Multidimensional Measurement of Poverty in Sub-Saharan Africa *

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Abstract

Since the seminal works of Sen, poverty is recognized as multidimensional phenomenon. Recently, there is a renewed interest in this approach since relevant databases became available. Several methods of aggregation have been suggested to measure poverty in this way. Up to now, there is no consensus on the best measure. However, a suitable measure should satisfy some useful properties. Alkire and Foster (2007) propose a multidimensional poverty measure using a counting approach. This method is applied to estimate multidimensional poverty in fourteen Sub-Saharan African countries. Poverty identification is based on four dimensions (*assets*, *health*, *schooling* and *empowerment*). The main results show important differences in poverty among the countries of the sample. The findings are compared with some standard measures such as Human Development indicators (HDI) and the income poverty among others. Comparisons show that consider additional dimensions leads to country rankings different from the standard-based rankings. Poverty is also decomposed by rural and urban location and by dimension. Rural areas are identified obviously as the poorest while *schooling* appear to be in general the most contributor in poverty. Finally, some robustness and sensitivity analyses are done.

Keywords: Multidimensional poverty, counting measurement, robustness analysis, Sub-Saharan Africa.

JEL Classification: C12,D31,I31,I32.

OPHI Working Paper 13

August, 2008

*I am grateful for significant comments of Sabina Alkire. I am also grateful for inputs of Ana Lugo, Yu Jiantuo, Maria Emma Santos, Anna Hiltunen and David Vazquez-Guzman

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1 Introduction

Fighting extreme poverty and improving health and education are among the main Millennium Development Goals (MDGs) agreed by 189 heads of state in 2000. To efficiently achieve these goals, it would be helpful to propose a suitable measurement of poverty that allows the targeting of priorities. Income poverty measures have been usually used to analyse poverty in developing countries including those from Sub-Saharan Africa. Several poverty indices[2], which follow from the utilitarian definition of welfare, are tremendously useful for estimating poverty levels and making inter-temporal and inter-country comparisons. However, some arguments suggest now to go beyond these money-metric measures and consider other poverty measurements. The first argument, a more practical one, relates to the fact that the quality (regularity and comparability) of income/expenditures data is often poor in many developing countries, especially in Sub-Saharan African ones which are generally regarded as those showing the most poverty and extreme poverty. A second argument, more theoretical and methodological, concerns the multidimensional nature of well-being. Indeed, since the seminal works of Sen (1976; 1985; 1992; 1995), well-being and poverty are now seen as multidimensional phenomena. The well-being of an individual depends thus not only on income, but also on several other dimensions or capabilities such as health, education, empowerment etc. Nowadays, there is a renewed interest in the multidimensional approach of poverty since relevant databases become increasingly available to enable comparative analyses[3].

Several aggregation methods have been suggested to capture multidimensional poverty. Some of these approaches are axiomatic and are extensions of the unidimensional poverty indices (Tsui 2002; Atkinson 2003; Bourguignon and Chakravarty 2003). Other numerous methods, mainly non-axiomatic, are developed in the literature. For example, among others the fuzzy set approach (Cerioli and Zani 1990; Cheli and Lemmi 1994; Chiappero-Martinetti 2006), the distance function method (Lovell, Richardson, Travers, and Wood 1994; Anderson, Crawford, and Liecester 2005; Deutsch and Silber 2005) and the entropy measures approach (Maasoumi 1993; Deutsch and Silber 2005; Maasoumi and Lugo 2008). Other approaches, more centered on statistical methods, were also adopted such as the inertia approach and the factor analysis (Klasen 2000; Sahn and Stifel 2000; Sahn and Stifel 2003a). More recently, some methods from psychometric literature (Structural Equation Model and MIMIC models) have been introduced in the well-being analysis (Wagle 2005; Di Tommaso 2007; Krishnakumar 2007; Krishnakumar and Ballon 2008).

[2]For instance, Foster, Greer, and Thorbecke (1984) and Watts (1968) indices.

[3]Demography and Health surveys (DHS) in Africa.

Few studies are carried out to measure multidimensional poverty in Sub-Saharan African countries. Sahn and Stifel (2000) used Demography and Health Surveys (DHS) to compare poverty through time within and between fifteen African countries. Well-being is measured by an asset index estimated from a factor analysis (FA) of various household socioeconomic indicators. Booysen, Von Maltitz, Van der Berg, Burger, and Du Rand (2008) extend the work of Sahn and Stifel (2000) by analysing poverty comparisons over time and across countries, using multiple correspondence analysis (MCA) rather than FA and adding more recent surveys. By using a household survey from South Africa, Klasen (2000) compares the standard expenditure-based poverty measure with a multidimensional deprivation measure and find that, although there is a strong correlation between them, this correlation decreases when the worst-off of the population are considered. Von Maltzahn and Durrheim (2008) also use different poverty measures and obtain, for instance, that household income levels are correlated with the living standard measure (LSM) scores, meaning that there is an underlying dimension of poverty common to all measures. Duclos, Sahn, and Younger (2006a) and Batana and Duclos (2008) extend the analysis by integrating the bivariate stochastic dominance analysis. The first applies the methods to two measures of well-being, namely household expenditures per capita and children's height-for-age z scores in three countries, while the second uses an asset index rather than the household expenditures for six West African countries.

Up to now, there is no consensus about the best multidimensional poverty measure. For example, which measure could allow better targeting of the poor and suggest more effective poverty-reduction policies? One issue a good poverty measure needs to address is identification. With multidimensional poverty, identification of the poor arises at two stages. The first stage is to identify the deprived individuals in each dimension while the second concerns the poverty definition across all dimensions. A measure also needs to satisfy useful properties such as the decomposability. By enabling decomposability across individuals and across dimensions, the measure is able to guide good policies. Alkire and Foster (2007) propose a counting approach for measuring the multidimensional poverty. This approach is appealing for several reasons. First, it is easy to implement so many people could use it. Secondly, it integrates the identification analysis using two cutoffs, where the first is the known dimension-specific threshold for identifying the individuals deprived in that dimension. The second cutoff is the number of dimensions (or weighted sum) in which an individual has to be deprived to be considered poor. Moreover, this approach satisfies several desirable properties including decomposability. One has also the latitude to assign different weights to each dimension.

The main objective of this paper is to apply the above multidimensional counting method to estimate multidimensional poverty in fourteen Sub-Saharan African countries. That is an interesting empirical contribution since the analysis goes beyond the previous on Africa by taking account simultaneously of additional dimensions such as *health*, *schooling* and *empowerment*. The main results show important differences in poverty among the countries of the sample. The findings are further compared with the ones from standard measures such as human development indicators (HDI), income poverty, gender related development index and asset index. Comparisons show that consider additional dimensions leads to country rankings different from the standard-based rankings. Decompositions are done by rural/urban location and by dimension. This enables us to identify rural areas, obviously, as the poorest and *schooling* as the most contributor in poverty. Finally, some robustness and sensitivity analyses are performed. The robustness analysis allows researchers to check whether the differences in poverty between countries significantly hold for a full range of between-dimensional cutoffs. The sensitivity analysis concerns the implications of possible changes in each dimensional threshold. The analysis undertaken in this paper seems relevant to capture multidimensional poverty more accurately and could provide tools for better poverty-reducing policies.

The paper is structured as follows. Section 2 describes the methodology for estimating the multidimensional poverty and adds some short extensions for doing decomposition and robustness analyses. Empirical applications consisting in data description, poverty estimation results and comparisons with alternative measures are found in section Section 3. Section 4 analyses the poverty decompositions by urban and rural location and by dimension, while section 5 presents some robustness and sensitivity results. Section 6 concludes.

2 Methodology

Alkire and Foster (2007) suggest a counting approach which follows the method of aggregation proposed by Foster, Greer, and Thorbecke (1984) in the sense that it is built on the same family of measures. This family satisfies a certain number of axioms such as *symmetry*, *replication invariance*, *decomposability*, etc.

Consider a population of n individuals. Let $d \geq 2$ be the number of dimensions and $x = [x_{ij}]$ the $n \times d$ matrix of achievements, where x_{ij} is the achievement of individual i $(i = 1, ..., n)$ in dimension j $(j = 1, ..., d)$. x is of the following form:

$$x = \begin{bmatrix} x_{11} & \cdot & x_{1j} & \cdot & x_{1d} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{i1} & \cdot & x_{ij} & \cdot & x_{id} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{n1} & \cdot & x_{nj} & \cdot & x_{nd} \end{bmatrix}$$

Let z be a row vector of dimension-specific thresholds z_j, x_i the row vector of individual i's achievements in each dimension, and x_j a column vector of dimension j achievements across the set of individuals. The first step for measuring the poverty is to identify who is poor.

2.1 Identification

For simplicity we assume all dimensions are equally weighted, an assumption we can relax later. Suppose that a matrix of deprivations $\widetilde{x}^0 = [\widetilde{x}^0_{ij}]$ is derived from x as follows:

$$\text{for all } i \text{ and } j,\ \widetilde{x}^0_{ij} = \begin{cases} 1 \text{ if } x_{ij} < z_j \\ 0 \text{ otherwise} \end{cases}$$

For example, $\widetilde{x}^0_{ij} = 1$ means that individual i is deprived in dimension j and $\widetilde{x}^0_{ij} = 0$ that individual i is not.

By summing each row of $\widetilde{x}^0$, we obtain a column vector c of deprivation counts containing c_i the number of deprivations suffered by individual i.

For identifying, consider the identification function $\rho(x_i; z)$ such that:

$$\rho(x_i; z) = \begin{cases} 1 \text{ if individual } i \text{ is multidimensionally poor} \\ 0 \text{ if not} \end{cases} \tag{1}$$

Let k be the cutoff. An individual i will be considered as poor or $\rho_k(x_i; z) = 1$ if $c_i \geq k$. $\rho_k(x_i; z)$ is the identification function relating to the cutoff k. The equation (1) could be rewritten:

$$\rho_k(x_i; z) = \mathbb{I}(c_i \geq k) = \begin{cases} 1 \text{ if } c_i \geq k \\ 0 \text{ if not} \end{cases} \tag{2}$$

$\mathbb{I}(c_i \geq k)$ is the standard indicator function taking the value 1 if the expression in brackets holds and the value 0 if not.

The central question here is following: which is the minimum number k of deprivations an individual should suffer to be considered poor?

