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Revealed Meta-Preferences: Axiomatic Foundations of Normative Assessments in the Capability Approach

ABSTRACT: This paper explores the possibility of defining a non-utilitarian normative standard for assessments of welfare and deprivation. The paper contributes to this end in three ways. First, the paper formalises a key aspect of Prof. Sen's critique of neoclassical welfare economics, regarding the assumption of consistent utility-maximisation in the revealed preference theory. Secondly, the paper explores alternative formulations of the axiom of revealed preferences that are consistent with Prof. Sen's critique and proposes a set of intuitive assumptions to characterise the relation between observed choices and underlying preferences in the absence of consistent utility-maximisation. Finally, we use these to construct two alternative normative ranking rules that can be used in empirical applications of Prof. Sen's so-called capability approach.

"It is a mistake to set up physics as a model and pattern for economic research. But those committed to this fallacy should have learned one thing at least: that no physicist ever believed that the clarification of some of the assumptions and conditions of physical theorems is outside the scope of physical research." (Ludwig von Mises, 1949, p. 4)

George Stigler described Adam Smith's *The Wealth of Nations* as a "stupendous palace erected on the granite of self-interest" (Stigler, 1982, p. 136)¹. Indeed, beyond its ubiquitous presence in predictive economic models, the assumption that individuals consistently behave as self-interested utility maximisers constitutes a necessary condition for upholding the normative framework that enables neoclassical welfare economics to claim, for instance, that competitive markets are "efficient" or that "market prices are defensible weights on quantities in measuring national income" (Ravallion, 2010, p. 3). This is why Prof. A.K. Sen's questioning of this apparently uncontroversial assumption constitutes such a fundamental challenge to modern economic theory (Sen A. K., *Rational Fools: A Critique of the Behavioural Foundations of Economic Theory*, 1977; Sen A. K., *Equality of What?*, 1979; Sen A. , *Rationality and Freedom*, 2002, p. 21; Sen A. K., *Choice, Welfare and Measurement*, 1997).

But if Prof. Sen has convincingly refuted the normative validity of some of the claims made by neoclassical welfare economics, he has refrained from suggesting a comprehensive alternative to these existing standards, insisting instead on the importance of allowing for normative issues to be settled through public discussion and reasoned argument (Sen A. K., *Dialogue, Capabilities, Lists, and Public Reason: Continuing the Conversation*, 2004). This deliberate vagueness may have left his so-called "capability approach" vulnerable to criticism, as some have questioned the theoretical grounding of high-profile applications of the capability approach, such as the Human Development Index and the more recent Multidimensional Poverty Index, in particular as regards their choices of weights and indicators (Ravallion, 2010, p. 3).

This paper will seek to address this issue by going back to the root of Sen's criticism of neoclassical welfare economics, and exploring the possibility of devising a normative standard that is compatible with Sen's critique, as well as with his open-ended proposal for an alternative normative standard. In particular, we will focus on the implications of relaxing the assumption of self-interested utility

¹ Although, as Sen (Sen A. K., *Democracy as a Universal Value*, 1999; Sen A. K., *Rationality and Social Choice*, 1995) points out, Smith's thought is probably the most nuanced of this long tradition. Smith (Smith, *The Theory of Moral Sentiments*, 1761, p. 34), for instance emphasised the importance of "self-command", which he opposes to our "own original and selfish feeling".

maximisation in the so-called revealed preference theory, which constitutes a key piece of theoretical engineering required to link the common variables that furnish economic theory (e.g. prices, markets, goods) with the normative concepts imported from utilitarian ethics (e.g. utility, efficiency, maximisation). Based on this analysis, we will then explore possible alternative formulations of the revealed preference theory, which can provide a stepping stone for devising a non-utilitarian normative standard. Finally, we will use this reformulated revealed preference theory to propose two alternative normative standards that can be used to define weights in the construction of indices of welfare and deprivation that apply Sen's "capability approach".

Sen's Critique of the Revealed Preference Theory

Revealed Preference Theory

The justification for using market prices as normative weights in assessments of welfare and deprivation has been formalised in the first welfare theorem (Arrow & Debreu, Existence of a Competitive Equilibrium for a Competitive Economy, 1954), which links the positive description of the General Equilibrium (Walras, 1874) with the normative concept of Pareto-efficiency (Pareto, 1906)². The use of Pareto-optimality allows the first welfare theorem to circumvent classical economics' reliance on the informationally demanding –and unobservable – notion of cardinal utility, which had been central to Smith's original formulation of the "invisible hand" argument, on which this line of thought is based (Smith, Book IV: Of Systems of Political Economy, 1776, p. 400). As Samuelson pointed out, however, despite its significant weakening of Smith's original normative claims, it is not clear that the neoclassical formulation would meet the minimum empiricist requirements of verifiability typically demanded by scientific methodology (Hicks, A Revision of Demand Theory, 1956, p. 6):

Consistently applied, however, the modern criticism [of classical utility theory] turns back on itself and cuts deeply. For just as we do not claim to know by introspection the behaviour of utility, many will argue we cannot know the behaviour of ratios of marginal utilities or of indifference directions. Why should one believe in the increasing rate of marginal substitution, except in so far as it leads to the

² Walras' description predicts that markets will, under specified conditions, reach a competitive equilibrium when consumers' marginal rates of (utility) substitution between goods have been equalised with producers' marginal rates of transformation for the same goods through the intermediate of market prices. The notion of Pareto-efficiency, on the other hand, defines an outcome as optimal if no-one can be made better off without making someone else worse off. Both Walras' positive description of market equilibria and the normative standard of Pareto-optimality grew out of the marginalist critique of classical economics (Jevons, 1871; Marshall, 1890), which had relied on aggregate values, including aggregate utility – defined as "the greatest happiness of the greatest number" (Bentham, 1789).

type of demand functions in the market which seem plausible? (Samuelson, A Note on the Pure Theory of Consumer's Behaviour, 1938, p. 61)

In order to address this flaw, Samuelson proposed a key piece of theoretical engineering which has come to be known under the term of revealed preference theory. As long as individuals behave consistently, Samuelson argued, we could infer from their choice of x over y , certain features of the underlying preference relation between x and y . Hence, while utility itself may be unobservable, sufficient fragments of it can be indirectly observed to allow us to make the type of ordinal claims needed for normative welfare assessments of Pareto-efficiency (Little, 1949; Houthakker, 1950). Later Samuelson, building on Little (Little, 1949), went on to show how indifference curves could be approximated through the iterative comparison of binary preference relations over choice bundles (Samuelson, Consumption Theory in Terms of Revealed Preference, 1948), thus paving the way for a reconstruction of a theory of consumer behaviour based solely on observable choices.

