditure as a measure of individual wellbeing. The problems associated with the use of GDP are clear: only market transactions are considered; quality is not computed; distributive aspects are ignored; and stocks or durable goods and infrastructures are practically out of the picture. GDP also leaves out activities of the informal sector that can be significantly large in developing countries, and public sector activities are valued at cost due to the lack of markets and prices (Spence 2009). The use of consumption expenditure as a measure of individual wellbeing is also far from unproblematic because it may leave out many factors – such as the quality of health or the value of knowledge – that are crucial for human flourishing but cannot be measured due to the lack of market prices. For a more detailed discussion of the flawed assumptions behind using per capita GDP as a measure of development, see Alkire and Deneulin (2009). Yet GDP and related indicators are used primarily because we have not yet found a better alternative that is so generally accepted.

GDP certainly captures a relevant part of the economic performance of a society, but it is far from being a complete measure of economic development and certainly further from being a sufficient measure of human development and social wellbeing. In fact, it is improbable that any single indicator can capture human development or social wellbeing, which is multifaceted by nature. This rather requires a multidimensional approach, which was recognised soon after GDP became a standard and has been discussed ever since.<sup>1</sup> Like human development, human

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<sup>1</sup> For example, the United Nations 1954 report on the standards of living, the “basic needs approach” fostered by

deprivation or poverty is also usually understood in terms of deprivation in income or consumption expenditure. However, as Ruggieri, Saith, and Stewart (2003) showed in case of India, non-deprivation in income does not necessarily mean non-deprivations in health and education. In fact, human deprivation or poverty, like human development, is also multifaceted and requires a multidimensional approach. A number of indices based on multiple dimensions have been developed in many areas of research, especially regarding inequality, poverty, subjective wellbeing, education, or health, to name a few. Some of these indices are composite indices and others are more sophisticated multidimensional indices. The relevance of this approach led the OECD to issue a manual on the construction of composite indices (Nardo et al. 2008).

Note that moving from one to several dimensions, when approaching the development of a society, creates a number of difficult issues that call for agreement and compromise. The key points are: (i) Which are the most relevant dimensions to be considered? (ii) How can we approximate those dimensions by means of specific variables whose data are available? (iii) How should those variables be aggregated into a single index in order to get a systematic evaluation criterion? The difficulty of tackling all those issues explains a good deal about the persistence of GDP as the main index for economic growth and development. In spite of the different proposals put forward, no general agreement was reached on the adoption of a new standard, at least until the launching of the Human Development Index in 1990.

Since its inception, the Human Development Index (HDI) has become the most successful index to use multiple dimensions that address economic development and social wellbeing. Besides the HDI, the United Nations Development Programme (UNDP) in subsequent Human Development Reports has introduced several other indices, of which the more well-known ones are the Human Poverty Index (HPI) for measuring poverty, the Gender-Related Development Index (GDI) for capturing inequality in human development across gender, and the Gender Empowerment Index (GEM) for measuring women's empowerment. Each of these measures has evolved over time in terms of the selection of indicators and methodology. In this paper, we shall restrict our analysis to the measures of human development and human deprivation or poverty, leaving aside the discussion of indices developed for other purposes.

The first Human Development Report (HDR) was launched in 1990 and since then global

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the International Labour Organization in 1974, the Physical Quality of Life Index (PQLI) put forth by Morris (1979) and reformulated by Ram (1982), or the proposals of the Daj Hammarskjöld Foundation (Max-Neef 1984). For more recent critiques, see Boarini et al. (2006), Stiglitz et al. (2009), or Fleurbaey (2009).

HDRs have been produced yearly. Indices in the HDR proposed by the United Nations have mostly been applications of Amartya Sen's idea of functionings and capabilities (Sen 1985). These indices have been used frequently to measure the development and poverty of nations. Many countries have also produced national Human Development Reports, whose indices contain sub-national level information. The indices in these reports gone through several changes over time. We divide this timeline into two segments: pre-2010 and post-2010, because in the 2010 Human Development Report titled *The Real Wealth of Nations: Pathways to Human Development*, all indices have gone through significant amendments.

## 2. Indices in the Human Development Reports 1990–2009

The global HDRs have introduced various indices of human development, poverty, gender inequality, gender empowerment, and a few others. In this paper, we focus on the indices of human development and poverty.

### 2.1. The index for measuring human development

The HDI was introduced in the first HDR in 1990. The HDI soon became popular, and each new edition had a large impact in the mass media because of its intuitive character and the large number of countries that entered the evaluation. This approach to measuring human development identified health, knowledge, and material wellbeing as the key dimensions for social and economic development. Achievement in health was measured by the indicator life expectancy at birth ( $H$ ), which is the number of years that a newborn is expected to live, according to the actual pattern of mortality rates within each country. Knowledge, understood as educational achievements, was approximated by a composite indicator: a mixture of literacy rate ( $E_1$ ) and gross enrolment rate ( $E_2$ ) (with weights of 2/3 and 1/3, respectively). Finally, material wellbeing was associated with the logarithm of the per capita GDP ( $Y$ ).

Each of these four indicators was normalized with respect to a maximum and a minimum possible performance. This was essential in order to make the performance across indicators comparable. The maximum possible performances for these four indicators were denoted by  $H^{max}$ ,  $E_1^{max}$ ,  $E_2^{max}$ , and  $Y^{max}$ , and the minimum possible performances were denoted by  $H^{min}$ ,  $E_1^{min}$ ,  $E_2^{min}$ , and  $Y^{min}$ , respectively. Each of the four indicators was normalized as  $i_N = (i - i^{min}) / (i^{max} - i^{min})$  for  $i = H, E_1$ , and  $E_2$  and  $Y_N = (\ln Y^{max} - \ln Y) / (\ln Y^{max} - \ln Y^{min})$  for material wellbeing. The Human Development Index was defined as the arithmetic

mean of the normalized values of those three dimensions and is expressed as

$$HDI = \frac{1}{3} \left( H_N + \frac{2}{3} E_{1N} + \frac{1}{3} E_{2N} + Y_N \right) = \frac{1}{3} (H_N + E_N + Y_N).$$

Although the life expectancy indicator has been used consistently to assess the health dimension, different indicators have been used to measure the knowledge dimension and different transformations of the same indicator have been used to gauge the material wellbeing dimension over time. In the first HDR, the knowledge dimension was assessed only by the adult literacy rate. From the second HDR onwards, the knowledge dimension was assessed by both the adult literacy rate and mean years of schooling. In the 1995 HDR, the mean years of schooling indicator was replaced by the combined enrolment ratio indicator. This pair of indicators was used until 2009. The wellbeing indicator, on the other hand, has been assessed throughout by per capita GDP, but with different transformations. In the first HDR, the logarithmic transformation was used, but in consecutive reports an equally distributed equivalent transformation (based on Atkinson 1970) was used – until the 1999 HDR when the transformation was switched back to the logarithmic scale following the suggestions of Anand and Sen (2000).