In the multidimensional context, there are two prominent criteria of identification. The first is known as *union* definition and considers an individual as poor if he or she is deprived in at least one dimension. In this case, the cutoff $k = 1$. This definition seems to strong and could overestimate the poverty, especially when the number of dimensions d is high enough with possible substitutability among some dimensions.

A second criterion is the *intersection* definition which considers an individual as poor only when he or she is deprived in all the dimensions, i.e. with $k = d$. This could on the other hand underestimate the poverty by not considering, for example, a healthy homeless as poor when health and housing are two of the dimensions.

A third alternative is to choose an *intermediate* definition with $1 < k < d$. In the case of only two dimensions, this criterion will be a combination of these dimensions as proposed by Duclos, Sahn, and Younger (2006b). When a cutoff k is retained, the next step consists in proposing a suitable poverty measure.

2.2 Multidimensional poverty measure

Let $M(x; z)$ be the class of multidimensional poverty measures suggested by Alkire and Foster (2007). The first measure is given by headcount ratio. Let q_k be the number of poor identified according to the thresholds vector z and the cutoff k, the headcount ratio H is following:

$$H = \frac{q_k}{n}, \tag{3}$$

with $q_k = \sum_{i=1}^{n} \rho_k(x_i; z) = \sum_{i=1}^{n} \mathbb{I}(c_i \geq k)$

The *share of possible deprivations* suffered by a poor individual i is given by:

$$\bar{c}_i(k) = \frac{1}{d}[c_i \rho_k(x_i; z)], \tag{4}$$

and the *average deprivation share* across the poor by:

$$A = \frac{1}{q_k d} \sum_{i=1}^{n} c_i \rho_k(x_i; z). \tag{5}$$

The second measure proposed by Alkire and Foster (2007) combines H and A to obtain an expression satisfying the *dimensional monotonicity* (unlike H). The new measure M_0 called *adjusted headcount ratio* is given by:

$$M_0 = HA = \frac{1}{nd} \sum_{i=1}^{n} c_i \rho_k(x_i; z). \tag{6}$$

When some dimensions are represented by cardinal data, it is then possible to derive a class of adjusted FGT measures. In this case, a new matrix $\widehat{x}^1$ will be derived from $\widetilde{x}^0$ by replacing 1 by the respective normalized gaps $g_{ij}^1 = \frac{(z_j - x_{ij})}{z_j}$ for all cardinal data j and for all individuals i. One could also consider the general case for a matrix $\widehat{x}^\alpha$, with $g_{ij}^\alpha = \frac{(z_j - x_{ij})^\alpha}{z_j^\alpha}$. Consider the following expression of G^α:

$$G^\alpha = \frac{1}{\sum\limits_{i=1}^{n} c_i \rho_k(x_i; z)} \sum_{j=1}^{d} \sum_{i=1}^{n} g_{ij}^\alpha \rho_k(x_i; z),$$

the adjusted FGT measure $M_\alpha = HAG^\alpha$ is finally given by:

$$M_\alpha = \frac{1}{nd} \sum_{j=1}^{d} \sum_{i=1}^{n} g_{ij}^\alpha \rho_k(x_i; z). \tag{7}$$

For $\alpha = 0$, $\sum\limits_{j=1}^{d} \sum\limits_{i=1}^{n} g_{ij}^\alpha \rho_k(x_i; z) = \sum\limits_{i=1}^{n} c_i \rho_k(x_i; z)$ and we obtain M_0.

For $\alpha = 1$, we obtain the adjusted poverty gap M_1.

A useful property satisfied by this measure is *decomposability*. Suppose that the n-size population is divided for example into two mutually exclusive subgroups of sizes n_1 and n_2 respectively. It is such a case when one considers urban and rural populations. The two subgroups are respectively represented by two matrices of achievements x_1 and x_2. Then we have:

$$M(x; z) = \frac{n_1}{n} M(x_1; z) + \frac{n_2}{n} M(x_2; z). \tag{8}$$

This decomposition could be generalized in a straightforward way to any number of exclusive subgroups.

2.3 Robustness analysis methods

There is no unambiguous way to choose dimensional thresholds (z_j) and a unique cutoff (k) to identify the poor. When we have to compare multidimensional poverty between two or more countries as it is the case in this paper, it may be useful to check that any ordered relation between them holds for the full range of k and for reasonable various threshold vectors z. This analysis, known in the unidimensional poverty context as stochastic dominance (Atkinson 1987; Foster and Shorrocks 1988a, 1988b), has been statistically applied to the bidimensional poverty by Duclos, Sahn, and Younger (2006a, 2006b) and Batana and Duclos (2008). Such comparisons are normatively robust since they are proved to be consistent with welfare dominance.

With Alkire and Foster (2007) approach, a straightforward way to make a quite similar analysis is to consider the range of k. In fact, if d is high enough, it is possible to make robustness analyses by describing the poverty at each point of the full range ok $k = 1, ..., d$. Then, comparing the multidimensional poverty in two populations for each k could be regarded as a dominance analysis. An increase in k could correspond to a decrease in the poverty line for an aggregated dimension derived from factor analysis over all dimensions. Atkinson (2003) suggests such a dominance analysis where the robustness is checked along the variation from an *union* definition of poverty to an *intersection* definition. Given two distributions A and B with F and G as respective cumulative distribution functions (cdf_s). The first comparison of poverty will be in terms of the headcount ratio. Let n^A and n^B be the respective size of A and B. Multidimensional poverty[4] is said to be greater in A than in B, for a given k, if $H^A > H^B$. B then dominates A in poverty if the previous condition holds for all k. Following Davidson and Duclos (2006), these relations could be tested by specifying the null hypothesis of non dominance as follows:

$$H_0 \quad : \quad \frac{1}{n^A}\sum_{i=1}^{n^A}\mathbb{I}(c_i^A \geq k) - \frac{1}{n^B}\sum_{l=1}^{n^B}\mathbb{I}(c_l^B \geq k) \leq 0 \text{ for some } k;$$

versus

$$H_1 \quad : \quad \frac{1}{n^A}\sum_{i=1}^{n^A}\mathbb{I}(c_i^A \geq k) - \frac{1}{n^B}\sum_{l=1}^{n^B}\mathbb{I}(c_l^B \geq k) > 0 \text{ for all } k. \tag{9}$$

[4]The dominance analysis here comes down to a simple unidimensional dominance since all dimensions are aggregated in a vector c of deprivation counts.

c_i^A and c_l^B are the numbers of deprivations suffered by individuals i and l respectively from the distributions A and B. The tests are based on the empirical likelihood ratio (Davidson and Duclos 2006). Given p_i^A and p_l^B the probabilities associated respectively to the individuals i and l, our statistic is obtained by maximizing the following empirical likelihood function (ELF):

$$\max_{p_i^A, p_l^B} \sum_i n_i^A \log p_i^A + \sum_l n_l^B \log p_l^B \tag{10}$$

subject to

$$\sum_i p_i^A = 1, \quad \sum_l p_l^B = 1$$

First, the unconstrained and maximized ELF is estimated according to (10). Then, the constrained maximized ELF is still estimated from (10) by adding a new constraint induced by the null hypothesis:

$$\sum_{i=1}^{n^A} p_i^A \mathbb{I}(c_i^A \geq k) - \sum_{l=1}^{n^B} p_l^B \mathbb{I}(c_l^B \geq k) = 0 \tag{11}$$

The test ratio for each k is given by the difference between the two ELF_s. If the minimum test ratio over the k is great enough, the null will be rejected. To conduct the same dominance analysis for M_0, the following constraint will be considered:

$$\sum_{i=1}^{n^A} c_i^A p_i^A \mathbb{I}(c_i^A \geq k) - \sum_{l=1}^{n^B} c_l^B p_l^B \mathbb{I}(c_l^B \geq k) = 0 \tag{12}$$

With this specification no analytical solution exists. The procedure used by Batana and Duclos (2008) can be followed to deal with this issue.

Another analysis could consist in checking the robustness of the poverty measures using different vectors z of thresholds. Such an exercise can not be done elegantly with dominance analysis like those presented previously, since a change in z will induce a change in our interest vector c. Any manner to proceed would be by defining a limited number of reasonable vectors z and analysing whether moving from a vector to another changes the poverty ranking. A more sensible and consensual z would be to choose the thresholds that reflect as far as is possible the MDGs. Two other sets of thresholds could be defined, one with conditions stronger than MDGs and the other with weaker conditions.

3 Empirical results

3.1 Data and thresholds selection

There are few databases in Africa providing information on income or expenditures. Fortunately, Demography and Health Surveys (DHS), which were conducted in many countries including most of African ones since the middle 1980s, contain a lot of information on the standard and quality of living. This information can enable researchers to suitably capture multidimensional poverty. DHS are nationally representative surveys which include three questionnaires: a household questionnaire, a women's questionnaire and a men's questionnaire[5]. More interesting, the same methodology is adopted is all countries, allowing international comparisons.

For this paper, the DHS data posterior to 2000 are selected for fourteen Sub-Saharan countries, including seven Francophone countries (Benin, Burkina, Guinea, Madagascar, Mali, Niger, Senegal), six English-speaking countries (Ghana, Kenya, Malawi, Nigeria, Tanzania, Uganda) and Cameroon which is a bilingual (French and English) country. Details on the surveys main characteristics are provided in table A1 in appendix. This choice of DHS is motivated by the availability of information we need to assess poverty.

The unit of analysis is the woman. Indeed, women's questionnaires allow us to consider an additional and uncommon dimension *empowerment*. Data were collected for women from fifteen to forty nine years old.

Other dimensions considered are *assets*, *schooling* and *health*. *Assets* are generally estimated from a set of nested dimensions or indicators on ownership of durable goods (radio, television, refrigerator, bicycle, motorcycle, car) and on access to services (electricity, phone, quality of floor, drinking water, sanitation). Such indicators are used to derive an asset index in Africa using factor or multiple correspondence analysis (Sahn and Stifel 2000; Batana and Duclos 2008; Booysen, Von Maltitz, Van der Berg, Burger, and Du Rand 2008). *Assets* are represented by eight nested dimensions, all of them listed in table 1. *Schooling* is given in single years attendance while *health* is measured by the *body mass index* (BMI). *Empowerment* is measured by the latitude a woman has to decide on her visits to family and relatives[6]. Here, rather than to sum all dimensions in a black box, the adopted poverty

[5] For more details, refer to the following website: http://www.measuredhs.com

[6] Ibrahim and Alkire (2007) propose several internationally comparable indicators of individual empowerment. From DHS, five indicators relating to the household decision-making could be considered. That is the final say on respectively (i) health care, (ii) making large household purchases, (iii) making household purchases for daily needs, (iv) visits to family and relatives and (v) food to be cooked each day. Preliminary

measure enables us to observe simultaneously what happens in each dimension.