Here, we will build on the notation used by Sen (Sen A. K., Choice Functions and Revealed Preference, 1971) and Arrow (Arrow, Rational Choice Functions and Orderings, 1959) to formalise the argument. Let X be the set of all possible options and K the set of all non-empty and finite subsets, S , of X . The non-empty "choice set" $c_i(S)$ represents elements of any given subset S of K that have been chosen by individual i . Henceforth, we will write i 's choice set over S simply as c_i for short. We will not make any particular assumptions about the functional form (or absence thereof) of c_i , and will treat it simply as a subset of S in our analysis. The decision maker i 's preferences are represented by transitive, reflexive and binary relations, R_i^U , defined over the elements of X contained in S . Let $N = \{i | i = (1, \dots, n)\}$ be the set of all individuals holding preferences R_i^U . Then $\Gamma_N^U = \{R_1^U, \dots, R_n^U\}$ represents the set of all possible preference relations over X .

The preference for x over y is designated by $xR_i^U y$, meaning that " x is at least as good as y " according to the preference relation R_i^U ". The superscript U indicates that preference relations are based on utility maximising calculus. Hence, $xR_i^U y$ is equivalent to saying that x confers i with at least as much utility as y , and can be interpreted in marginalist terms, if x and y represent units of x and y for given quantities. The asymmetric and symmetric elements of R_i^U will be designated, respectively, by P_i^U and I_i^U .

The axiom of revealed preference (ARP) then states in its original, weak, form:

$$\forall x, y \in X \text{ and } \forall i \in N: x \in c_i \text{ and } y \in S \setminus c_i \Rightarrow \neg (yR_i^U x)$$

(1)

Plainly put, if both x and y are part of i 's opportunity set, but only x is chosen, we can conclude, at the least, that i does not prefer y to x .

In order to create this direct relation between observable choices and underlying preferences and utility, the revealed preference theory is forced to impose the simplifying, albeit implicit, assumption of a single motivational source, namely that of self-interested utility maximisation. Hence utility appraisals are consistently translated into preference orderings, which are consistently translated into choices. In practice, however, Sen argues that although self-interest maximisation undoubtedly a most powerful driving forces of human conduct, it is but one of the factors that motivate action, alongside things like commitment, compliance with moral rules, religious principles or social norms.

There have been attempts to circumvent this critique by expanding the definition of preferences to include anything sought by the individual, be it for religious reasons, moral reasons, pathological desires or some other form of "want" (Hicks, Value and Capita, 1939). Such an expansion of the definition of preferences, however, would quickly empty the term of its normative content by equating it with the descriptive notion of "behaviour" (Harsanyi, 1997). As Sen has pointed out, from a normative point of view, "[t]he rationale of the revealed preference approach lies in this assumption of revelation" (Sen A. K., Behaviour and the Concept of Preference, 1973, p. 244), and not in "dropping off the last vestiges of the utility analysis", as Samuelson had initially claimed (Samuelson, A Note on the Pure Theory of Consumer's Behaviour, 1938, p. 62). Indeed, if behaviour didn't reveal anything about underlying preferences, and if preference said nothing about utility, price ratios would not tell us anything about marginal rates of substitution and market equilibria would thus tell us nothing about Pareto-optimality.

Plural Motivations

In this subsection, we will look at what happens to ARP when we relax the implicit assumption of consistent and exclusive utility maximising behaviour. For the purpose of the present argument, we will make a distinction between two fundamental types of motivations described by Sen, namely (1) self-centred motivations, R_i^U which can include altruism, sympathy, etc., and (2) reasoned motivations, which we denote by $R_i^M \in \Gamma_N^M = \{R_1^M, \dots, R_n^M\}$ ³. Sen refers to these as "preferences

³ It may be worth noting that the rationale for adopting one or the other of these representations of human nature cannot be based on empiricist criteria of "verifiability" since the claim that individuals are driven exclusively by self-interested utility-maximisation is as unobservable and unverifiable as the claim that there may be a reasoned component to human behaviour (Ameriks, 2000, p. 42). Therefore our justification for choosing this particular set of assumptions must be based on logical criteria, which take us far beyond the scope of this paper. The interested reader is referred to Kant's critique of utilitarian and Hobbesian philosophy

over preferences” or “ ‘metarankings’, reflecting what [an individual] would like his preference to be” (Sen A. K., *The Journal of Philosophy*, 1983, p. 25).

The former type of motivations are idiosyncratic in the sense that they are defined by a person’s particular interests, inclinations, desires, fears, etc. , with regards to her own utility (e.g. in the case of sympathy, a person’s utility is negatively affected by the sight of someone else’s suffering). The latter type of motivations is here defined as those that allow us to disregard our own particular circumstances and act, as Sen calls it, “as if” we are maximising someone else’s utility. The latter is derived from our capacity for universalisation, that is, the faculty of reason to abstract from our own circumstances in order to act in accordance with a law or a moral principle⁴. Sen describes this as the capacity of individuals to “suspen[d] calculations geared towards individual rationality” (i.e. self-interest maximisation) and to “behave as if they are maximizing a different welfare function from the one that they actually have” (i.e. to see a problem from someone else’s perspective) (Sen A. K., *Behaviour and the Concept of Preference*, 1973, p. 252). It is this capacity to disregard one’s own narrow self-interest and act in the interest of the common good that allows individuals to avoid suboptimal outcomes in problems such as the prisoners’ dilemma, as well as numerous other social problems, such as taxation, provision of public goods, etc. (Sen A. K., *Behaviour and the Concept of Preference*, 1973, p. 250), and even, arguably, in market transactions (Sen A. K., *Markets and freedoms*, 1991).

While we do not exclude the possibility that different individuals’ idiosyncratic preferences may coincide, due to sympathy or convergence of interests, for instance, we must recognise the possibility that they may differ due to divergence of interests and tastes. Therefore, we define R_i^U as:

Definition 1: $\Gamma_N^U = \{R_i^U \mid \exists i, j \in N, i \neq j, s.t. R_i^U \neq R_j^U\}$.