An index of human development, however, measures the progress of an entire society. The index ensures that any overall progress in human development is supported by an increase in its value and any decline is evaluated by a decrease. An index of human development nevertheless ignores the underlying causes of progress or decline. Progress may occur with continual improvements in the lives of those already enjoying high levels of human development, while neglecting the lives of those actually needing improvement. In other words, progress may take place despite a large section of the population remaining deprived of basic needs, capabilities, and public services. The pursuit for progress in human development remains incomplete until existing deprivations in the population are successfully eradicated.

## 2.2. Indices for measuring human deprivations

Although the first attempt to assess deprivations using a poverty index was made in the 1996 Human Development Report (HDR), the first four HDRs presented the HDI as a complement of the country's deprivation. A country's deprivation in three dimensions was understood as a shortfall in that country's performance from the best possible performance in that dimension. A deprivation score was assigned to each of the four indicators as  $D_i = (i^{max} - i)/(i^{max} - i^{min})$  for  $i = H, E_1,$  and  $E_2,$  and  $D_Y = (\ln Y^{max} - \ln Y)/(\ln Y^{max} - \ln Y^{min})$ . Then the overall

deprivation score was obtained as  $D = (D_H + D_E + D_Y)/3$ . The HDI was the complement of the overall deprivation score, such that  $HDI = (1 - D)$ . It is straightforward to verify that this formulation is equivalent to the traditional HDI formulation – the simple average of performances in three dimensions. Thus, in the early Human Development Reports, an effort was made to link the HDI to the concept of deprivation albeit at the country level.

However, deprivation at the country level may not necessarily be sensitive to deprivations at the individual level within countries. The HDI, even when presented as the complement of country-level deprivation may not be sensitive to individual deprivations. Anand and Sen (1997) refer to measuring human development as a ‘conglomerate approach’ and measuring poverty as a ‘deprivation approach’. A poverty index, unlike an index of development, is solely focused on those who fail to meet the deprivation cut-off. Every poverty index is supposed to satisfy the ‘focus axiom’, which requires that the poverty index should not be sensitive to the performance of those who are non-deprived or non-poor.<sup>2</sup>

Like human development, human deprivation is also multidimensional. Reducing deprivation in one dimension – such as income – may not necessarily translate to the reduction of deprivations in other dimensions. The earliest attempt to introduce a poverty index – referred to as the Capability Poverty Measure (CPM) – was made in the 1996 Human Development Report. The CPM was a composite index or a simple average of the basic capability shortfalls in three dimensions: living a healthy and well-nourished life, having the capability of safe and healthy reproduction, and being literate and knowledgeable. The corresponding indicators were the percentage of children under five years who were underweight, the percentage of births not attended by trained health personnel, and the percentage of women aged 15 years and above who were illiterate. Note that the three chosen indicators did not capture the deprivations of the entire population, only deprivations among women and children.

In the 1997 HDR, two different poverty indices were introduced: one for the developing countries, referred to as HPI-1, and another for the industrialized countries, referred to as HPI-2. The HPI-1 consisted of three dimensions (like the HDI): (i) a long and healthy life, (ii) knowledge, and (iii) a decent standard of living. Deprivation in the long and healthy life dimension was measured by the percentage of people not expected to survive to the age of forty

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<sup>2</sup> We discuss in section 2.3 that the terms ‘deprived’ and ‘poor’ are not synonymous when multiple dimensions are involved in the construction of a poverty index.



( $P_1$ ). Deprivation in the knowledge dimension was assessed by the percentage of adults who were illiterate ( $P_2$ ). Finally, deprivation in the standard of living dimension was an average of deprivations in three indicators: the percentage of people without access to safe water ( $P_{31}$ ), the percentage of people without access to health services ( $P_{32}$ ), and the percentage of moderately and severely underweight children under the age of five years ( $P_{33}$ ). Thus, the third dimension – a decent standard of living – was measured as  $P_3 = (P_{31} + P_{32} + P_{33})/3$ . However, given the lack of frequent data on access to health services, from the 2001 HDR onwards, this third dimension has been measured by the average of the first and the third indicators only, such that  $P_3 = (P_{31} + P_{33})/2$ . The HPI-1 was a composite index of the three dimensions using the well-known formulation of the general mean of order three and can be expressed as  $HPI-1 = [(P_1^3 + P_2^3 + P_3^3)/3]^{1/3}$ .

The choice of indicators for the HPI-1 was, however, not suitable for the much richer industrialized countries because there would not be any deprivation in any of these indicators. An alternative index consisting of four dimensions, referred to as HPI-2, was developed for the industrialized countries. The first dimension related to the survival of citizens to a relatively early age, as measured by the percentage of people not expected to survive to the age of sixty years ( $P_1$ ). The second dimension was knowledge, which was assessed by the percentage of people who were functionally illiterate as defined by the OECD ( $P_2$ ). The third related to a decent standard of living, measured by the percentage of people living below the income poverty line, which was 50 percent of the median disposable household income ( $P_3$ ). The final and fourth related to non-participation or exclusion as gauged by the rate of long-term (12 months or more) unemployment of the labour force ( $P_4$ ). The HPI-2 also used the generalized mean to formulate the HPI-2 such that  $HPI-2 = [(P_1^3 + P_2^3 + P_3^3 + P_4^3)/4]^{1/3}$ .

### 2.3. A critical evaluation of the pre-2010 indices

Certainly, these additional indicators were novel and represented progress towards a more comprehensive measure of development and poverty. Yet, these indices of human development and poverty have also received many criticisms.<sup>3</sup> Let us first discuss the criticisms attributed to the HDI. The main ones refer to

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<sup>3</sup> We follow here Herrero, Martínez and Villar (2010b). See also the contributions in Anand and Sen (1994 a, b), Hicks (1997), Sagar and Najam (1999), Osberg and Sharpe (2002), Philipson and Soares (2001), Pinilla and Goerlich (2003), Foster, López-Calva and Székely (2005), Becker, Philipson and Soares (2005), Stiglitz, Sen and Fitoussi (2009), Seth (2009), or Herrero, Soler and Villar (2010).