Table 1: Selection of dimensional thresholds

Dimension	Weight	Poverty threshold
1. Asset	1	
i) Electricity	0.125	Yes/**no**
ii) Radio and/or TV	0.125	If does not have at least a radio
iii) Refrigerator	0.125	Yes/**no**
iv) Vehicle	0.125	If does not have at least a motorcycle
v) Floor	0.125	Natural or rudimentary
vi) Phone	0.125	Yes/**no**
vii) Drinking water	0.125	MDG definitions
viii) Sanitation	0.125	MDG definitions
2. Schooling	1	If highest education level < 6 (MDG definitions)
3. BMI	1	If BMI < 18.5 (WHO standard)
4. Empowerment	1	If has no say on visits to family and relatives

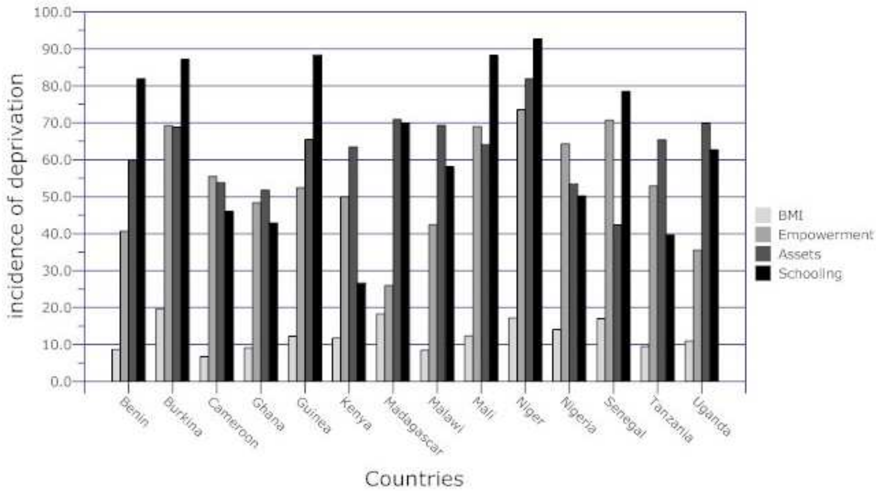
The weight 1 is assigned to each of the four dimensions. The asset weight is then equally divided into its eight nested dimensions. The ownerships respectively of bicycle, motorcycle and car are grouped together in one indicator called vehicle ownership. This indicator is ordered from having nothing to having the three goods. The selection of thresholds is illustrated in table 1. For dichotomous indicators like electricity, refrigerator and phone, an individual will be considered as deprived in each indicator when she lacks it. The indicator "having radio and/or TV "is ordered from having nothing to having both. It is judged that an individual who does not have either is deprived. For vehicle, a threshold is the owning of at least a motorcycle. The quality of floor is dichotomized so that this indicator is 0 for deprived individuals (natural and rudimentary floor) and 1 for not deprived. The deprivations in drinking water, sanitation and *schooling* follow the MDG definitions while the one in *BMI* is based on the World Health Organization (WHO) standard. For *empowerment*, it seems reasonable to consider that a woman is deprived if she has no say regarding her visits to family and relatives.

findings have shown high correlations between these indicators, which has led us to retain only one of them for multidimensional poverty analysis

3.2 Poverty estimation results

By estimating an average incidence of deprivation of *Assets*, we plot the figure 1 which shows the deprivation in the four dimensions for the fourteen countries.

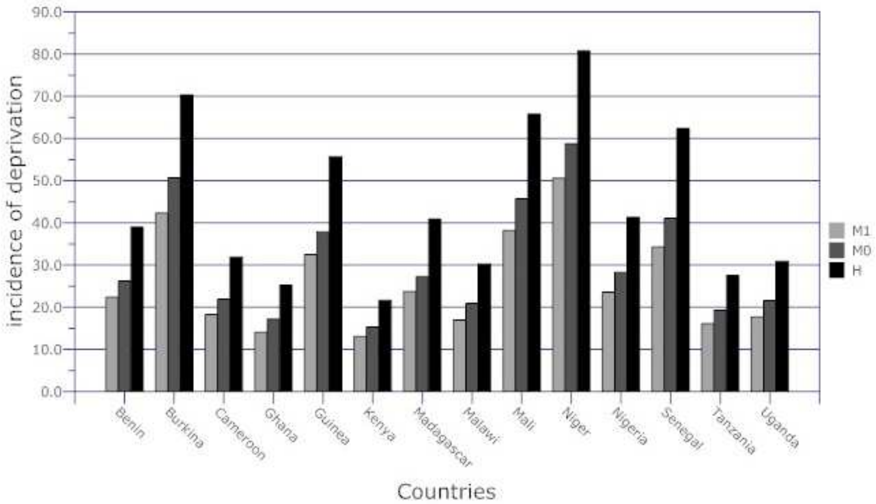
Figure 1: Incidence of deprivation in the four dimensions for all countries



Information on all dimensions, including the nested dimensions, will be found in tables A2 and A3 in appendices. We can notice that *schooling* is the dimension in which people are deprived the most, followed by *assets*. The differences between countries are very considerable. Thomas, Wang, and Fan (2001) show that African countries present a high concentration of deprivation in *schooling* because of their low education levels. In our sample of countries, the proportion of women who did not complete primary education is greater than 70% for six countries (Benin, Burkina, Guinea, Mali, Niger and Senegal). All the Francophone countries appear to be deprived the most in *schooling*. Women in Ghana, Kenya and Tanzania seem to be relatively well off with proportions around 40% or less. For better understanding the matter of this situation, note that one of the objectives in the monitoring of MDGs is to achieve, by 2015, universal primary education for both boys and girls (United Nations 2003).

The best performance of English-speaking countries in *schooling* does not hold in the other dimensions. For instance, Senegal appears to display the least deprivation in *assets*, and Madagascar is the best in the *empowerment* dimension. In fact, for *assets*, only Senegal displays a proportion of deprivation less than 50%. Using earlier DHS data on several African countries including eight countries from our sample, Sahn and Stifel (2000) find that Senegal and Ghana are less poor than the other countries when an asset index is considered as a poverty measure. In another study also based on asset index, Booysen, Von Maltitz, Van der Berg, Burger, and Du Rand (2008) find that Senegal is better than Ghana, Mali, Kenya and Tanzania. The data used in the present study are more recent than those in the previous studies. Concerning *empowerment*, only 26% of women have no say on visits to family and relatives in Madagascar against more than 40% for the remaining countries except for Uganda where the proportion is about 36%. On the whole, *BMI* is the dimension in which women are deprived the least. As in Alkire and Foster (2007), the health problems that could be posed by the high levels of *BMI* (obesity) are ignored.

Figure 2: Multidimensional poverty indicators for cutoff $k = 2$

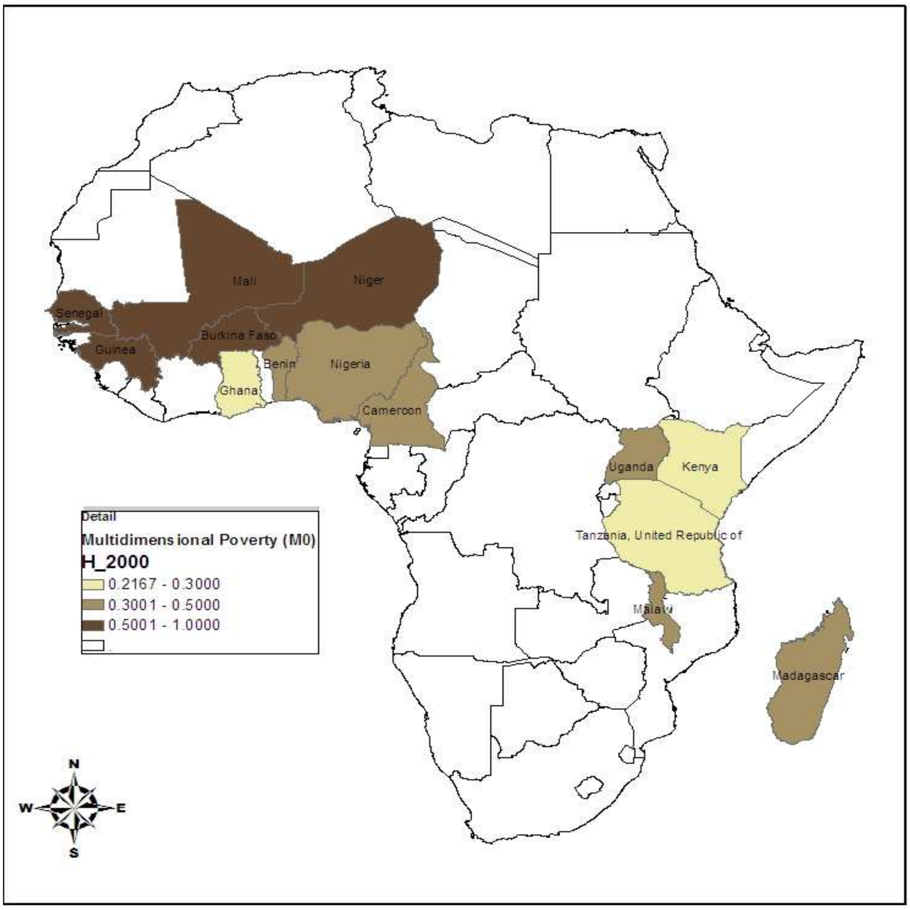


Tables A2 and A3 in appendices show marked differences in deprivation between the nested dimensions of *assets*. The most prevalent deprivations are a lack of electricity, refrigerator, vehicle and phone. Even if refrigerator and vehicle could be seen as luxury goods to a certain extent, electricity and phone, which may be public utilities, are likely to influence economic growth and poverty reduction.

The multidimensional poverty measures are estimated from all dimensions and reported in the figures 2 and 4 for $k = 2$ and $k = 3$ respectively.