By contrast, meta-preferences, R_i^M , will be defined here by their convergence across individuals within a group. Without going as far as claiming that the latter motivations must concord with a universal law of reason, we will define them as the ability of individuals, who are part of a group, to take a bird’s eye view on a situation, and recognise that there is a common interest that transcends

(Kant, *Le conflit des facultés. Der Streit der Facultäten.*, 1798, p. 208; Kant, *Review of Gottlieb Hufeland's Principles of Natural Law. Recension von Gottlieb Hufeland's Versuch über den Grundsatz des Naturrechts*, 1786, p. 1321).

⁴ Formally, Kant defines pure reason as the “faculty of the unity of the rules of understanding under principles” (Kant, *Kritik der reinen Vernunft*, 1781, p. A302/B359), that is the capacity for abstraction or universalisation. Judgement, then is the capacity “of thinking the particular as contained under the universal” (Kant, *Critique of the Power of Judgement*, 1790, p. 5:179), that is to link a particular situation to a universal rule.

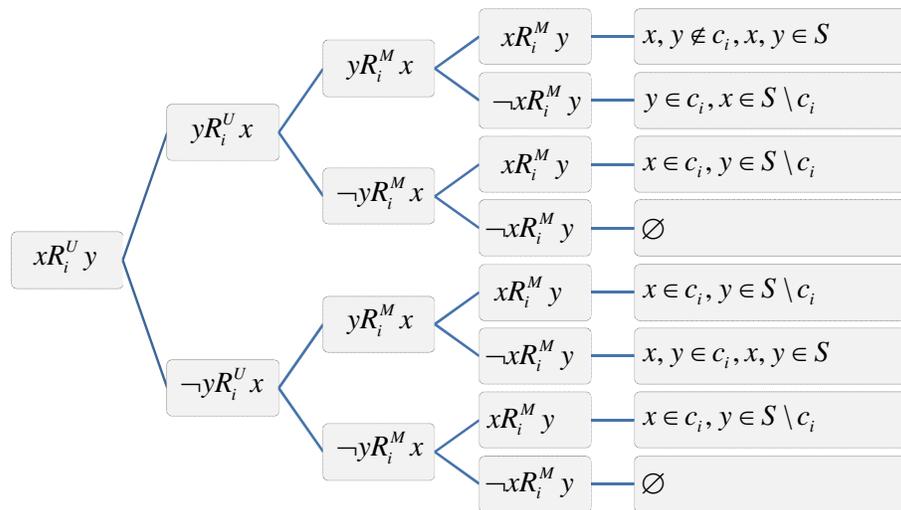
their individual interests⁵. Let Q be the set of all non-empty subsets of N and let $I \in Q$ designate a group of individuals $i = (1, \dots, n)$:

$$\text{Definition 2: } \Gamma_N^M = \{R_i^M \mid \forall I \in Q: i, j \in I \Rightarrow R_i^M = R_j^M\}.$$

We will assume that R_i^U and R_i^M each, independently of each other, satisfy the normal properties of completeness, reflexivity and transitivity over X .

Once we admit the possibility of plural motivations, the technical trick of revealed preferences, which allowed us to draw inference about underlying utility from observable behaviour, ceases to be effective, since observed behaviour could equally well have been caused by moral commitment, norm following or even self-sacrifice, neither of which tell us anything about a person's level of utility – from which all value must ultimately be derived in the utilitarian framework. The figure below indicates various possible outcomes that can result from a combination of utility- and meta-preferences over the outcomes x and y ⁶.

Figure 1: Event tree associated with a utility and meta-preference over x and y



⁵ This formulation will allow for reasoned disagreements between groups of individuals, which is important to Sen (Sen A. K., Elements of a Theory of Human Rights, 2004), while at the same time recognising the possibility of existence of universal truths (see corollary 1).

⁶ Figure 1 excludes the cases which violate the assumption of complete preferences such as: $-xR_i^U y$ and $-yR_i^U x$, for instance.

In this example, we have assumed that an individual will choose both x and y in cases where she has strict preferences for both simultaneously, and will choose neither if she is indifferent both in terms of utility and reasoned preferences. While it is possible to conceive of other plausible combinations of preference relations to outcomes, the precise form of the relation will not matter here. Instead, we are interested in showing that, in the presence of dual motivations it will no longer be possible to infer underlying preference relations with certainty from observed choices, as was done in ARP, since each choice pattern will be associated with, not one, but several plausible combinations of underlying preference relations.

Reformulating the Revealed Preference Theory

Modified Axioms of Revealed Preferences

Depending on the assumptions we choose about the form of underlying decision making mechanism, we will still be able to make certain weaker claims based on observed choices. .

Figure 1, for instance, allows us to make the following claim, which is a close but weaker relative to the ARP:

$$\forall x, y \in X \text{ and } \forall i \in N: x \in c_i \text{ and } y \in S \setminus c_i \Rightarrow \neg (y P_i^U x \vee y P_i^M x)$$

(2)

The axiom now states that if i is observed to choose x over y , he cannot possibly have a strong preference for y over x . Given the uncertainty we now face about underlying preference relations, it may be more appropriate to represent these claims as probabilities. We will therefore refer to inferred motivational sources as events, with ψ_i^U and ψ_i^M describing the events $y P_i^U x$ and $y P_i^M x$, respectively and χ_i^U and χ_i^M describing the events $x P_i^U y$ and $x P_i^M y$. In this perspective, axiom (2) can be reformulated as :

$$\forall x, y \in X \text{ and } \forall i \in N: p(\psi_i^U \cup \psi_i^M | x \in c_i \text{ and } y \in S \setminus c_i) = 0$$

(3)

Henceforth, for notational simplicity, we will write the probability that a given event has occurred, given observation that option x is chosen over y (i.e. given $x \in c_i$ and $y \in S \setminus c_i$) simply as $p(\cdot | c_i)$.