- (a) *The nature of the selected dimensions:* Some relevant aspects of human development were missing, such as social integration and sustainability.
- (b) *The choice of indicators:* Even though the choice of indicators was significantly affected by the availability of data, it was not clear that the indicators used for approximating health, education, and material wellbeing were the most sensible ones. Moreover, the nature of the three variables involved made the interpretation of the HDI as a summary statistic of a representative agent difficult.
- (c) *The absence of time-consistent data:* Due to frequent data revisions of indicators between subsequent years, inter-temporal comparisons using HDI became difficult.
- (d) *The lack of concern for distributive issues:* It is only natural to think that the measurement of human development should compute not only “the size of the cake,” but also the way in which it is distributed.
- (e) *The additive structure of the index:* Aggregating different components by the arithmetic mean had strong implications on their substitutability (linear indifference curves) and makes the index dependent on the normalization methods applied to different indicators.
- (f) *The lack of theoretical justification of the formula.* This makes it difficult to analyse the suitability of this index vis-à-vis other alternatives. Moreover, it induces the use of the HDI as an ordinal measure (a criterion to produce a ranking) and not as a cardinal measure that would help in evaluating the size of the differences between countries.

Like the HDI, the pre-2010 poverty indices could also be subject to criticism. Let us start with the Capability Poverty Measure or CPM, which faced two major criticisms. The first related to the selection of indicators. The three chosen indicators did not capture the deprivations of the entire population – only deprivations among women and children. Indeed, women and children should receive particular attention in any poverty eradication policy, but a poverty index for a country should not be restricted to a particular section of the population. The second criticism was due to the particular functional form used to aggregate and obtain the composite poverty index. Like the HDI, the CPM used the arithmetic mean, ensuring that any increase in the deprivation in one dimension could be compensated by an equal-sized reduction in another dimension.

The HPIs, however, had one methodological improvement over the CPM in that the HPIs used a different order of general mean for aggregation rather than the arithmetic mean. The general mean of order  $a \geq 1$  of any  $n$  real values  $x_1, \dots, x_n$  is defined as  $([x_1^a + \dots + x_n^a]/n)^{1/a}$ . The arithmetic mean is also a general mean with order  $a = 1$  and is equal to  $(x_1 + \dots + x_n)/n$ . As

the value of parameter  $\alpha$  increases, more emphasis is given to the larger values. In the HPI formulation, the use of a higher order of  $\alpha$  places more emphasis on the larger deprivations. This ensures that an increase in deprivation in one indicator that has a relatively larger deprivation should be compensated by a much larger improvement in another indicator with a relatively lower deprivation. Also, a more equal distribution of deprivations across indicators is rewarded.

Another improvement appears to be in the selection of indicators. Unlike in the CPM, the indicators in the HPIs were not biased towards a particular section of the population – such as women and children; they captured deprivations across a wider range of population. Different indicators were, however, still based on different set of population. Consider the HPI-1 for example. The indicator for a long and healthy life was based on the living population, but was also affected by the number of people who died. The indicator for knowledge captured deprivations among the adult population only. Finally, one indicator for standard of living captured deprivations among the entire population while the other captured deprivations only among children. Because the population sets were different across indicators, it was not possible to capture the multiplicity or the extent of deprivations for a particular group of people. In other words, the HPIs were not useful for understanding who within a country was more or less poor. This criticism actually could also be partially attributed to the use of composite indices for measuring poverty in general.

A composite index is built by first obtaining a comprehensive deprivation score for each indicator across the population and then aggregating these comprehensive deprivation scores to obtain the index. We can clearly see that, like the HDI, for the CPM and the HPIs, the data for different indicators were collected from different sources and for different population subgroups. For any multidimensional index of poverty, however, information on all indicators ought to be collected from the same dataset so that the information on each indicator is available for each person or each household. Thus, the construction of a multidimensional poverty index involves two stages: identification of those who are poor and aggregation of the deprivation information of the poor to obtain the overall index. For composite indices of poverty, there is no difference between the terms ‘deprived’ and ‘poor’ because people are separately identified as poor in each indicator in order to obtain the comprehensive deprivation score for that indicator. In multidimensional poverty analysis, however, the terms ‘deprived’ and ‘poor’ have a clear distinction. A person is considered deprived in an indicator if the person fails to meet the threshold in that indicator. By being deprived a person may not necessarily be considered poor

though. It is the identification function based on the joint deprivations that identifies a person as poor or non-poor (See example in section 3.3).<sup>4</sup>

One clear distinction between composite indices of poverty and multidimensional indices of poverty is, thus, the consideration of joint deprivations at the identification stage. The second major difference is that a multidimensional index requires the information on all indicators to be available from the same dataset whereas a composite index may be constructed by collecting information from different sources. This second difference may make a composite index appear more flexible and the multidimensional index more demanding. However, if it is feasible to capture joint deprivations then the ability to capture them may outweigh the flexibility of composite indices. Certainly, the construction of the CPM and HPIs were innovative at a time when the measurement of poverty was dominated by the income approach – such as \$1 a day and \$2 a day – but they were merely composite indices and fell short of being truly multidimensional indexes of poverty.

### 3. Indices in the Human Development Report 2010 and Onwards

The twentieth anniversary of the HDR was taken as the right occasion to refurbish these indices, after launching an open discussion among specialists concerning possible improvements.<sup>5</sup> As a result, some substantial changes were introduced in the design of the HDI, a complementary index known as the Inequality-adjusted HDI (IHDI) was introduced, and the HPI-1 was replaced by a completely new index of poverty – the Multidimensional Poverty Index (MPI). Let us first discuss the modifications in the HDI and then outline the MPI.

#### 3.1. The Human Development Index

The 2010 Human Development Index is a more solid construct than its predecessor, even though it keeps most of the essential traits of the traditional HDI. In particular, (i) it maintains the three-dimensional nature of the index; (ii) continues to consider health, education, and material wellbeing to be the only key dimensions to evaluate human development; (iii) holds the equal-weight assumption for those variables; (iv) keeps the normalization convention already adopted and the evaluation of material wellbeing in terms of logs, and (v) recurs to a mean in

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<sup>4</sup> See however the discussion in Villar (2013).

<sup>5</sup> See the research papers 2010 series of United Nations Development Program and, particularly, the contributions by Alkire and Foster (2011), Alkire and Foster (2010), Herrero, Martínez and Villar (2010b), Kovacevic (2010), Alkire and Santos (2010).

order to aggregate the normalized variables into a single number. There are, however, three major modifications in the 2010 Human Development Index that improve its analytical power. First, the indicators for measuring the achievements in material wellbeing and education were replaced. Table 3.1 presents the pre-2010 and 2010 HDI indicators for the three dimensions. Second, a time-consistent series for each indicator, using 1980 as a starting point, was developed, allowing systematic inter-temporal comparisons to be made. Third, instead of using the arithmetic mean, achievements in the three dimensions are aggregated using the geometric mean, thus adopting the following formula:

$$HDI_{2010} = \left( \frac{H - H^{min}}{H^{max} - H^{min}} \times \frac{E - E^{min}}{E^{max} - E^{min}} \times \frac{\ln Y - \ln Y^{min}}{\ln Y^{max} - \ln Y^{min}} \right)^{1/3};$$

where  $H$ ,  $E$ , and  $Y$  are the indicators measuring achievements in health, knowledge, and material wellbeing dimensions, respectively, and the minimum and maximum goalposts are used in order to normalize each variable within the  $[0, 1]$  interval. Let us now devote some time to conducting a more in-depth analysis of the improvements to the 2010 HDI over the pre-2010 HDI.