When $k = 2$, the country rankings remains the same across the three measures namely the headcount ratio (H), the adjusted headcount ratio (M_0) and the adjusted poverty gap (M_1). It is obvious that $H > M_0 > M_1$ for all k, due to the way they are computed. For instance, M_0 is derived from H by multiplying the latter by the average deprivation share. Unless all poor are deprived in all dimensions for some k, in which case $M_0 = H$ for these k, M_0 will be always lower than H. M_1 is computed in the presence of cardinal dimensions by multiplying M_0 by the average poverty gap. Since the average poverty gap is lower than 1, M_1 will be lower than M_0. For convenience, the sample of countries could be shared

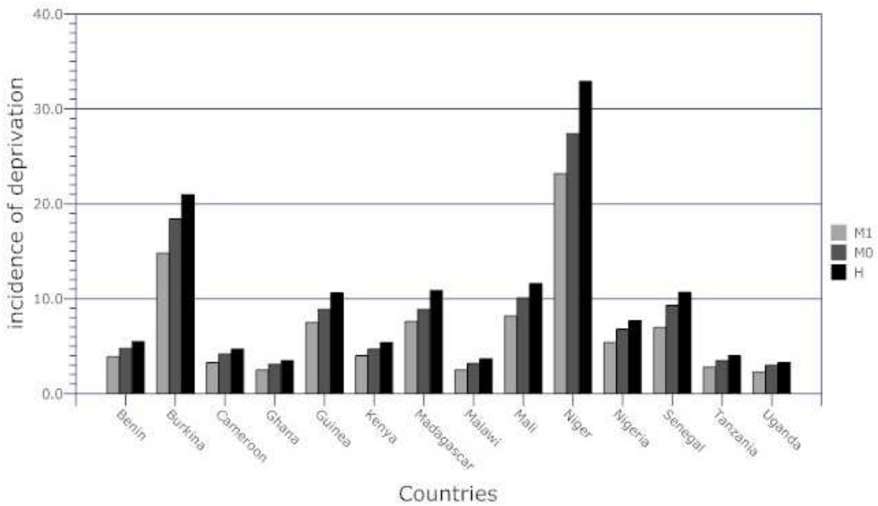
Figure 3: Poverty classification in Africa based on Multidimensional headcount and for $k = 2$



out in three groups. This classification is presented in the figure 3. The first group includes the poorest countries in which the multidimensional headcount ratio exceeds 50%. These countries, all of them being francophone, are Guinea (56%), Senegal (62%), Mali (66%), Burkina (70%) and Niger (81%). Apart from Guinea, the remaining countries are Sahelian ones, which could explain in a certain extent the relative poverty of these countries.

The second group is composed by the medium poor countries whose multidimensional headcount ratio is between 30% and 50%. That are Malawi (30%), Uganda (31%), Cameroon (32%), Benin (39%), Madagascar (41%) and Nigeria (41%). The remaining group is made up by the three relatively richest countries, Kenya, Ghana and Tanzania whose ratios are below 30%. Note that the proportion of poor is almost four times higher in the poorest (Niger) than in the richest (Kenya). By taking account of education, health and empowerment as additional dimensions, our results differ from those obtained by considering only the asset index. For instance, Senegal was often identified as a relatively wealthy country when our results suggest the opposite.

Figure 4: Multidimensional poverty indicators for cutoff $k = 3$



The previous rankings change a little when $k = 3$. The poorest group remains the same except that Madagascar and Guinea switch their group, Madagascar becoming poor and Guinea becoming medium. Some changes are also seen in the top group since Kenya and Tanzania are replaced by Uganda and Malawi. Burkina and Niger remain by far at the bottom, which is consistent with the usual view that these two countries are among the poorest and the least developed in the world. Here there is a little bit of difference between the rankings from H and those from M_0 since Senegal and Madagascar switch their rank.

Table 2: Poverty measures as cutoff k is varied, for Benin

cutoff k	Number of people	Headcount ratio H	Adjusted headcount ratio M_0	Adjusted poverty gap M_1	Average deprivations $A(k)$
0.5	15254	0.943	0.475	0.423	2.012
1	14885	0.920	0.471	0.420	2.047
1.5	12530	0.775	0.425	0.379	2.193
2	6314	0.390	0.262	0.224	2.685
2.5	4788	0.296	0.211	0.180	2.845
3	896	0.055	0.048	0.039	3.440
3.5	489	0.030	0.029	0.022	3.776
4	68	0.004	0.004	0.003	4.000

For analysing the behavior of poverty measures when k varies, we consider the case of Benin. Benin is a medium poor country and is represented by the largest sample with

16172 observations. The main results are presented in table 2. As expected, all measures (H,M_0,M_1) decrease when k increases. Indeed, an increase in k is seen as a decrease in poverty line. The variable $A(k)$ in the last column is obtained by multiplying A in equation (5) by d the number of dimensions. It represents the average number of deprivations in the poor people. For any k, as the poor individuals are identified as those with at least k deprivations, then $A(k) \geq k$. If there is at least any individual deprived in all dimensions, then $A(k) > k$ when $k < d$ and $A(d) = d$. When $k = 0$, $A(k)$ becomes simply the average number of deprivations in the pooled populations. The estimated $A(0)$ is 1.93. M_0 and M_1 differ from H by taking account of this relative depth of deprivation in poverty measurement. For instance, M_0 will be close to H only if poor people tend to be deprived in all dimensions.

For $k = 1$, the poverty is overestimated since the multidimensional headcount ratio H, which is the same as the FGT incidence of poverty, varies from 66% in Kenya to 98% in Niger. For $k = 3$, that is the opposite case. The incidence of poverty varies from 3% in Uganda to 33% in Niger. Poverty appears underestimated since it is unthinkable that, in a developing country like Uganda, only 3% of women can be considered as poor. On the other hand, for $k = 2$, H take values between 22% in Kenya and 81% in Niger. This result seems more reasonable and is in accordance with the previous results, including the World Bank findings that, in average, about 50% of individuals are poor in Sub-Saharan Africa. Then, the cutoff $k = 2$ may be considered as suitable enough for doing some analyses including the comparisons with the standard poverty measures.

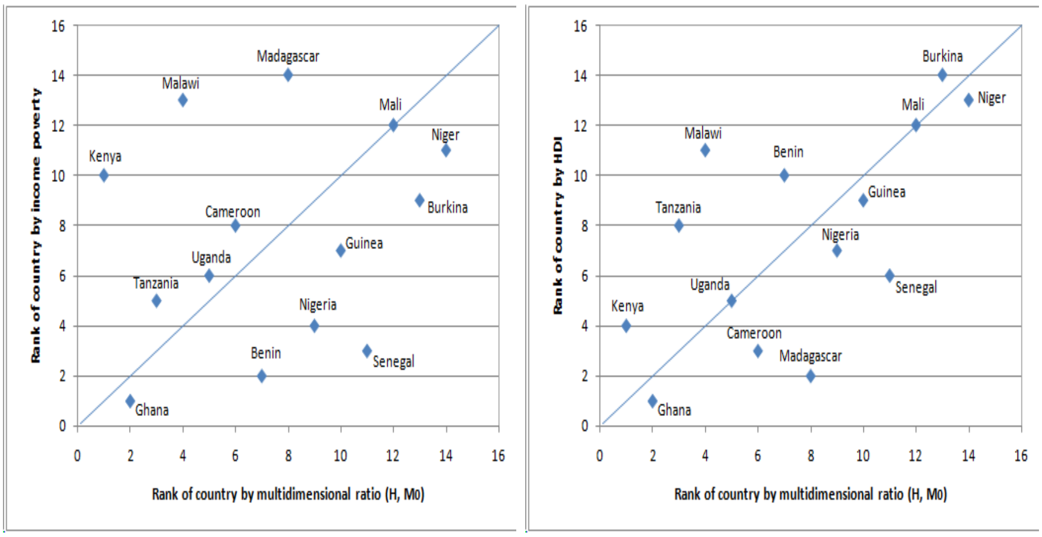
3.3 Comparisons with some standard measures

We now compare these findings with two standard poverty measures: income poverty and human development index (HDI). This is useful to capture the changes in the poverty assessment induced by the current multidimensional measure. The country rankings from the current multidimensional poverty measure are plotted against the rankings obtained when using the two alternative measures. Figure 5 presents the comparison results for $k = 2$.

When rankings according two measures are identical, all the points in the graph, representing each country, should be on the 45-degree line. Since the measures used in this paper go beyond the standard ones in the sense of considering additional and uncommon dimensions, we are not expecting to find strong correlations between them. Moreover, since the woman is the unit of analysis, the current estimated measures could reflect in a certain extent gender differences. Figure 5 shows that differences in rankings from M_0 and those based on income poverty are substantial. Except for Mali and Ghana and Uganda in a little measure, most countries shifted noticeably. These differences could also be partially due to the incompatibility between dates. The date of income poverty estimation varies from 1992 to 2006 according countries when M_0 is estimated from DHS data posterior to 2000 for all selected countries. A quite similar result is obtained by Sahn and Stifel (2000) who find substantial differences between the asset index rankings and the rankings based on GDP per capita.

The differences are less marked between HDI and M_0. All the points are closer to the 45-degree line. Indeed Mali, Uganda, Ghana, Guinea, Burkina and Niger are on the line or very close to it.