In order to characterise the relation between observed choices and underlying preferences in the absence of exclusive utility maximising behaviour, we will assume that the following intuitive properties, which are consistent with definitions 1 and 2, are satisfied:

$\forall i, j \in N, i \neq j, \text{ and } \forall x, y \in X:$

1. Coherence: $p(\psi_i^U \cup \psi_i^M | c_i) = 0$
2. Completeness: $p(\chi_i^U \cup \chi_i^M | c_i) = 1$
3. Uncertainty: $p(\chi_i^U | c_i) = p(\chi_i^M | c_i)$
4. Anonymity: $p(\chi_i^U | c_i) = p(\chi_j^U | c_j)$
5. Independence : $p(\chi_i^U \cap \chi_j^U | c_i \cap c_j) = p(\chi_i^U | c_i) \times p(\chi_j^U | c_j)$

Property 1 is a re-statement of axiom (3). Property 2 states that a given act, c_i , must have been motivated either by utility or by reason, or both. Property 3 captures the fact that, with no additional information about underlying preferences, we cannot know whether a given behaviour is motivated by self-interested utility-based considerations or by moral concerns. Property 4 states that the probability that a given act is motivated by selfish motives is equal across all individuals (i.e. there are no a priori more virtuous individuals). Property 5, finally, makes the point that idiosyncratic motivations are independent across individuals⁷.

Axiom of Revealed Meta-Preferences

In the non-utilitarian perspective advocated by Prof. Sen⁸, it is not underlying levels of utility that constitute a legitimate standard of value, but rather the reasoned assessment that individuals make of their own preferences, i.e. their metarankings (Sen A. , Rationality and Freedom, 2002). Indeed, utility-based preferences may be determined by culture, upbringing, genetics or other factors over which the individual has no control (Nussbaum, 2003) and they may be adaptive, in the sense of being moulded by prejudice, fear or simply resignation to an unjust fate (Sen A. K., Resources, Values

⁷ This assumption is consistent with definition 1 and includes cases in which the individuals exhibit sympathy or altruism. If i feels sympathy for j , he does not include j 's actual utility function in his own, but rather constructs a mental representation of the elements that he thinks will affect j 's utility. Actual motivations are thus independent across individuals, even though presumptions about other people's motivations may affect decisions. By contrast, meta-preferences, R_i^M , are defined by the ability that individuals have to achieve a common understanding within a given group. By this definition, the members of this group must therefore be able to hold the exact same meta-rankings.

⁸ Sen's has referred elsewhere to his approach as being part of the "rationalist" tradition, including non-utilitarian thinkers, such as Kant, Rousseau among others (Sen 1999, p.255).

and Development, 1984)⁹. The question that is of interest for the purpose of operationalisation of the capability approach is thus whether meta-preferences, R_i^M , can be, if not observed, at least inferred from looking at choices c_i . in the same way as Samuelson's revealed preference theory had allowed us to infer unobservable utility-preferences from observed choices?

At the individual level, the answer to this question is negative, since we do not exclude the possibility of a convergence between idiosyncratic and moral preferences due to chance or sympathy. This implies that a same behaviour c_i , e.g. charitable giving, could be caused by either self-interested motives (e.g. attempting to secure loyalties or relieving one's own displeasure at the sight of poverty) or moral motivations (e.g. a social obligation towards other less fortunate members of the group), as indicated in property 3. However, at the aggregate level, we argue that it will be possible to infer some properties of underlying meta-preference based on the definitions adopted here. The rationale is simple: as neoclassical economics accurately predicts, the probability of achieving a convergence of interests over a large group of individuals is typically small. Looking at the problem from the opposite angle, therefore, and accepting the existence of reasoned meta-preferences, we can conclude that in the event that individuals actually do reach agreement, it is likely that something other than the simple pursuit of self-interest must have been at work.

This idea is captured in the following lemma, which follows from definitions 1-2 and properties 1-5 (See Annex 1 for a proof of Lemma 1):

Lemma 1: $\forall I, A \in Q$, s. t. $1 < |I| < \infty$ and $A \neq \emptyset$, with individual preference defined as 1-2, and satisfying properties 1-5: $I \supset A \Rightarrow p(\chi_I^M | c_I) \geq p(\chi_A^M | c_A)$.

We use the notation χ_I^U to describe the event that $xR_i^U y$ occurs simultaneously for all $i \in I$. In other words it describes the case in which there is a concordance of interests. For a group $I \in Q$ of n individuals, $i = (1, \dots, n)$, we thus have $p(\chi_I^U) = p(\bigcap_{i=1}^n \chi_i^U)$. Similarly, on the output side, instead of looking at choice sets, we are now looking at consensus sets, c_I , which we define simply as the non-empty intersection between individual choice sets: $c_I \equiv \bigcap_{i=1}^n c_i \neq \emptyset$. The set of all possible non-empty and finite consensus choice sets, c_I , held by groups $I \in Q$ is called Z (this includes individual choice sets). The cardinality of a consensus, c_I , achieved by a group I of individuals $i = (1, \dots, n)$, is

⁹ For a more general critique of utilitarian moral philosophy, see (Sen & Williams, *Utilitarianism and Beyond*, 1982).

designated by $|I|$. At the moment we do not make any assumptions about the way that consensus is reached, other than the fact that each individual, $i \in I$ who counts towards $|I|$, has equal influence over the consensus c_I . This formulation can therefore accommodate all possible decision making mechanisms, ranging from absolute dictatorship (in which case we would have $|I| = 1$), to unanimity rule (in which case $|I| = n$, i.e. the consensus size is equal to the number of individuals in the population). In the most common case of absolute majority rule, we have $|I| = \frac{1}{2}n + 1$.

In the limit, as $|I|$ tends to ∞ , we can think of a hypothetical consensus involving an infinite number of individuals with all possible combinations of preference relations. Definition 1 implies that the probability of utility-based interests converging over an infinite number of individuals is 0.

Consequently, such a consensus would, by construction, need to coincide with a universal law of reason since that is the only law that any reasonable individual could voluntarily be expected to subject themselves to, regardless of their personal circumstances, inclinations and interests. This idea is represented by the following corollary, which follows from Lemma 1 (see Annex 2 for a proof of Corollary 1):

Corollary 1: $\forall I \in Q, 1 < |I| < \infty$, with individual preference defined as 1-2, and satisfying properties 1-5: $\lim_{|I| \rightarrow \infty} p(\chi_I^U \cup \chi_I^M | c_I) = p(\chi_I^M | c_I) = 1$.