**Table 3.1: The Dimensions and Indicators of the Old and New HDI**

Dimensions	Indicators	
	Pre-2010 HDI	2010 HDI
Health ( $H$ )	Life expectancy at birth	Life expectancy at birth
Knowledge ( $E$ )	Adult Literacy Rate	Mean Years of Schooling
	Gross Enrolment Ratio	Expected Years of Schooling
Material Wellbeing ( $Y$ )	GDP Per Capita (PPP USD)	GNI Per Capita (PPP USD)

The first significant modification in the 2010 HDI is the amendment of the indicators. Indeed, the choice of the indicators that approximate the achievements in the three selected dimensions is a key element of the construction of the index. Life expectancy at birth was kept as the indicator for assessing the health dimension, so there is no novelty regarding this dimension. The normalization of this indicator is obtained by taking  $H^{max} = 83.2$  and  $H^{min} = 20$ . The 2010 version of the HDI, however, measures material wellbeing in terms of the logarithmic transformation of per capita Gross National Income rather than that of the per capita GDP. This entails taking into account the incomes of nationals living abroad and the proceeds of firms operating in other countries. This is a minor improvement in the design of the index. The normalization of this indicator is obtained by taking  $Y^{max} = \$108,211$  and  $Y^{min} = \$163$ .

The change in the variable that measures educational achievements was a major one and was really needed. The excessive weight given to the literacy rate in the traditional HDI made it unsuitable for capturing differences in human capital, particularly in developed countries. Among the several alternatives for measuring educational achievements, the 2010 HDI selected yet another composite variable: the geometric mean of “mean years of education” (adults) and “expected years of schooling” (children), suitably normalized. Getting the normalized variable for education requires first normalizing each partial index and then taking the square root of its product. To normalize mean years of education the max value is set equal to 13.2 years and to normalize the expected years of schooling the max value is set equal to 20.6 years, whereas the minimum goalpost is set equal to zero in both cases. The resulting value is normalized again with  $E^{max} = 0.951$  and  $E^{min} = 0$ .

The second major improvement in the 2010 HDI is the reconstruction of time-consistent values for the HDI according to the new method. This allows comparing the evolution of this index and yields interesting results on the dynamics of the different countries.

The third major improvement is the use of geometric mean as an aggregator rather than the arithmetic mean. Using the arithmetic mean to aggregate achievements in the three dimensions into a real-valued indicator has a number of drawbacks, despite the appeal of its intuitive character. The arithmetic mean is an additive aggregation procedure that implies assuming perfect substitutability between components (linear indifference curves). It amounts to admitting that we can substitute, for instance, expected life years for education at a constant rate, no matter the average level of health. A constant rate of substitution independent of the level of the variable is hard to justify in many contexts and particularly in this one. Moreover, an additive index of this sort generates a ranking that is sensitive to the normalization of the different indicators. Namely, a change in the arbitrary normalization of the raw variables induces changes in the ranking that the index produces (because changing the normalization amounts to modifying the weights with which those variables enter the index).

The need for a change in the aggregation process was widely recognised in the literature. Many authors agreed on the need to replace the arithmetic mean with a more general nonlinear type of mean, most particularly the geometric mean (see Chakravarty 2003; Foster, López-Calva and Székely 2005; Herrero, Martínez and Villar 2010a; Seth 2009, 2011, among others, for a discussion). The geometric mean is a well-known aggregator in economics. It corresponds to the familiar symmetric Cobb-Douglas formula for production and utility functions and exhibits

much better properties regarding substitutability among the variables. Also note that the geometric mean penalizes the dispersion of variables that are aggregated whereas the arithmetic mean is insensitive to the distribution of the variables being averaged.<sup>6</sup>

How does our vision of human development change with the new index? The 2010 Human Development Report says on this respect (p. 217): “The methodological improvements in the HDI, using new indicators and the new functional form, result in substantial changes.... Adopting the geometric mean produces lower index values, with the largest changes occurring in countries with uneven development across dimensions. The geometric mean has only a moderate impact on HDI ranks.” Indeed, the new HDI discriminates more than the old one (the coefficient of variation is 40% higher) and yields a good deal of shifts in the ranking, mostly due to the change in the variable that measures education. See Klugman, Rodriguez, and Choi (2011) for a detailed discussion.

### 3.2. Adjusting inequality in human development

There is a general consensus on the need to take into account distributive considerations when evaluating economic growth or human development. This can now be easily accomplished because there are statistics on income inequality for many countries and we have a well-established theory that permits linking the evaluation of the size and the distribution of income. It is therefore striking that the human development report waited for twenty years to include distributive considerations into the HDI, in spite of several proposals being put forward (e.g. Anand and Sen 1994b; Hicks 1997; Foster, López-Calva, and Székely 2005; Herrero, Martínez, and Villar 2010 a, b; Seth 2009, 2011).

The 2010 Human Development Report includes a new index that addresses the distribution of the different variables: the Inequality-adjusted Human Development Index (IHDI). This index has the same structure as the 2010 HDI, but each constituent variable has previously been adjusted by a discount rate that measures the inequality of its distribution within each country. That is, the IHDI is the geometric mean of the inequality-adjusted values of the variables for health, education, and material wellbeing:

$$I(H) = f_H(H)(1 - A_H), I(E) = f_E(E)(1 - A_E), I(Y) = f_Y(Y)(1 - A_Y)$$

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<sup>6</sup> Given the small values of the normalized variables, changes in rankings induced by substituting the arithmetic mean by the geometric mean will be small. The changes in the relative values, though, are more relevant and reflect the dispersion of the partial indices.

where  $A_C$ , for  $C = H, E, Y$ , is the inequality measure of the corresponding variable, and  $f_C(C)$  describes the transformation of the original values into normalized values (with logs in the case of the per capita GNI). According to this formulation, inequality reduces the achievements in each variable. The term  $f_C(C)A_C$  is a measure of the loss due to inequality.