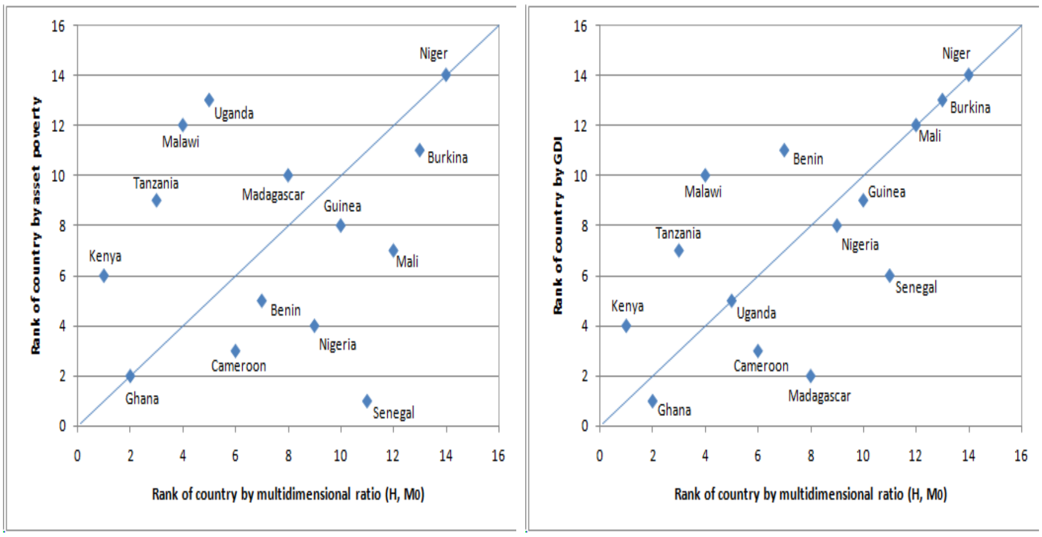
Figure 5: Countries ranking comparisons for $k = 2$



(a) Against income poverty (b) Against human development index

Our results are also compared with two additional measures: asset poverty and gender related development index (GDI). Figure 6 shows the comparison results for $k = 2$. Asset poverty is estimated from the current method by considering only the eight indicators of *Assets*. Each indicator is weighted to 1 and the poverty measured by M_0 for $k = 4$. The results, illustrated in the part (a) of the figure 6, suggest significant differences in rankings. This is due to the fact that the dimensions are uncorrelated or weakly correlated. For example, Senegal which is the best with asset poverty appears among the poorest countries when we take account of the three other dimensions. When the correlations between dimensions are perfect, the findings will be unchanged whatever the number of dimensions which are considered. Then, the differences in rankings are more important when the correlations decrease. The part (b) of figure shows the comparison with GDI. The differences are less

Figure 6: Countries ranking comparisons for $k = 2$



(a) Against asset poverty (b) Against gender related development index

important than in the part (a). The result is almost identical to the one obtained in part (b) of figure 5 because HDI and GDI induce very closed rankings. The detailed results are provided in tables A4, A5 and A6 in appendices.

On the whole, the adding of *schooling*, *health* and *empowerment* allows us to better capture multidimensional poverty in the sense of Sen (1976; 1985; 1992; 1995). This creates poverty measures more suitable and a little different from the standard and previous indicators. Indeed, restricting the poverty analysis only to the *assets*, like in previous studies on Africa, leads often to incorrectly capture poverty since there is many other well-being dimensions. Moreover, the consideration of a larger number of dimensions is likely to reduce the effect of the discontinuities in each dimension on the aggregate poverty measure.

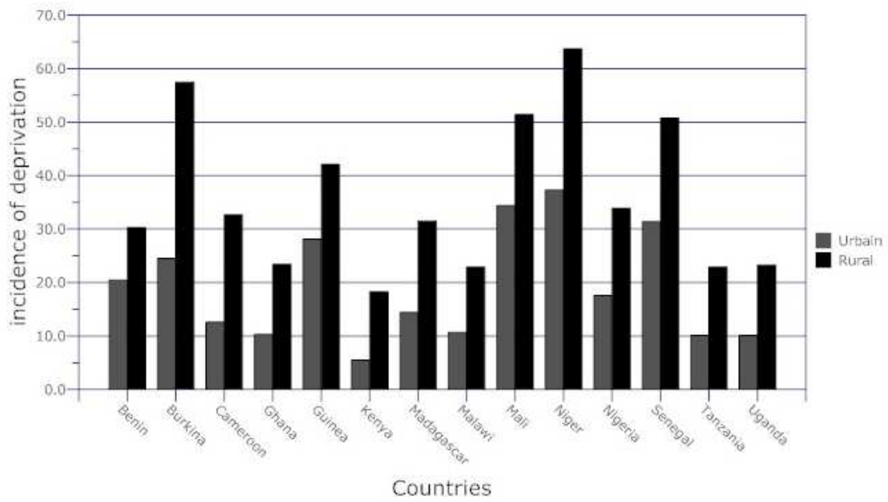
4 Decomposing poverty

4.1 Decomposing by location

Decomposition is a useful property allowing to analyse poverty by subgroups. It is then straightforward to implement better poverty-reducing policies by targeting the kinds of deprivations affecting each subgroup. The poverty measures, when $k = 2$, are decomposed by urban and rural location in all the countries and findings are illustrated in figure 7.

This figure compares rural and urban poverty for the adjusted multidimensional headcount ratio. It suggests unambiguously a difference in poverty between urban and rural locations. In all countries, rural people appear to be more poorer than urban people. This is not surprising since such outcomes were ready observed in the previous studies. Using various welfare indicators (asset poverty, enrollment, infant mortality rate, adult malnutrition, etc.), Sahn and Stifel (2003b) show that standards of living in rural areas are lower than those in urban areas in African countries. Booysen, Von Maltitz, Van der Berg, Burger, and Du Rand (2008) obtain a quite similar results with asset poverty. Using household expenditures per capita and children's height-for-age z score as the two dimensions of well-being, Duclos, Sahn, and Younger (2006a) test stochastic dominance relations between rural and urban areas in Ghana, Madagascar and Uganda. They also find that rural areas are poorer than the urban areas.

Figure 7: Decomposition of adjusted headcount M_0 by urban and rural area, for $k = 2$

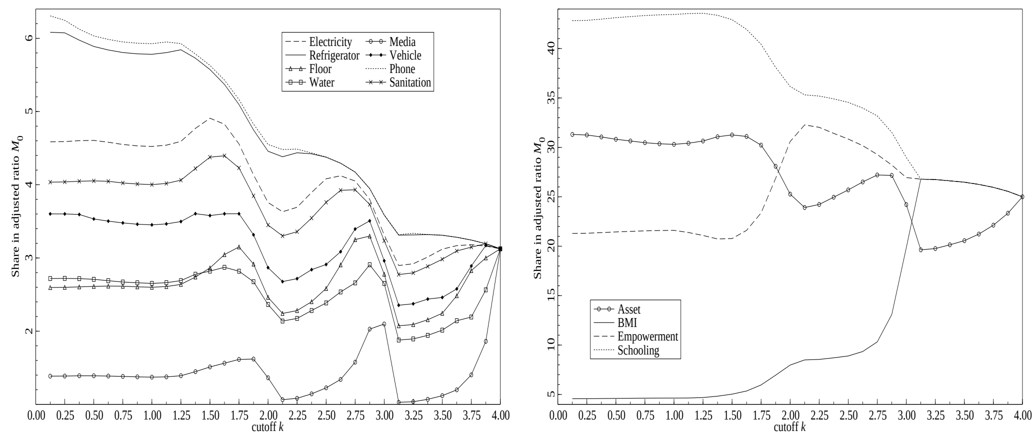


But the inequality in poverty between rural and urban areas seems to differ across countries. From figure 7, one can notice that, for some relatively less poor countries like Kenya, Ghana, Malawi, Cameroon, Tanzania and Uganda, the incidence of poverty is at least twice as high in rural areas as in urban ones. This is not the case for the poorest countries (Mali, Niger, Senegal, Guinea). Burkina is the only poor country where the imbalance is marked like in the former countries. Benin, which is a medium poverty country, displays one of the most balanced poverty situations since the difference between rural poverty and urban poverty is relatively low. The appreciable urban-rural inequality observed in Burkina is underlined by Batana and Duclos (2008) from a dominance analysis. However, this inequality, in our multidimensional measurement context, could be overestimated a little since some dimensions such as land and livestock owning, which is generally strongly valued in rural areas, are missing in the DHS database.

4.2 Decomposing by dimension

For assessing how the poverty is affected by the deprivations in each dimension, one can decompose the adjusted ratio M_0 into dimensions. The straightforward way for doing it is to disaggregate A in equation (5) into the four dimensions and eleven indicators. Let A_j be the contribution of dimension j in A. A_j could be interpreted as the *average deprivation share* across the poor in dimension j. The dimensional adjusted ratio M_{0j} is then obtained like in equation (6), by multiplying H and A_j. Dimensional decomposition is done when $k = 2$ for all countries, and the main results are reported in tables A7 and A8 in appendices. It is obvious that such a decomposition reflects a lot the differences in incidence of deprivation as shown in tables A2 and A3 and in figure 1.

Figure 8: Decomposition of adjusted ratio M_0 by dimension as k is varied for the pool of countries



(a) Nested components of *assets* (b) Four dimensions

Figure 8 describes the behavior of the dimensional adjusted ratio M_{0j} for Benin when k is varied. Concerning asset indicators, whose shares are plotted in the part (a), it is clear that deprivations in refrigerator and telephone contribute the most to the poverty for any k. The other most influential indicators, according to their poverty contribution, are in order electricity, sanitation and vehicle owning. The floor quality and the drinking water access are a third group of contributors less important than the previous. Finally, radio and/or TV (media) owning appears to contribute the least to poverty. The ranking between the above four groups of contributors is robust to the variation in k.

The sum of the shares of nested dimensions gives the total share in M_0 for *assets*. The part (b) of figure 8 shows that the rankings in contribution of poverty depend lightly on k. For $k \leq 1.5$, the contributions remain stable for the four dimensions. *Schooling* has the biggest contribution (44%), followed by *assets* (31%) and *empowerment* ((21%)) respectively, while *BMI* has a negligible contribution (4%). From $k = 1.5$, the contributions of *schooling* and *assets* begin to decline while those for *empowerment* and *BMI* start to increase. From $k = 1.875$, *empowerment* become the second contributor behind *schooling*. Finally, the contribution of *BMI* which is almost marginal with low levels of k, starts to increase significantly from $k = 1.75$. This coincides with a decline in other dimensions. Indeed, since the sum of contributions is 100, then a decrease in any dimension is balanced by an increase in another dimension.