This analysis allows us to propose the following general axiomatic formulation, which we will call the Axiom of Revealed Meta-preferences (ARM):

$$\forall x, y \in X \text{ and } \forall I \in Q: p(\chi_I^M | c_I) = f(\cdot, |I|)$$

(4)

Where $f(\cdot, |I|)$ is a monotonically non-decreasing function of $|I|$. The axiom of revealed meta-preferences simply states that the probability that an observed consensus, c_I , has been generated by a meta-preferences will be a function of the number of individuals reaching consensus. An outcome x over which there is a consensus among $|I|$ members of a group I will thus be said to be revealed meta-preferred to outcome y with probability $p(\chi_I^M | c_I)$. In this perspective, the case covered by the original revealed preference axiom becomes a special case of this general axiom, in which $|I| = 1$. In this case, we know from property 3 that $p(\chi_i^M | c_i) = p(\chi_i^U | c_i)$, meaning that the observed choice has equal probabilities of having been caused by either motive. Other special cases that have been

discussed above, include the case in which $|I| \rightarrow \infty$, in which case corollary 1 tells us that $p(\chi_I^M | c_I) = 1$. A final special case is that in which there is no consensus, that is $|I| = 0$. By definition 2, if there is no consensus, we know that individuals have been unable to overcome their divergence of interests in order to agree on a common objective. Hence, in this case, we have $p(\chi_I^M) = 0$. All other cases, such that $1 < |I| < \infty$ are covered by Lemma 1.

Normative Meta-Rankings

Probability-Based Ranking

At the moment, the axiom of revealed meta-preferences does not make any normative claim. It simply states that broader consensuses are more likely to have been reached through reasoned agreement. If, like Sen, we consider that value stems from reason rather than from utility, it is logical to make the normative value of a given choice directly proportional on the probability that it is caused by R_i^M rather than R_i^U . We do this by turning the probabilistic logic of Lemma 1 into a normative rule, using the following axiom:

Probability-Based Ranking (PBR): $\forall I, J \in Q$ and $\forall c_I, c_J \in Z$:

1. $c_I > c_J \Leftrightarrow p(\chi_I^M | c_I) > p(\chi_J^M | c_J)$
2. $c_I \sim c_J \Leftrightarrow p(\chi_I^M | c_I) = p(\chi_J^M | c_J)$

(5)

Where \succcurlyeq , meaning “is at least as good as”, describes the normative ranking of social states– not to be confused with the positive description of individuals’ actual preferences over those states, noted R_i^U and R_i^M . Hence, whereas $x R_i^M y$ indicated that a group I meta-preferred x to y , $x \succcurlyeq y$ indicates that it is also, and for that reason, preferable from a normative point of view (with asymmetric and symmetric elements of \succcurlyeq noted \succ and \sim , respectively).

The PBR axiom captures Sen’s (Sen A. K., Dialogue, Capabilities, Lists, and Public Reason: Continuing the Conversation, 2004) notion of normative validation through a process of inter-rational testing – e.g. through public discussion or reasoned argument (see also (Popper, 1980, p. 111)). Although we can never reach certainty that one normative standard is universally and unconditionally superior to the other, the logic developed in this paper implies that the international consensus reached about fundamental human rights, for instance, will carry more normative weight than, say, the norms that govern a small isolated community or tribe. This is because the former involves agreement between

a large number of different individuals with widely divergent views, cultures and interests, whereas in the latter case it is more likely that agreement may have been facilitated by a strong convergence of interests and experiences¹⁰. By the same logic, a national constitution should carry more normative weight than national legislation, insofar as it is protected by additional procedural guarantees (e.g. two thirds majority or double majority) that mean that a broader consensus (i.e. a more thorough vetting process) would be needed to reach agreement on the former than on the latter.

This idea is captured by the following proposition, which follows from definitions 1-2 and properties 1-5 (see Annex 3 for a proof of proposition 1)¹¹:

Proposition 1: If \succsim is a binary transitive and reflexive ranking over Z satisfying PBR, and if properties 1-5 hold, then $\forall c_I, c_J \in Z$ and $\forall I, J \in Q$, s. t. $1 < |I|, |J| < \infty : |I| > |J| \Rightarrow c_I \succsim c_J$.

A potentially controversial implication of Proposition 1 is that the decisions of a large democracy, such as India, would, assuming comparable political systems, automatically carry more normative force than that of almost any other country. While this may seem like an uncomfortable conclusion, it is a logical consequence of the normative standard that relies on a process of inter-rational validation that gives equal weight to the views of each individual. To be convinced of this, it suffices to consider the alternative of a population-neutral normative standard, which would give equal weights to the consensus of Lichtenstein as to that of the United States. One way of avoiding these two extremes would be to devise a more restrictive ranking rule, contained in corollary 2, that only allows us to compare groups that have been able to agree on a common (higher) standard (the proof for corollary 2 is provided by the first step of the proof for proposition 1):

¹⁰ This need not mean that lower order norms are unfit for normative assessments. When judging each others' actions, members of the tribe or the village may be content with, and indeed prefer, doing so in accordance with their own set of rules and norms that they have all consented to. However, when comparing conflicting tribal laws, we will need to appeal to higher order consensus, such as a national legislation or international human rights.

¹¹ It is important to note that we are ranking choice sets, rather than the options that may be contained in the choices that have been made. Hence, if groups I and J both agree on the public provision of healthcare, for instance, we could have $c_I \succ c_J$ if $|I| > |J|$, which simply means that the normative force of I 's preference for healthcare is greater than that of J 's preferences for healthcare. The ranking will therefore be transitive over Z but may not be transitive over X .

Corollary 2: If \succsim is a binary transitive and reflexive ranking over Z satisfying PBR, and if properties 1-5 hold, then $\forall c_I, c_A \in Z$ and $\forall I, A \in Q : I \supset A \Rightarrow c_I \succsim c_A$.

Such a ranking rule would thus not allow us to compare, for instance, Denmark and the United States on their fulfilment of the right to healthcare implied by Art. 12.2.d of the international covenant of social, economic and cultural rights, since the United States is one of a handful of countries in the world that have not ratified the international covenant (although it is a signatory since 1977). Under the current set of assumptions, however, the more restrictive ranking provided by corollary 2 would only be a special case of the more general ranking provided by Proposition 1.