The report adopts Atkinson's (1970) inequality index for the value of the inequality aversion parameter  $\varepsilon = 1$ , which yields an inequality-adjusted measure for each indicator corresponding to the geometric mean of individual achievements (see Foster, López-Calva, and Székely (2005) and Alkire and Foster (2010) for details). The IHDI is given by the formula:

$$IHDI = \sqrt[3]{I(H) \times I(E) \times I(Y)}.$$

However, the variables used for computing the inequality measures are not necessarily computed from the same the variables used for computing the partial indices for the HDI. Thus,  $f_H(H)$ ,  $f_E(E)$ , and  $f_Y(Y)$  are the partial indices for the HDI such that  $f_H(H)$  is the life expectancy at birth,  $f_E(E)$  is the geometric mean of expected years of schooling and mean years of schooling, and  $f_Y(Y)$  is the corresponding partial index based on GNI per capita. In contrast, the inequality measure for the knowledge dimension has been computed using years of schooling among adults only as it is not possible to capture inequality across the expected years of schooling variable. Similarly, the inequality measure for the material wellbeing dimension has been computed from various variables such as per capita disposable income, per capita consumption expenditure, or income imputed from asset indices.<sup>7</sup> The inequality measure for the health dimension is, however, computed using the same indicator used for constructing the corresponding partial indicator, yet the computation of the inequality measure is not straightforward. The measure is not computed by capturing inequality across the health status of the entire population. Rather inequality is computed across the mortality rates for different age groups (for a detailed discussion, see Kovacevic 2010).

### 3.3. A new index for measuring human deprivations

In the 2010 Human Development Report, UNDP introduced a new index of multidimensional poverty referred to as the 'Multidimensional Poverty Index (MPI)' for developing countries, which was proposed by Alkire and Santos (2010). This shows the UNDP's willingness to mark a clear departure from the use of composite indices to multidimensional indices that are able to

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<sup>7</sup> Income inequality is calculated with respect to the original distribution without logarithmic transformations.



capture joint distributions across the population. Like the HDI and the HPI-1, the MPI also has three dimensions – education, health, and standard of living, but it consists of ten indicators. The indicators and their deprivation cut-offs are reported in Table 3.2. The health dimension and the education dimension consist of two indicators each, and the standard of living dimension consists of six indicators. Thus, two indicators in the MPI, child mortality and access to safe drinking water, are the same as those in the HPI-1.

**Table 3.2: Dimensions, Indicators, Deprivation Cut-offs, and Weights of the MPI**

Dimension	Indicator	A Person in a Household is Deprived if ...
Health	Nutrition	Any woman or child in the household with nutritional information is undernourished
	Mortality	Any child has died in the household
Education	Schooling	No household member has completed five years of schooling
	Attendance	Any school-aged child in the household is not attending school up to class 8
Standard of Living	Electricity	The household has no electricity
	Sanitation	The household's sanitation facility is not improved or it is shared with other households
	Water	The household does not have access to safe drinking water or safe water is more than a 30-minute walk round trip
	Flooring material	The household has a dirt, sand, or dung floor
	Cooking fuel	The household cooks with dung, wood, or charcoal
	Assets	The household does not own more than one of these items: radio, telephone, TV, bike, motorbike, or refrigerator; and does not own a car or truck

Source: Alkire, Roche, Santos, and Seth (2011)

The method of the MPI is an adaptation of the Adjusted Headcount Ratio proposed by Alkire and Foster (2007, 2011). Unlike the HPI-1, the MPI is computed directly from the survey dataset rather than the indicators being computed from different sources. Let us provide a brief outline of the method with an exemplary country with  $n$  individuals and  $d$  indicators. In case of the MPI,  $d = 10$ . Let us refer to the performance of a person in an indicator by achievement. The achievements of all  $n$  persons in  $d$  dimensions is represented by the  $n \times d$ -dimensional matrix  $X$ . The achievement of any person  $i$  in indicator  $j$  is denoted by  $x_{ij}$ . The weight attached to indicator  $j$  is denoted by  $w_j > 0$  such that  $\sum_j w_j = 1$ . Each indicator has its own deprivation cut-off. A person failing to meet the cut-off is identified as deprived in that dimension. The deprivation cut-off of indicator  $j$  is denoted by  $z_j$ . Subject to the deprivation cut-off, person  $j$  is assigned a deprivation status score in indicator  $j$ , which is denoted by  $g_{ij}$  such that  $g_{ij} = 1$  if person  $i$  is deprived in indicator  $j$  and  $g_{ij} = 0$ , otherwise. In the next step, a deprivation score is obtained for each person  $i$  such that  $c_i = \sum_j w_j g_{ij}$ . The deprivation score of each person is the weighted average of deprivation status scores. Note that at the two extremes,  $c_i = 0$  if person  $i$

is not deprived in any indicator and  $c_i = 1$  if person  $i$  is deprived in all indicators, and so  $c_i \in [0,1]$ .

Not all those who are deprived in any indicator are identified as poor, however. The identification step involves a poverty cut-off  $k$ . A person is identified as poor whenever  $c_i \geq k$  and non-poor whenever  $c_i < k$ . If the value of  $k$  is positive but lower than the minimum weight assigned to any indicator such that  $0 < k < \min\{w_1, \dots, w_d\}$ , then the identification approach is referred to as the union approach. By union approach, a person is identified as poor, even when the person is deprived in a single indicator. On the other extreme, an intersection approach identifies a person as poor only if the person is deprived in all indicators or when  $k = 1$ . Both of these approaches may be too stringent and in that case an alternative middle ground may be found by using an intermediate approach, such that  $\min\{w_1, \dots, w_d\} < k < 1$ . Once individuals are identified as poor and non-poor, then a censored distribution of deprivation scores is obtained, such that  $c_i(k) = c_i$  if  $c_i \geq k$  and  $c_i(k) = 0$  for all  $c_i < k$ . The adjusted headcount ratio, denoted by  $M_0$ , is computed from the censored distribution scores as  $M_0 = [\sum_i c_i(k)]/n$ .

The MPI uses a particular set of indicators and deprivation cut-offs, a particular set of weights, and a certain value of poverty cut-off. The three dimensions, ten indicators, and the corresponding deprivation cut-offs are already outlined in Table 3.2. Like the HDI and the HPI-1, the MPI weighs each dimension equally and furthermore the weight within each dimension is equally distributed across indicators. For example, the mortality indicator in the health dimension is assigned a weight equal to 1/6; whereas the assets indicator in the standard of living dimension is assigned a weight equal to 1/18. The poverty cut-off for the MPI is equal to one third of the weighted indicators or  $k = 1/3$ . Thus, a person within a household is identified as poor if the household's deprivation score is equal to or larger than 1/3. Note that the identification takes place at the household level but not at the individual level because it is difficult to obtain data at the individual level. Because the identification takes place at the household level, it is not possible to capture the difference in achievements that may exist within a household. Despite this shortcoming, the construction of the MPI is a big leap forward in the measurement in poverty.