Figure 9: Decomposition of multidimensional poverty by dimension for each country and for $k = 2$

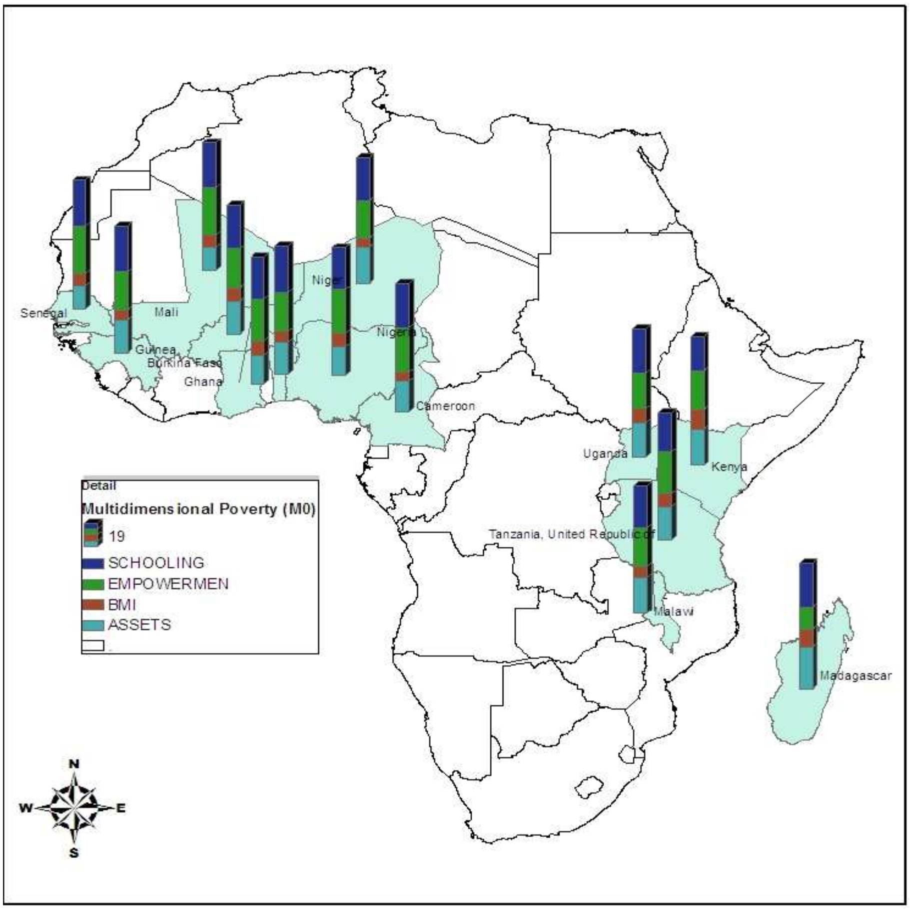


Figure 9 presents, for $k = 2$, the contributions of the four dimensions to the poverty. Burkina, Guinea and, in the little extent, Malawi display results close to Benin since *schooling* appears to be the most contributor, followed respectively by by *empowerment* and *as-*

sets. In countries like Cameroon, Ghana, Mali, Nigeria, Senegal and Tanzania, the contributions of *schooling* and *empowerment* are almost the same and both are higher than the *assets* contribution. The contributions of these three dimensions are quiet equal in Kenya while only *empowerment* and *assets* have the same contribution, a little lower than the *schooling* contribution, in Niger and Uganda. Finally, the poverty in Madagascar is explained mainly by *schooling* and *assets*. On the whole, BMI has a weak contribution with a relative high level in Kenya and Madagascar. This decomposition of contribution by country could be helpful for suggesting better sectorial poverty-reducing policies.

5 Robustness analysis

5.1 Variations in cutoff k

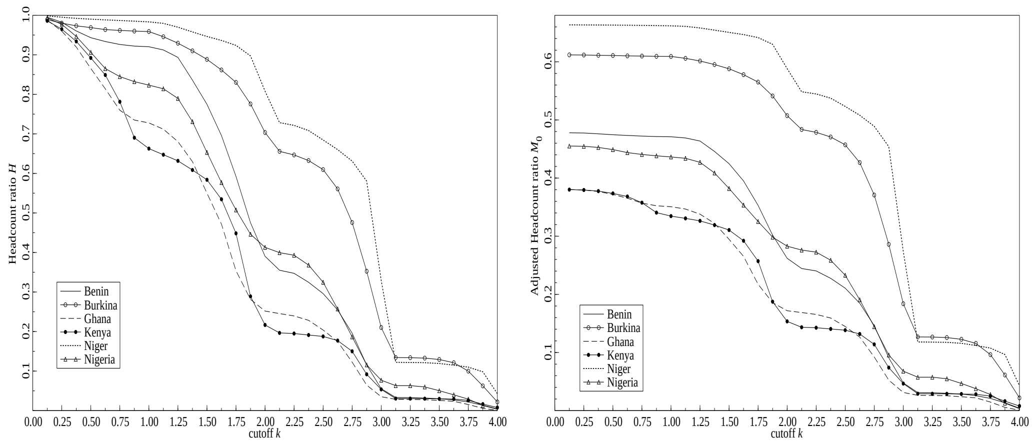
To analyse whether the country rankings are robust across k, we implement the methodological approach in subsection 2.3. Figure 10 illustrates some dominance relations from a sub-sample of countries including two highest poverty countries (Burkina, Niger), two medium countries (Benin, Nigeria) and two countries with relatively low poverty level (Ghana and Kenya). Each curve in the figure describes the poverty level in the country when k is varied. Dominance is then possible between two countries when any curve lies above or below another for all possible values of k. When two curves cross, there is no possibility of dominance. In the two graphs of the figure, no dominance relation exists between Ghana and Kenya, Benin and Nigeria or Burkina and Niger since all these pairs of curve intersect. There are yet possibilities of dominance between any country from one pair and any country from another pair.

Several relations are identified from the full sample of countries and are tested under the null hypothesis of non dominance against the alternative of dominance. The results are reported in tables 3 and 4, respectively for H and M_0.

In the case of H, 42 dominance relations are proved to be significant at 5% level, out of the combination of $C_{14}^2 = 91$ possible relations. Niger is dominated by all countries except Burkina. Moreover, Ghana, Kenya and Cameroon dominate the five other poorest countries that are Nigeria, Senegal, Guinea, Mali and Burkina. Some of these dominance relations are restricted in the sense that they are found to be significant only for a reasonable restricted range of k. There is no dominance between the eight first countries ranked as by the table A5.

The non dominance relations, especially when curves intersect, could partly denote

Figure 10: Poverty comparisons for some countries as k is varied



(a) Comparisons in H (b) Comparisons in M_0

Table 3: Results of dominance in headcount ratio H for all countries

	Gha	Ken	Cam	Tan	Mwi	Uga	Ben	Mad	Nga	Sen	Gui	Mli	Bur	Ngr
Ghana	–	ND	ND	ND	ND	ND	ND	ND	D	$D*$	D	D	$D*$	D
Kenya		–	ND	ND	ND	ND	ND	ND	$D*$	$D*$	$D*$	$D*$	$D*$	D
Cameroon			–	ND	ND	ND	ND	ND	$D*$	$D*$	$D*$	D	$D*$	D
Tanzania				–	ND	ND	ND	ND	ND	ND	$D*$	D	$D*$	D
Malawi					–	ND	ND	ND	ND	ND	$D*$	D	$D*$	D
Uganda						–	ND	ND	ND	ND	$D*$	$D*$	$D*$	$D*$
Benin							–	ND	ND	ND	$D*$	$D*$	$D*$	D
Madagascar								–	ND	ND	ND	ND	$D*$	D
Nigeria									–	ND	ND	D	$D*$	D
Senegal										–	ND	ND	$D*$	D
Guinea											–	ND	ND	D
Mali												–	ND	$D*$
Burkina													–	ND
Niger														–

ND: non dominance; D: dominance; $D*$: restricted dominance obtained for at least $0.5 \leq k \leq 3.5$.

possible discontinuities in deprivations along k. These discontinuities are illustrated by the sharp bends observed in the curves of figure 10. Since the headcount curves could be seen as cdf_s for a decreasing series of k, this means that density curves are lumpy. The lumps are induce by the nested dimensions. In fact, the decimal values of k describe four superimposed distributions respectively for the intervals $[0, 1]$, $[1, 2]$, $[2, 3]$ and $[3, 4]$. Except for the discontinuities, the non dominance could be obviously explained by the main moments of the distribution that are mean and variance in the case of a normal distribution. The curves representing M_0 appear to be smoother than those for H, particularly for k lower than 2. This leads us to expect more dominance relations in the case of M_0.

Indeed, with M_0, 14 additional dominance relations are obtained, that is a total of 56

Table 4: Results of dominance in adjusted headcount ratio M_0 for all countries

	Gha	Ken	Tan	Mwi	Cam	Uga	Ben	Mad	Nga	Sen	Gui	Mli	Bur	Ngr
Ghana	–	ND	ND	ND	$D*$	ND	ND	D	D	$D*$	D	D	D	D
Kenya		–	ND	ND	ND	ND	ND	ND	$D*$	$D*$	$D*$	$D*$	D	D
Tanzania			–	ND	ND	ND	ND	ND	$D*$	$D*$	D	D	D	D
Malawi				–	ND	ND	ND	D	ND	$D*$	D	D	D	D
Cameroon					–	ND	ND	ND	D	$D*$	D	D	D	D
Uganda						–	ND	ND	ND	ND	$D*$	D	D	D
Benin							–	ND	ND	$D*$	$D*$	$D*$	D	D
Madagascar								–	ND	ND	D	D	D	D
Nigeria									–	$D*$	ND	D	D	D
Senegal										–	ND	ND	D	D
Guinea											–	$D*$	D	D
Mali												–	D	D
Burkina													–	ND
Niger														–

ND: non dominance; D: dominance; $D*$: restricted dominance obtained for at least $0.5 \leq k \leq 3.5$.

relations out of the 91. Here, Burkina and Niger are dominated by all other countries. This result confirms that both countries are the worst in term of poverty and well-being. Our dominance results differ from those obtained by Sahn and Stifel (2000). For instance, in their findings, Senegal is in the top with Ghana and dominates countries such as Kenya, Tanzania, Cameroon, Uganda and Benin. That is understanding since Senegal is relatively better in asset-based well-being. On the whole, as in subsection 3.2, five francophone countries appear to be the poorest.

5.2 Changes in thresholds selections

For analysing the effect of a change in thresholds on poverty measures, we modify some definitions from table 1. Most of indicators remain unchanged, but the poverty cutoffs for five indicators changed, including four nested dimensions (Media, vehicle, drinking water, sanitation) and the dimension *schooling*. This choice is explained by the possibility to make some sensible changes in the poverty line. The previous set of thresholds could be seen as the MDG definition. Two additional sets of thresholds are defined, the first like a stronger definition than MDG one and the second like a weaker definition. For example, in strong definition, a woman is considered as being deprived in *schooling* if her education level in single years is low than 10. More details on the new definitions can be found in the table 5.