Consensus-Based Ranking

Finally, we propose an alternative normative ranking rule that captures the rationale of the above discussion, but presents the advantage of being based entirely on the external and thus observable properties of the consensus reached between different agents. Needless to say that the normative power of this ranking ultimately is derived from the assumed link of consensus to underlying meta-preferences, much in the same way as the normative power of choice in the neoclassical framework was derived from its assumed link to underlying utility. By avoiding references to unobservable properties of the individual decision-making mechanism, however, we make it possible to centre the axiomatic discussion on the desirable political properties of a consensus-based ranking. We propose the four following simple axioms to characterise our ranking:

Anonymity (ANO):

$$\forall i, j \in N: c_i \sim c_j$$

(6)

Monotonicity (MON):

$$\forall I, A \in Q, \quad s.t. (c_I, c_A) \neq \emptyset: I \supset A \Rightarrow c_I > c_A$$

(7)

Independence (IND):

$$\forall I, J \in Q, \text{ and } \forall h \in Q \setminus (I \cup J), \quad s.t. c_I \cap c_h \neq \emptyset, c_J \cap c_h \neq \emptyset: c_I \succsim c_J \Rightarrow c_I \cap c_h \succsim c_J \cap c_h$$

(8)

Indifference between Non-consensual Groups (ING):

$$\forall I, A \in Q, \quad s. t. I \supset A \quad \text{and} \quad \forall h \in Q \setminus A: c_A \cap c_h = \emptyset \Rightarrow c_A \cap c_h \sim c_I \cap c_h$$

(9)

The Anonymity axiom (ANO) states that the choice of any given individual has the same normative force as that of any other individual. The Monotonicity axiom (MON) states that the consensus reached by any given group of individuals has more normative force than the consensus reached by any of its subgroups. The Independence axiom (IND) states that a ranking between two consensuses will not be affected by the inclusion of an additional individual who agrees with both. These axioms echo fairly closely the axioms proposed by Pattanaik and Xu (Pattanaik & Xu, 1990) to characterise a cardinal ranking rule. However, they exclude cases in which individuals are unable to reach consensus. To complete the ranking we therefore need to add a fourth axiom, to deal with those cases. We do this by using the Indifference between Non-consensual Groups axiom (ING), which states that all groups that have been unable to reach consensus will have the same normative force regardless of their size.

Based on these four axioms, we are able to construct a ranking rule that is dependent solely on the number of individuals reaching consensus (see Annex 4 for a proof of theorem 1):

Theorem 1: If \succsim is a binary, reflexive and transitive ranking over Z , satisfying ANO, MON, IND and ING, then $\forall I, J \in Q$ and $\forall c_I, c_J \in Z : c_I \succsim c_J \Leftrightarrow |I| \geq |J|$.

It is easiest to think of I and J as sets of individuals (e.g. electorates). However, since the consensus-based ranking is not directly dependent on the properties of individual motivational structures, it is also possible to think of a different unit, such that I and J representing sets of countries, for instance. This would be relevant for the cases discussed under proposition 1, where it may be necessary to give equal weight to countries regardless of their population, as is currently the case in international human rights legislation, for instance.

Before concluding, we should note that by taking the existing consensus as the starting point of the assessment, and thus disregarding the decision-making process, we are implicitly imposing a number of assumptions that will need to be clarified. In particular, we have assumed that the decisions

reached reflect a genuine consensus in the sense that no individual has been subjected to any form of coercion, intimidation or deceit. We may also need to add assumptions regarding the nature of the decision makers. In particular, we need to assume that no individual has more power than other and can influence the decisions of his followers. Finally, we are, of course, assuming that everyone has access to the same level of information and has sufficient information to make free and informed decisions. All of these assumptions may be optimistic and none may in fact ever be met in reality. However, they are not fundamentally less plausible than the basic set of assumptions needed to prove the fundamental theorems of welfare economics. In fact these assumptions echo quite well the ones that characterise the Weberian general equilibrium, such as perfect and competitive markets with perfect information and no monopolies. As such, they may provide an adequate basis for exploring the conditions under which these ideals may be realised and the reasons for which they fail to do so.

Conclusion

The assumption that individuals consistently act to maximise their own utility has become so central to normative and positive economic theory that it has come to be seen as an integral and inalienable part of economic theory in the eyes of many of its supporters as well as its detractors¹². However, as we have tried to show in this paper, there is nothing intrinsically “economic” about this assumption, any more than there is any inherent reason why market equilibria should be considered “efficient” when they maximise utility – or balance marginal utilities – rather than, say, when they fulfil social and economic rights¹³. The conclusions reached in this paper suggests that the relaxation of this unverified axiom could have far reaching implications for the normative conclusions reached in economic assessments, and could allow to envisage a welfare economics based on non-utilitarian premises. This analysis has immediate and practical implications for the way in which we can and should measure welfare and deprivation:

First of all, the argument developed in this paper allows us to discard the use of market prices as acceptable normative weights in such assessments. Market prices are the product of an iterative

¹² Nowhere is this confusion of the discipline for its axioms more evident than in Becker’s exploration of the “economics” of issues pertaining to the fields of biology, criminology and even religion (Becker, 1976).

¹³ As Prof. A.K. Sen has shown, the reason for which both positive and normative economic theory have embraced utilitarian ethics as well as its descriptive model of human behaviour, is primarily a historic one – namely the fact that many of the founders of modern economic theory, starting with Adam Smith himself, were utilitarian moral philosophers (Sen A. K., On ethics and economics, 1987).

process involving a very large number of individual decisions taken by consumers, individually, based on tastes, needs and perhaps even moral concerns (and/or producers individually, based on production costs, etc.). At no point does this process require individuals to reach out and seek compromise with other consumers who have different interests and inclinations. Consequently, market prices do not tell us anything about people's reasoned assessments of the various options they face. From the perspective of non-utilitarian framework, such as the one proposed by Prof. Sen, this thus undermines the normative justification for using measures, such as GDP or income poverty, which effectively rely on market prices to weigh different goods or dimensions of wellbeing.

Let us now consider another measure that assigns relative monetary values to different goods, namely a public budget that has been voted by a democratically elected parliament. Unlike market prices, a budget will require lengthy discussions and a process of inter-rational validation in order to agree on which priorities the community should invest in and in what proportions. The weights provided by public spending ratios in a national budget could therefore, unlike market prices, constitute a valid normative standard in the construction of an aggregate index.