The MPI also has certain useful properties. First, it can be expressed as a product of two terms. One is the multidimensional headcount ratio ( $H$ ), which is the proportion of the population living in households that are deprived in one-third of weighted indicators or with deprivation

scores equal to or larger than one-third. If we denote the number of poor by  $q$ , then  $H = q/n$ . The other term is the average deprivation score among the poor ( $A$ ). By definition,  $H$  lies between zero and one: it is equal to one when everyone is identified as poor and is equal to zero when there are no poor at all. The range of  $A$  is, however, not as straightforward. Whenever there is at least one poor person,  $A$  lies between  $k$  and one, but if there is no poor person in the society, then  $A$  cannot be defined. The second useful feature is that the MPI can be expressed as a weighted average of the censored headcount ratios of the ten indicators. The censored headcount ratio of an indicator is the proportion of the population that is identified as multidimensionally poor and is simultaneously deprived in that indicator. The third useful property is that the MPI is decomposable across any population subgroup, which means that the overall MPI can be expressed as the weighted average of subgroup MPIs where the weight attached to each subgroup is equal to its relative population share.

#### 4. A Critical Evaluation of the 2010 Indices

Although the indices introduced in the 2010 HDR overcame many limitations of the indices introduced in the previous HDRs, there is still room for improvement. In this section, we devote some time to critically assess these 2010 indices.

##### 4.1. The Human Development Index

The new HDI is regarded as containing major improvements with respect to measuring educational achievements, the construction of time-consistent data series, the new aggregation formula, and the introduction of distributive considerations. Yet it has also opened an important discussion about some methodological issues, which refer to the nature of the index and its internal structure. From a purely theoretical viewpoint, there are some flaws in the design of the new HDI that should be addressed (see Herrero, Martínez, and Villar 2012). From an empirical perspective, the 2010 HDI received a number of criticisms related to the apparent performance of African countries and the questionable trade-offs between variables, according to the new method (see Ravallion 2010).

Most of the criticisms are related to the change in the aggregation procedure. Some argue that the change in the aggregation formula does not alter significantly the ranking of the countries (most of the changes are actually due to the new education variable) whereas the geometric mean is a less intuitive concept than the arithmetic mean. Moreover, there are empirical outcomes regarding the resulting rates of substitution between dimensions that have shed some doubts

about the advantage of the new formulation (see the discussions in the blog “Let’s Talk Human Development,” Ravallion 2010; Zambrano 2011; Zambrano 2016; and Klugman, Rodríguez and Chow 2011). Finally, there are also criticisms on some modelling choices, particularly regarding the use of logs for the income variable and the normalization strategy (see Herrero, Martínez, and Villar 2010b, 2012). Let us critically evaluate the new HDI in more detail.

#### 4.1.1. Choice of weights

The 2010 HDI preserves the number and the nature of the selected dimensions and the equal-weight principle of the traditional HDI. There is no novelty in those respects, and the criticisms that applied to the pre-2010 HDI also apply here. The choice of equal weights for the different dimensions was made essentially on the basis that there were no rationale to give more weight to any of those essential aspects of human development (see HDR 1995, p. 48). The 2010 edition of the HDR keeps this weighting system. Yet, as Anand and Sen (1997) have pointed out, one may well consider that “the weights in the HDI should be traced either to individual preferences, some collective social choice process, or to a strong normative argument.” In an empirical paper regarding 1975–2005, using principal components techniques, Klasen, Nguefack, and Zucchini (2012) provide a statistical justification for the HDI weighting scheme.<sup>8</sup>

Any precise weighting scheme, however, is difficult to agree upon universally, and, therefore, instead of just debating the selection of a particular weighting scheme, it is important to understand how robust the comparisons are with respect to the choice of the initial weighting scheme to possible alternatives. Different tools for sensitivity and robustness analyses have been developed in order to test the robustness of rankings generated by the composite indices. For example, Foster, McGillivray, and Seth (2009, 2013) propose a tool for testing the robustness of pair-wise comparisons with respect to the initial weighting scheme. Applying the tool to the HDI ranking for various years, they find that nearly 70 percent of all the HDI pair-wise comparisons to be fully robust. Fully robust meant that 70 percent of the pair-wise comparisons did not alter no matter which alternative weighting schemes were selected when the weights were strictly positive and summed up to one.<sup>9</sup>

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<sup>8</sup> For a discussion on different techniques for setting weights for multidimensional indices, see Decancq and Lugo (2013).

<sup>9</sup> For further discussions on robustness and sensitivity analyses of the composite indices, see Nardo et al. (2008), Cherchye et al. (2008), and Permanyer (2011).

### 4.1.2. Choice of variables

Do the three dimensions of HDI sufficiently cover all facets of development? Certainly, the answer is no. In particular, the question of sustainability is essential and should be incorporated into the index – the sooner the better.<sup>10</sup> Also, including some aspects of social exclusion may improve the measurement of the differential impact of economic fluctuations. However, caution needs to be taken when increasing the number of dimensions because (i) the larger number of dimensions makes the aggregation procedure more difficult and (ii) the larger number of variables makes choosing a precise weighting scheme more complex.

The precise dimensions that should be added is a matter of great debate, which is beyond the scope of this paper. However, we provide a critical evaluation of the variables that have been used to measure the three dimensions of the new HDI.

#### 4.1.2.1. The health variable

Life expectancy at birth is an estimate of the average number of years for a newborn in a given society at a given point in time. It is obtained from the mortality tables of the existing population as follows. First, one determines the probability of death at age  $\gamma$ ,  $p_\gamma$ , and then computes the corresponding survival probability at that age, given by  $\bar{p}_\gamma = 1 - p_\gamma$ . The number of survivors at age  $\gamma$  in a given year,  $S_\gamma$ , is simply  $S_\gamma = S_{\gamma-1}\bar{p}_\gamma$ , under the convention of starting from a fictitious population  $S_0 = 100,000$ . Life expectancy at age  $\gamma$  (assuming that agents live during half of the year in which they die) is calculated as

$$e_\gamma = \frac{1}{2} + \frac{1}{S_\gamma} \sum_{i=\gamma+1}^{\infty} S_i = \frac{1}{2} + \sum_{i=\gamma+1}^{\infty} \left( \prod_{j=\gamma+1}^i \bar{p}_j \right).$$

Life expectancy at birth is simply  $e_0$ , that is, the average number of years that a person born in the year of reference will live.

It is clear from the formulation at the right in the above equation that life expectancy at any reference age  $\gamma$  is independent of the demographic structure of a country, which allows comparing consistently health in countries with different population pyramids.<sup>11</sup> Life expectancy

<sup>10</sup> The 2011 edition of the *Human Development Report* pays particular attention to this issue. The UN team responsible for the report is actively working on this topic, trying to find ways of incorporating the sustainability dimension in the HDI. See also Eurostat (2005), Costantini and Monni (2005), Neumayer (2011), or Llavador, Roemer and Silvestre (2011).