We analyse whether the country rankings are affected by adopting on of the two alternative definitions. For $k = 2$, the results are illustrated in figure 11. From MDG definition, choose strong definition induces a re-ranking of country except for the three bottom coun-

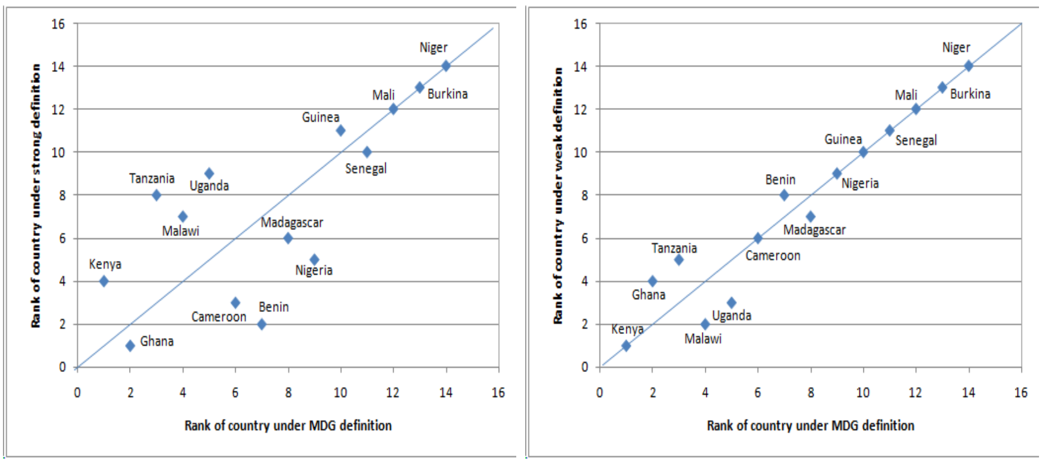
Table 5: Alternative definitions of thresholds

Dimension	Strong definition	Weak definition
Media	If does not have TV and radio	–
Vehicle	If does not have at least a car	If does not have at least a bycicle
Water	If does not have at least a piped water	If does not have at least a dug well water
Sanitation	If does not have at least a flush toilet	–
Schooling	If highest education level < 10	If highest education level < 3

tries which are Mali, Burkina and Niger. In general, this re-ranking seems stronger within the least poor countries. The comparisons between MDG and weak definition shows some re-rankings, less important than in the previous case. The situation remains unchanged for most countries, especially the poorest.

Changes in poverty thresholds definition are likely to re-rank the countries and, so, could change the previous dominance analysis. This finding does not matter since it depends on the multivariate distribution of poverty indicators in each country. The same conclusion may be found in univariate distribution, when two distributions intersect for any unidimensional poverty line $\overline{z}$.

Figure 11: Country rankings sensitivity to changes in thresholds definition, with $k = 2$



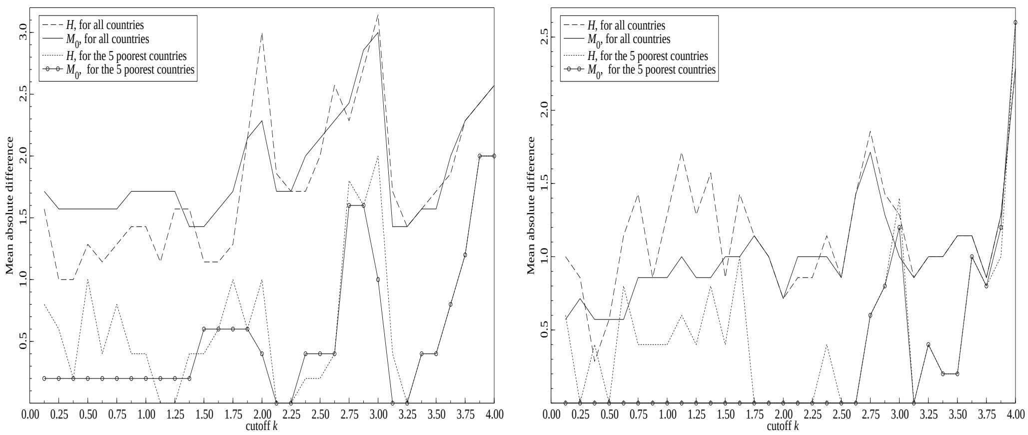
(a) Against strong definition (b) Against weak definition

For analysing the evolution of the rankings when k is varied, a mean absolute difference (MAD) is calculated as follows:

$$MAD_k = \frac{1}{J}\sum_{j=1}^{J}|R_j(k) - \overline{R}_j(k)|,$$

where j represents each country, J the total number of countries, $R_j(k)$ the rank of country j from MDG definition and $\overline{R}_j(k)$ the rank of country j from alternative definition.

Figure 12: Evolution of mean absolute difference as k is varied



(a) MDG vs strong definition (b) MDG vs weak definition

The MAD is estimated for each k and is illustrated in figure 12. Each graph presents four curves for H and M_0, for all countries and for only the five bottom countries according the MDG definition. In the whole, the poverty measures appear to be sensitive to the thresholds definition since MAD is strictly positive for most k. However, the depth of the re-ranking varies according k. In the case of MDG versus strong definition, it appears relatively low and stable for $k \leq 1.75$. The curves for M_0 are smoother than the ones for H. This figure also confirms that the bottom countries depict less change in rankings than the others. For instance, in the case of MDG versus weak definition, for $k \leq 2.625$, there is no re-ranking for these countries, which is shown in figure 11 for $k = 2$. This means that there are high likelihoods so that some dominance relations obtained in subsection 5.1 hold across changes in thresholds definition, when these relations imply the domination of one of the bottom countries.

The sensitivity of this poverty measure suggests that, for identifying the poor, one should select a set of thresholds reasonable enough so that a deprivation in any dimension reflects ethically and consensually an aspect of poverty. In this way, MDG could be useful, as in this paper, for appealing to a suitable poverty identification.

6 Conclusion

Since poverty is recognized as being multidimensional, many methods of aggregation are proposed in the literature. An important issue that arises is to identify the poor. This identification concerns both the deprivation in each dimension and the poverty definition across all dimensions. Alkire and Foster (2007) suggest an intuitive method using counting approach that deals with this issue in the two stages of identification. Moreover, this measure satisfies several useful properties which allow, for instance, targeting and poverty comparisons over time and across countries or regions.

Basing on such an approach, this paper estimated multidimensional poverty in fourteen sub-Saharan African countries. The findings suggest, for convenience, three groups of countries, the first including the poorest countries with headcount ratio exceeding 50%, the second composed by the medium poor countries with ratio between 30 and 50%, and the third by countries with ratio less than 30%. The differences between countries seem important since, for instance, poverty in the bottom country is many times higher than poverty in the top country. Comparisons with some standard measures such as HDI, income poverty, GDI and asset poverty show that consider additional dimensions leads to country rankings different, in a certain extent, from the standard-based rankings. Decompositions by rural/urban location and by dimension are done, showing both that the poverty is higher in rural areas than in urban areas, and that the contributions in poverty of deprivations in *schooling*, *empowerment* and *asset* depend on each country. Some robustness and sensitivity analyses are implemented to capture the effect of variations in cutoff k and changes in thresholds. In the first case, many dominance relations are obtained denoting that a difference in poverty between two countries is robust along the full range of k. The second analysis reveals the sensitivity of poverty measures to changes in thresholds.

On the whole, the proposed method seems to be suitable for measuring poverty in developing countries such as those from Sub-Saharan Africa. The sensitivity issue suggests to select the thresholds in an ethically and consensually reasonable way. In this paper, that is done by selecting some poverty lines using as far as is possible the MDG criterions. Another issue which arises with this approach is relating to the weighting. None of the weighting methods is proved to be the best. The poverty measurement approach used here does not provide a suitable method to address this matter. However, it gives the latitude to assign weights to each dimension in positive or normative way. In this paper, four dimensions are equally and normatively weighted by 1, and one of them (*assets*) also equally divided into eight nested dimensions. Future extensions could consist in exploring the effects of change in the weighting on the poverty measures.

Appendices

Table A1: Characteristics of samples from DHS

Country	Survey years	Obs number	Rural area (%)
Benin	2006	16172	57.9
Burkina	2003	11864	76.6
Cameroon	2004	4738	51.4
Ghana	2003	5235	58.9
Guinea	2005	3757	71.3
Kenya	2003	6980	68.3
Madagascar	2003	7541	35.4
Malawi	2004	10864	86.5
Mali	2006	13622	64.9
Niger	2006	4396	66.7
Nigeria	2003	7121	59.9
Senegal	2005	4220	56.6
Tanzania	2004	9598	75.9
Uganda	2006	1742	87.7

Table A2: Incidence of deprivation (in percentage) in all dimensions

	Benin	Burkina	Cameroon	Ghana	Guinea	Kenya	Madagascar
Electricity	70.1	87.3	50.3	50.1	79.0	82.3	77.0
Media	21.2	29.3	28.8	23.9	30.1	20.0	35.7
Refrigerator	93.0	93.4	83.7	76.9	90.5	93.6	96.1
Vehicle	55.1	69.1	82.7	90.6	83.9	92.3	96.8
Floor	39.7	58.1	46.8	11.8	53.1	61.4	66.9
Phone	96.4	94.8	97.6	90.0	91.7	83.6	93.6
Water	41.6	51.2	34.5	47.4	67.5	59.4	59.7
Sanitation	61.7	67.2	6.1	23.5	28.2	14.9	41.7
Asset	59.9	68.8	53.8	51.8	65.5	63.5	70.9
BMI	8.7	19.7	6.7	9.1	12.2	11.8	18.3
Empowerment	40.7	69.2	55.5	48.4	52.5	50.0	26.0
Schooling	81.9	87.1	46.0	42.9	88.4	26.7	69.9

Table A3: Incidence of deprivation (in percentage) in all dimensions (end)

	Malawi	Mali	Niger	Nigeria	Senegal	Tanzania	Uganda
Electricity	91.7	80.1	88.7	47.0	46.3	86.8	91.6
Media	31.7	21.7	40.3	21.8	7.8	35.3	31.6
Refrigerator	96.2	93.9	96.0	80.0	73.4	94.2	96.8
Vehicle	96.5	61.5	89.7	74.3	80.2	96.0	95.8
Floor	76.8	71.8	85.4	33.4	30.5	70.8	78.5
Phone	93.4	94.6	98.9	94.5	75.1	87.8	99.4
Water	54.8	71.0	76.6	53.0	7.8	38.7	53.3
Sanitation	13.9	18.4	79.1	23.7	17.5	13.5	12.5
Asset	69.4	64.1	81.9	53.5	42.3	65.4	69.9
BMI	8.5	12.3	17.2	14.1	17.0	9.5	11.0
Empowerment	42.5	69.0	73.6	64.3	70.7	52.9	35.5
Schooling	58.1	88.2	92.7	50.2	78.5	39.7	62.7