It is possible to think of other standards that may carry information about weights. The Millennium Declaration, for instance, describes an implicit weighting system, as it defines eight equally important goals, each of which is composed of a number of targets, which in turn are measured by various indicators. The Universal Declaration of Human Rights also makes reference to weights and even to marginal rates of substitution when it states, for instance, that all rights are indivisible and inalienable (implying zero elasticity of substitution between rights). More importantly, it also implicitly assigns a weight of zero to a large number of rights, namely those that are not included in the declaration.

Which of these standards should be used in constructing a normative index of welfare will depend on the purpose and the scope of the assessment. A national budget may, for instance, provide an adequate standard for a punctual assessment of national policies, whereas a more demanding standard, such as the Universal Declaration, may be required when making a comparison between various countries and over several years. Finally, an assessment geared specifically towards development issues, may be content with a more focused normative standard, such as the Millennium Declaration.

Annex 1: Proof of Lemma 1

Let $A, I \in Q$ be two non-empty groups of individuals, such that $A \subset I$ and $1 < |I| < \infty$. Further, let $B = I \setminus A$.

$A \neq \emptyset$ implies $|A| > 0$, whereas $A \subset I$ implies $|B| = |I| - |A| > 0$, given that $1 < |I| < \infty$. There is therefore at least one individual in each subgroup.

Furthermore, since $B = I \setminus A$, we can use definition 1 to infer that $\chi_A^U \cap \chi_B^U \subseteq \chi_A^U$, given that there is at least one pair of individuals, i and j , $i \neq j$, such that $R_i^U \neq R_j^U$.

Consequently, $p(\chi_A^U \cap \chi_B^U) \leq p(\chi_A^U)$.

(10)

By contrast, since $A, B \in I$, definition 2 implies that $\chi_A^M \cap \chi_B^M = \chi_A^M = \chi_B^M$, since $R_i^M = R_j^M$ for all $i, j \in I$.

Consequently, $p(\chi_A^M \cap \chi_B^M) = p(\chi_A^M)$

(11)

Let us now turn to the conditional probabilities $p(\chi_A^M \cap \chi_B^M | c_A \cap c_B)$ and $p(\chi_A^M | c_A)$:

By definition of a conditional probability, we have:

$$p(\chi_A^M \cap \chi_B^M | c_A \cap c_B) = \frac{p(\chi_A^M \cap \chi_B^M \cap c_A \cap c_B)}{p(c_A \cap c_B)}$$

(12)

And:

$$p(\chi_A^M | c_A) = \frac{p(\chi_A^M \cap c_A)}{p(c_A)}$$

(13)

Property 2 combined with property 1 implies that $\chi_i^M \subseteq c_i$, since it means that c_i must have been caused by either χ_i^M or χ_i^U and cannot have been caused by any of the other motives, ψ_i^M or ψ_i^U .

Consequently: $\chi_A^M \cap c_A = \chi_A^M$ and thus $p(\chi_A^M \cap c_A) = p(\chi_A^M)$.

We can therefore re-write (12) and (13) as:

$$p(\chi_A^M \cap \chi_B^M | c_A \cap c_B) = \frac{p(\chi_A^M \cap \chi_B^M)}{p(c_A \cap c_B)}$$

(14)

And:

$$p(\chi_A^M | c_A) = \frac{p(\chi_A^M)}{p(c_A)}$$

(15)

We know from (11) that : $p(\chi_A^M \cap \chi_B^M) = p(\chi_A^M)$. The magnitudes of (14) and (15) will therefore depend on $p(c_A \cap c_B)$ and $p(c_A)$.

Since $B = I \setminus A$, it follows from the definition of $c_I \equiv \bigcap_{i=1}^{|I|} c_i$ that $c_A \cap c_B \subseteq c_A$.

Consequently:

$$p(c_A \cap c_B) \leq p(c_A)$$

(16)

Combining equations (14) and (15) with results (11) and (16), finally, we get:

$$p(\chi_A^M \cap \chi_B^M | c_A \cap c_B) = \frac{p(\chi_A^M \cap \chi_B^M)}{p(c_A \cap c_B)} \geq \frac{p(\chi_A^M)}{p(c_A)} = p(\chi_A^M | c_A)$$

(17)

By construction of χ_I^M and c_I , finally, $B = I \setminus A$ implies $\chi_A^M \cap \chi_B^M = \chi_I^M$ and $c_A \cap c_B = c_I$. We can therefore re-write (17) as:

$$p(\chi_I^M | c_I) \geq p(\chi_A^M | c_A)$$

(18)

Annex 2: Proof of corollary 1

By the general property of additivity of probabilities, we have:

$$p(\chi_I^U \cup \chi_I^M | c_I) = p(\chi_I^U) + p(\chi_I^M) - p(\chi_I^U \cap \chi_I^M)$$

(19)

By construction of χ_I^M and c_I , we can re-write equation (19) in its generalised for the case of a group I of cardinality $|I|$ as:

$$p(\chi_I^U \cup \chi_I^M | c_I) = p\left(\bigcap_{i=1}^{|I|} \chi_i^U | c_i\right) + p\left(\bigcap_{i=1}^{|I|} \chi_i^M | c_i\right) - p\left(\bigcap_{i=1}^{|I|} (\chi_i^U \cap \chi_i^M) | c_i\right)$$

(20)

Let us now consider what happens to (20) when n tends to ∞ :

From definition 2, first, we know that R_i^M is equal for all $i \in I$, which means that $\bigcap_{i=1}^{\infty} (\chi_i^M) = \chi_i^M = \chi_I^M$.

$$\text{Consequently, } \lim_{|I| \rightarrow \infty} \left[p\left(\bigcap_{i=1}^{|I|} \chi_i^M | c_I\right) \right] = p(\chi_I^M | c_I).$$

Definition 1, on the other hand, tells us that $\bigcap_{i=1}^{\infty} (\chi_i^U) = \emptyset$, since $R_i^U \neq R_j^U$ for some $i, j \in I$.

$$\text{Hence, as } |I| \text{ tends to infinity, } \lim_{|I| \rightarrow \infty} \left[p\left(\bigcap_{i=1}^{|I|} \chi_i^U | c_I\right) \right] = p(\emptyset) = 0$$

$$\text{And } \lim_{|I| \rightarrow \infty} \left[p\left(\bigcap_{i=1}^{|I|} (\chi_i^U \cap \chi_i^M) | c_I\right) \right] = p(\emptyset \cap \chi_I^M | c_I) = p(\emptyset) = 0.$$

Consequently, in the limit, equation (20) will boil down to:

$$\lim_{|I| \rightarrow \infty} p(\chi_I^U \cup \chi_I^M | c_I) = p(\emptyset) + p(\chi_I^M | c_I) - p(\emptyset \cap \chi_I^M | c_I)$$

(21)

which is equivalent to:

$$\lim_{|I| \rightarrow \infty} p(\chi_I^U \cup \chi_I^M | c_I) = p(\chi_I^M | c_I)$$

(22)

By the property of completeness (property 2), finally, we have :

$$\lim_{|I| \rightarrow \infty} p(\chi_I^U \cup \chi_I^M | c_I) = p(\chi_I^M | c_I) = 1$$

(23)

Appendix 3: Proof of Proposition 1

Let $I, J \in Q$ be two groups of individuals, such that $|I| > |J|$ and let $A \subset I$ such that $|A| = |J|$.