<sup>11</sup> The independence of life expectancy from the demographic structure is a way of avoiding the “composition effect” that appears when using the average mortality rates. Indeed, it might be the case that country A has a

at birth is a variable that provides a sensible approximation of the measurement of a long and healthy life. Although the data is available for most countries, this variable is rather elementary in construct. The data show that developed countries exhibit very high values of life expectancy at birth, with a small variance, while they exhibit more relevant differences in the demographic structure. Therefore, life expectancy at birth tends to overestimate the development capacities of those countries with a relatively older population and to underestimate the development capacities of those countries with a younger population (typically developing countries). In future revisions of the health variable, quality of life (e.g. in terms of quality-adjusted life years [QALYs] or self-perceived health states) and the population structure (e.g. the relative size of the working age population) should be considered in addition to the quantity of life. Finally, there is also a more essential consideration as to whether this is the type of indicator that fits best to evaluate human development.

#### 4.1.2.2. The education variable

The change in the variable measuring educational achievements was probably the most needed one. The HDI 2010 substituted the old combination of literacy rate and gross enrolment rates with another composite variable that consists of the geometric mean of “mean years of education” (among adults) and “expected years of schooling” (among children), suitably normalized. These two new partial indicators are certainly more informative and capture the essential differences in the level of human capital among countries.

Yet the way in which the composite education variable has been constructed presents two disadvantages worth considering. The use of the geometric mean in order to create the composite variable of education makes the indicators’ impact on the HDI less transparent. Moreover, the geometric mean of the two partial indicators of education amounts to unjustifiably penalising improvements in the educational expectations of the young, as the geometric mean fosters the equalization of the component variables. Consequently, improvements in children’s schooling are only partially reflected in the index of education, which makes incentives to invest in education less apparent. From a different perspective, it is worth taking into account that the empirical evidence suggests that, after some minimal threshold, it is the quality of education and not the years of schooling indicator that better explains differences in development. The Programme for International Student Assessment (PISA) studies provide a

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lower average mortality rate than country B while A exhibits higher mortality rates at all age intervals. The reason is that specific mortality rates vary a lot across cohorts and the relative size of the different cohorts may induce this counter-intuitive outcome. That is why life expectancy is preferred.

rich database that may be considered for the future incorporation of the quality of education (e.g. by computing quality-adjusted years of schooling).

#### **4.1.2.3. The income variable**

As in the traditional HDI, the new version measures material wellbeing in terms of logs of an income variable (per capita GDP before 2010 and per capita GNI afterwards). The use of logs implies that the effect of one additional unit of a given variable decreases with the level at which this happens. This is the conventional way of describing how the consumption of a given good relates to personal welfare and is an expression of the “decreasing marginal utility” principle. The Human Development Report provides this type of explanation to justify the use of logarithms when measuring material wellbeing: an additional euro has a different impact depending on the level of wellbeing at which it is gained. The obvious question is: Why is this principle applied to the income variable and not to the other ones in the HDI? Why is it that an additional year of life or education has the same value no matter the level at which it occurs?

A reasonable explanation of this asymmetry goes along the lines of the axiomatics provided in Zambrano (2014) (see also Klugman, Rodríguez, and Chow (2011) for a wider discussion). When raw variables are interpreted as estimates of capabilities, direct indicators should be distinguished from indirect indicators of capabilities and thus variables should receive differential treatment. Even though this is a consistent approach that might justify the use of logs for the income variable, it involves some drawbacks. First, it does not fit very well with some of the novelties of the HDI 2010 (especially with respect to the inequality-adjusted measures discussed in the next subsection). Second, it imposes restrictions on the normalization formula, as one cannot take minimum values below one for the logged variable, no matter the units in which we measure income. Third, it has a relevant impact on the substitutability of the underlying primary variables reported in Table 3.1 as the meaning of substitutability is not very clear (or, alternatively, prevents sensible calculations of marginal rates of substitution).

#### **4.1.2.4. Rights**

We shall be extremely brief on this point as this is a well-known problem: the HDI does not take into account human rights. People in charge of the Human Development Reports are well aware of this. As an example, the heading of Figure 4.1 in the 2010 HDR reads as follows: “A high Human Development Index does not mean democracy, equality or sustainability.” Equality has already been addressed, for example by the introduction of the IHDI, and dealing with sustainability is on the agenda. What about democracy? (e.g. the compliance with some basic

rights, as those identified in the Universal Declaration of Human Rights of 1948).

The objective of involving as many countries as possible has led to the neglect of this basic question in the existing HDI and forced those interested to look elsewhere when seeking a measure of the basic rights of citizens. This is an arguable strategy, because it seems to send the message that those aspects are not as important.

### 4.1.3. Modelling flaws

The preceding discussion regarding the choice of variables is partly a matter of judgement on which are the best options for approaching the different dimensions. We now discuss two aspects that can be regarded as conceptual flaws in the way the HDI is modelled.

#### 4.1.3.1. The normalization formula

As the variables that measure the three human development dimensions are measured in different units, some normalizing is required. Following the procedure already used in the traditional HDI, the normalization formula chosen by the 2010 HDI preserves the maximum and minimum goalposts. However, the interpretation of maximum and minimum goalposts is somewhat different. Maximum goalposts are mostly regarded as a technical device to keep the range of variables within a compact interval. Minimum goalposts, on the contrary, are given an ethical content and interpreted as minimum admissible values (a kind of subsistence level).

We have already seen that a given raw variable,  $X$ , is transformed into a normalized variable,  $x$ , according to the formula

$$x = \frac{X - X^{min}}{X^{max} - X^{min}},$$

where  $X^{max}$  and  $X^{min}$  are the corresponding goalposts. That is, we transform the original values into relative gains so that all the transformed variables move into the  $[0, 1]$  interval and the normalized variables are all unit-free. Note that the above formula can be regarded as a linear transformation of the original variable,  $x = aX'$ , where  $a = \frac{1}{X^{max} - X^{min}}$  defines the units in which the variable is measured and  $X' = X - X^{min}$  is the net value of the variable.

This way of normalizing the variables, however, has three negative implications.

1. It makes the whole construction of the HDI dependent on the arbitrary choices of the



normalization parameters. In particular, changing the minimum goalpost can revert (and indeed it does) the ranking and modify the relative valuations.<sup>12</sup> This is unfortunate because the dependence of the ranking on the arbitrary choice of normalization values was one of the main criticisms of the arithmetic mean. Note that the multiplicative formula of the 2010 HDI implies that changing the maximum goalposts only affects the units of measurement and therefore it alters neither the ranking nor the relative values of any two countries.<sup>13</sup>

2. Deducting any positive value from the original variables (i.e. using minimum goalposts) worsens the picture we get of lower performing countries while having practically no impact on those countries with higher values. As a consequence, the gap between top and bottom countries increases artificially. A simple example using the health variable illustrates this: a minimum goalpost of 20 years implies computing one half of Afghanistan's life expectancy and 3/4 of Japan's corresponding value.
3. The use of a minimum goalpost in the normalization may have a very large effect on the marginal rates of substitution due to the behaviour of the slope of a Cobb-Douglas function when a given component approaches zero. Therefore, subtracting whatever amount to an already very close to zero magnitude will increase substantially (and again artificially) the associated marginal rates of substitution. This is the main reason behind the polemic substitution rates found in Ravallion (2010); see Herrero, Martínez, and Villar (2012) for a discussion and a calculation of marginal rates of substitution without using minimum goalposts.