Table A4: Countries ranking according several indicators for $k = 1$

	Adjust. head. ratio M_0		Income poverty* ratio		HDI (2007, UNDP)		GDI (2007, UNDP)		Asset poverty** for $k = 4$	
Country	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
Kenya	0.335	1	0.520	10	0.521	4	0.521	4	0.589	6
Ghana	0.351	2	0.280	1	0.553	1	0.549	1	0.424	2
Cameroon	0.375	3	0.410	8	0.532	3	0.524	3	0.429	3
Tanzania	0.378	4	0.360	5	0.467	8	0.464	7	0.619	9
Malawi	0.414	5	0.650	13	0.437	11	0.432	10	0.675	12
Uganda	0.415	6	0.380	6	0.505	5	0.501	5	0.677	13
Nigeria	0.436	7	0.340	4	0.470	7	0.456	8	0.435	4
Madagascar	0.441	8	0.710	14	0.533	2	0.530	2	0.653	10
Benin	0.471	9	0.290	2	0.437	10	0.422	11	0.527	5
Senegal	0.518	10	0.330	3	0.499	6	0.492	6	0.307	1
Guinea	0.543	11	0.400	7	0.456	9	0.446	9	0.610	8
Mali	0.582	12	0.640	12	0.380	12	0.371	12	0.595	7
Burkina	0.609	13	0.460	9	0.370	14	0.364	13	0.610	11
Niger	0.662	14	0.630	11	0.374	13	0.355	14	0.794	14

* Provided by World Development Indicators (WDI) online.

** Calculated by considering the asset indicators only, each indicator weighted to 1.

Table A5: Countries ranking according several indicators for $k = 2$

Country	Adjust. head. ratio M_0		Income poverty* ratio		HDI (2007, UNDP)		GDI (2007, UNDP)		Asset poverty** for $k = 4$	
	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
Kenya	0.153	1	0.520	10	0.521	4	0.521	4	0.589	6
Ghana	0.172	2	0.280	1	0.553	1	0.549	1	0.424	2
Tanzania	0.193	3	0.360	5	0.467	8	0.464	7	0.619	9
Malawi	0.209	4	0.650	13	0.437	11	0.432	10	0.675	12
Uganda	0.216	5	0.380	6	0.505	5	0.501	5	0.677	13
Cameroon	0.219	6	0.410	8	0.532	3	0.524	3	0.429	3
Benin	0.262	7	0.290	2	0.437	10	0.422	11	0.527	5
Madagascar	0.273	8	0.710	14	0.533	2	0.530	2	0.653	10
Nigeria	0.283	9	0.340	4	0.470	7	0.456	8	0.435	4
Guinea	0.379	10	0.400	7	0.456	9	0.446	9	0.610	8
Senegal	0.411	11	0.330	3	0.499	6	0.492	6	0.307	1
Mali	0.457	12	0.640	12	0.380	12	0.371	12	0.595	7
Burkina	0.507	13	0.460	9	0.370	14	0.364	13	0.610	11
Niger	0.588	14	0.630	11	0.374	13	0.355	14	0.794	14

* Provided by World Development Indicators (WDI) online.

** Calculated by considering the asset indicators only, each indicator weighted to 1.

Table A6: Countries ranking according several indicators for $k = 3$

Country	Adjust. head. ratio M_0		Income poverty* ratio		HDI (2007, UNDP)		GDI (2007, UNDP)		Asset poverty** for $k = 4$	
	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank
Uganda	0.030	1	0.380	6	0.505	5	0.501	5	0.677	13
Ghana	0.031	2	0.280	1	0.553	1	0.549	1	0.424	2
Malawi	0.032	3	0.650	13	0.437	11	0.432	10	0.675	12
Tanzania	0.035	4	0.360	5	0.467	8	0.464	7	0.619	9
Cameroon	0.042	5	0.410	8	0.532	3	0.524	3	0.429	3
Kenya	0.047	6	0.520	10	0.521	4	0.521	4	0.589	6
Benin	0.048	7	0.290	2	0.437	10	0.422	11	0.527	5
Nigeria	0.068	8	0.340	4	0.470	7	0.456	8	0.435	4
Guinea	0.089	9	0.400	7	0.456	9	0.446	9	0.610	8
Senegal	0.093	11	0.330	3	0.499	6	0.492	6	0.307	1
Madagascar	0.089	10	0.710	14	0.533	2	0.530	2	0.653	10
Mali	0.101	12	0.640	12	0.380	12	0.371	12	0.595	7
Burkina	0.184	13	0.460	9	0.370	14	0.364	13	0.610	11
Niger	0.274	14	0.630	11	0.374	13	0.355	14	0.794	14

* Provided by World Development Indicators (WDI) online.

** Calculated by considering the asset indicators only, each indicator weighted to 1.

Table A7: Decomposing M_0 by dimension and country for $k = 2$

	Electricity	Media	Refrigerator	Vehicle	Floor	Phone
Kenya						
M_{0j}	0.0064	0.0027	0.0067	0.0066	0.005	0.0064
Share (%)	4.199	1.780	4.351	4.310	3.7108	4.189
Ghana						
M_{0j}	0.0057	0.0023	0.0071	0.0074	0.0015	0.0076
Share (%)	3.293	1.332	4.104	4.284	0.866	4.417
Tanzania						
M_{0j}	0.0081	0.0037	0.0084	0.0084	0.0074	0.0081
Share (%)	4.211	1.913	4.353	4.363	3.837	4.211
Malawi						
M_{0j}	0.0092	0.0039	0.0093	0.0093	0.0084	0.0092
Share (%)	4.416	1.868	4.476	4.457	4.040	4.420
Uganda						
M_{0j}	0.0095	0.0044	0.0096	0.0096	0.0087	0.0097
Share (%)	4.403	2.037	4.458	4.444	4.049	4.480
Cameroon						
M_{0j}	0.0077	0.0040	0.0094	0.0083	0.0071	0.0099
Share (%)	3.497	1.812	4.272	3.774	3.231	4.507
Benin						
M_{0j}	0.0098	0.0036	0.0117	0.0075	0.0065	0.0119
Share (%)	3.757	1.364	4.458	2.868	2.462	4.550
Madagascar						
M_{0j}	0.0120	0.0079	0.0127	0.0127	0.0113	0.0126
Share (%)	4.394	2.898	4.640	4.635	4.134	4.629
Nigeria						
M_{0j}	0.0082	0.0039	0.0120	0.0105	0.0065	0.0127
Share (%)	2.905	1.384	4.238	3.727	2.311	4.494
Guinea						
M_{0j}	0.0146	0.0060	0.0163	0.0149	0.0108	0.0164
Share (%)	3.845	1.589	4.291	3.932	2.843	4.321
Senegal						
M_{0j}	0.0105	0.0016	0.0156	0.0163	0.0071	0.0156
Share (%)	2.566	0.393	3.800	3.968	1.724	3.803
Mali						
M_{0j}	0.0175	0.0050	0.0197	0.0132	0.0159	0.0198
Share (%)	3.826	1.100	4.319	2.898	3.477	4.337
Burkina						
M_{0j}	0.0206	0.0076	0.0214	0.0164	0.0144	0.0215
Share (%)	4.062	1.494	4.213	3.236	2.835	4.230
Niger						
M_{0j}	0.0233	0.0120	0.0248	0.0232	0.0225	0.0251
Share (%)	3.964	2.037	4.209	3.947	3.830	4.274
The whole						
M_{0j}	0.0114	0.0047	0.0129	0.0115	0.0099	0.0130
Share (%)	3.812	1.576	4.300	3.825	3.301	4.324

Table A8: Decomposing M_0 by dimension and country for $k = 2$ (end)

	Water	Sanitation	BMI	Empowerment	Schooling	All
Kenya						
M_{0j}	0.0050	0.0028	0.0239	0.0459	0.0414	0.1535
Share (%)	3.232	1.799	15.577	29.901	26.952	100
Ghana						
M_{0j}	0.0046	0.0037	0.0182	0.0573	0.0565	0.1719
Share (%)	2.696	2.149	10.607	33.365	32.888	100
Tanzania						
M_{0j}	0.0038	0.0021	0.0193	0.0632	0.0602	0.1928
Share (%)	1.972	1.106	10.002	32.804	31.227	100
Malawi						
M_{0j}	0.0059	0.0023	0.0178	0.0629	0.0705	0.2088
Share (%)	2.842	1.125	8.504	30.113	33.740	100
Uganda						
M_{0j}	0.0057	0.0019	0.0228	0.0593	0.0743	0.2155
Share (%)	2.654	0.891	10.597	27.511	34.477	100
Cameroon						
M_{0j}	0.0043	0.0013	0.0149	0.0761	0.0766	0.2195
Share (%)	1.951	0.599	6.772	34.681	34.905	100
Benin						
M_{0j}	0.0062	0.0090	0.0209	0.0802	0.0948	0.2621
Share (%)	2.365	3.450	7.977	30.595	36.156	100
Madagascar						
M_{0j}	0.0100	0.0088	0.0394	0.0467	0.0990	0.2731
Share (%)	3.653	3.216	14.441	17.106	35.255	100
Nigeria						
M_{0j}	0.0058	0.0038	0.0321	0.0947	0.0926	0.2829
Share (%)	2.066	1.327	11.338	33.463	32.751	100
Guinea						
M_{0j}	0.0126	0.0061	0.0300	0.1150	0.1362	0.3787
Share (%)	3.322	1.609	7.913	30.367	35.968	100
Senegal						
M_{0j}	0.0019	0.0040	0.0405	0.1508	0.1466	0.4106
Share (%)	0.474	0.986	9.863	36.733	35.692	100
Mali						
M_{0j}	0.0157	0.0047	0.0300	0.1540	0.1614	0.4572
Share (%)	3.440	1.032	6.571	33.687	35.313	100
Burkina						
M_{0j}	0.0127	0.0166	0.0486	0.1548	0.1728	0.5074
Share (%)	2.501	3.277	9.5744	30.516	34.063	100
Niger						
M_{0j}	0.0209	0.0212	0.0427	0.1732	0.1993	0.5882
Share (%)	3.557	3.601	7.259	29.444	33.879	100
The whole						
M_{0j}	0.0086	0.0051	0.0278	0.0929	0.1023	0.3003
Share (%)	2.880	1.685	9.275	30.950	34.074	100

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