#### 4.2. The Inequality-adjusted Human Development Index

The 2010 edition of the HDI took (at last!) distributive issues into account. It not only introduced inequality measures regarding income but also with respect to the two other dimensions. Distributive considerations are introduced by calculating the egalitarian equivalent worth of a given value. That is, if  $f_C(C)$  is the mean value of a reference variable  $C$  and  $A_C$  is an inequality measure, then the egalitarian equivalent value is given by  $f_C(C)[1 - A_C]$ . This is a well-founded conceptual construction, provided two requirements are fulfilled: (i) both the

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<sup>12</sup> When calculating the HDI 2010 by normalizing the raw variables as shares of the same max values used in the report (i.e., letting minimum goalposts equal to zero), some 30% of the countries change their ranking by five or more positions.

<sup>13</sup> A change in the minimum goalpost modifies both the units in which the variables are measured and the net value. The first only affects the scale and does alter the ranking in a multiplicative formula (as happens with changes in the maximum goalposts). The second impact is indistinguishable from a change in the level of the raw variable and that is why the ranking is altered.

inequality measure and the mean value should refer to the very same variable, and (ii) perfect equality is the best possible world. Unfortunately, the IHDI does not seem to satisfy those requirements.

Take the income variable first. The egalitarian-equivalent income fits neither with the use of logs nor with the normalization choice. If we measure inequality over the income distribution vector, as is done in the reports since 2010, we cannot use consistently the log of income as the reference variable. Moreover, the normalization used (with or without logs) is also inconsistent, as the chosen inequality measure is sensitive to the choice of minimum goalposts in the normalization.<sup>14</sup> If we want to keep the interpretation of the capability approach proposed in Zambrano (2011a), one should measure inequality over the vector of log income values. Be that as it may, the inconsistency with the normalization choice remains, unless one measures inequality over the distribution of the normalized individual logged variables. But the meaning of that exercise is far from clear. Measuring income in logs for the HDI and measuring inequality without logs or normalization, as is done in various HDRs, violates requirement (i) stated above.

In order to understand why the second requirement is not fulfilled, let us consider the health dimension, where it is not clear if perfect equality is the most desirable state. Recall that inequality in the health dimension is not based on the inequality in health conditions of the population in a given time period, but it is based on the inequality in mortality rates across different age groups in a given time period. If we consider two countries with the same life expectancies at birth, one with equal mortality rates across all age groups and the second with higher mortality rates among the elderly and much lower mortality rates among children, the latter country would be penalized only because it had made significant improvements in providing health services to the younger generation over, say, the past ten or fifteen years. Even acknowledging that it is quite tough to find a proper indicator to reflect health conditions of the citizens of a country, the present way of computing health inequality is hard to justify and requires further research for improvement.

Another area of concern for the IHDI is the ignorance of the joint distribution of wellbeing in various dimensions. In terms of measuring poverty, the UNDP has moved in the right direction by introducing a poverty measure that captures the joint distribution of deprivations rather than computing a composite index of poverty. The same, however, has not been true for the HDI or

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<sup>14</sup> We consider here the family of relative inequality measures, which is the one taken as reference in the 2010 report.

the IHDI. The chosen indicators have not been obtained from a single survey and thus not from the same set of individuals or households within a country. In this sense, the IHDI (and certainly the HDI) still remains a composite of dashboard indices and is being prevented from graduating to a multidimensional index capturing joint deprivations. Is it possible to capture the joint distribution of achievements? The answer is yes, but this requires the data on all indicators to be available from a single survey and across all individuals or households as in case of the MPI. An extension of the class of indices developed by Foster, López-Calva, and Székely (2005) has been proposed by Seth (2009, 2011), which captures the joint distribution of achievements and has been used to compute the UNDP's Gender Inequality Index in 2010. This index is in the same class of indices to which the IHDI belongs and may be used when the data are available.

### **4.3. The Multidimensional Poverty Index**

The UNDP's efforts towards upgrading the measurement of poverty from using a composite index (HPI) to a multidimensional index (MPI) capturing joint distribution is indeed novel. However, there is still room for improvement in certain aspects. One is the consideration of inequality across the poor, and the second is the different types of robustness of the ranking or pair-wise comparisons with respect to different parameters, such as alternative choices of deprivation cut-offs, poverty cut-offs, and weighting schemes.

Consideration of inequality while measuring poverty has been customary following Sen (1976). By construction, the MPI is an average of weighted deprivations that the poor experience and is not sensitive to inequality across the poor. There are various alternative poverty measures that use binary indicators as the MPI does, but are sensitive to inequality across the poor (see Bossert, Chakravarty, and D'Ambrosio 2009; Jayaraj and Subramanian 2009; and Rippin 2011). However, there is a crucial trade-off that should be taken into consideration. The inequality-sensitive poverty indices do not allow the overall indices to comprehend the contribution of each indicator or dimension to overall poverty, which is crucial for policy analysis (Alkire and Foster 2016). Given that both properties are important, whether it is possible to find a framework to incorporate both is a subject for further research.

A second improvement should come from developing a tool that may be used to test the robustness of rankings and country comparisons with respect to the choice of parameters in the MPI's construction. Although Alkire and Santos (2010, 2014) tested the robustness of country rankings with respect to a few alternative weighting schemes, a range of poverty cut-offs, and a

few different alternative sets of deprivation cut-offs, a sounder and more concrete approach is required.

## **5. Concluding Remarks**

In this paper, we present a critical evaluation of the indices for measuring human development and poverty in various Human Development Reports. We show how these indices have evolved over time to capture various aspects of wellbeing and deprivations. The introduction of simplified indices in the early reports was required to catch the attention of the mass media and policy makers to put the concept of human development on the agenda. However, a simplified index is not sufficient for capturing the complexity of human lives and their development and deprivations. These make the construction of these indices more complex. More complex indices, however, make their interpretation difficult. Hence, further research is required to amend the indices in a direction that maintains the intuitive interpretations of the indices and, at the same time, captures the complex realities of human development and deprivations.

Another important issue with these indices is the requirement of data. We have reiterated that the consideration of joint distribution is imperative in order to graduate an index of wellbeing or poverty from its composite index status to a truly multidimensional index status. The UNDP has moved in this direction by introducing a multidimensional measure of poverty. However, a move in the same direction has not been possible for the measurement of human development, primarily due to the lack of appropriate data. Our proposals for theoretical improvements cannot be materialized without solving the data constraints first.

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