By Lemma 1, we have:

$$I \supset A \Rightarrow p(\chi_I^M | c_I) \geq p(\chi_A^M | c_A)$$

(24)

By definition of \succcurlyeq and \geq , $>$ implies \succcurlyeq and $>$ implies \geq .

We can thus apply PBR1 to get $p(\chi_I^M | c_I) \geq p(\chi_A^M | c_A) \Rightarrow c_I \succcurlyeq c_A$.

Further, we know that $|A| = |J| > 1$. We can therefore choose two pairs of distinct individuals $(h, i), (j, k) \in N$, such that $h, i \in A, h \neq i$, and $j, k \in J, j \neq k$.

By property 4, we have:

$$\forall i, j \in N: p(\chi_i^U | c_i) = p(\chi_j^U | c_j) \text{ and } \forall (h, k) \in N: p(\chi_h^U | c_h) = p(\chi_k^U | c_k)$$

(25)

Furthermore, by property 3, we have $\forall i \in N$ (including j, h and k):

$$p(\chi_i^U) = p(\chi_i^M)$$

(26)

By property 2, we have $\forall i, j \in N$ (and thus also for h and k):

$$p(\chi_i^U \cup \chi_i^M) = 1 = p(\chi_j^U \cup \chi_j^M)$$

(27)

By the general property of additivity, (27) is equivalent to:

$$\begin{aligned} p(\chi_i^U \cup \chi_i^M | c_i) &= p(\chi_i^U | c_i) + p(\chi_i^M | c_i) - p(\chi_i^U \cap \chi_i^M | c_i) \\ &= p(\chi_j^U | c_j) + p(\chi_j^M | c_j) - p(\chi_j^U \cap \chi_j^M | c_j) = p(\chi_j^U \cup \chi_j^M | c_j) \end{aligned}$$

(28)

Solving for $p(\chi_i^U \cap \chi_i^M | c_i)$, we get:

$$p(\chi_i^U \cap \chi_i^M | c_i) = p(\chi_j^U \cap \chi_j^M | c_j) + p(\chi_i^U | c_i) - p(\chi_j^U | c_j) + p(\chi_i^M | c_i) - p(\chi_j^M | c_j)$$

(29)

From (25) and (26), we know that: $p(\chi_i^U | c_i) = p(\chi_j^U | c_j)$ and $p(\chi_i^M | c_i) = p(\chi_j^M | c_j)$.

Consequently, (29) boils down to:

$$p(\chi_i^U \cap \chi_i^M | c_i) = p(\chi_j^U \cap \chi_j^M | c_j)$$

(30)

Using the same method, we can show that the same holds for $p(\chi_h^U \cap \chi_h^M | c_h) = p(\chi_k^U \cap \chi_k^M | c_k)$.

By property 5, we can combine these two pairs to obtain:

$$p(\chi_i^U \cap \chi_h^U | c_i \cap c_h) = p(\chi_i^U | c_i) \times p(\chi_h^U | c_h) = p(\chi_j^U | c_j) \times p(\chi_k^U | c_k) = p(\chi_j^U \cap \chi_k^U | c_j \cap c_k)$$

Similarly, by using properties 4 and 5, we can add all remaining elements in A and J without affecting the equality, to obtain:

$$p(\chi_A^U | c_A) = p(\chi_J^U | c_J)$$

(31)

Using property 2, we then have:

$$p(\chi_A^U \cup \chi_A^M | c_A) = 1 = p(\chi_J^U \cup \chi_J^M | c_J)$$

(32)

Which is equivalent to:

$$\begin{aligned} p(\chi_A^U \cup \chi_A^M | c_A) &= p(\chi_A^U | c_A) + p(\chi_A^M | c_A) - p(\chi_A^U \cap \chi_A^M | c_A) \\ &= p(\chi_J^U | c_J) + p(\chi_J^M | c_J) - p(\chi_J^U \cap \chi_J^M | c_J) = p(\chi_J^U \cup \chi_J^M | c_J) \end{aligned}$$

(33)

Solving for $p(\chi_A^M | c_A)$:

$$p(\chi_A^M | c_A) = p(\chi_J^M | c_J) + p(\chi_J^U | c_J) - p(\chi_A^U | c_A) - p(\chi_J^U \cap \chi_J^M | c_J) + p(\chi_A^U \cap \chi_A^M | c_A)$$

(34)

From (31), we know that $p(\chi_A^U | c_A) = p(\chi_J^U | c_J)$. Furthermore, we know from (30) that $p(\chi_i^U \cap \chi_i^M | c_i) = p(\chi_j^U \cap \chi_j^M | c_j)$ for all $i, j \in N$, which implies $p(\chi_A^U \cap \chi_A^M | c_A) = p(\chi_J^U \cap \chi_J^M | c_J)$.

Consequently, (34) reduces to:

$$p(\chi_A^M | c_A) = p(\chi_J^M | c_J)$$

(35)

We can now apply PBR2, to get: $p(\chi_A^M | c_A) = p(\chi_J^M | c_J) \Rightarrow c_A \sim c_J$.

Finally, since \succsim is transitive over Z : $c_I \succsim c_A$ and $c_A \sim c_J \Rightarrow c_I \succsim c_J$.

Appendix 4: Proof of Theorem 1.

It is easy to see that a ranking of consensus sets in terms of the number of individuals reaching consensus would be transitive and would satisfy (ANO), (POC), (IND) and (ING).

In order to prove the theorem, we need to prove that for any binary transitive ranking \succsim satisfying the four axioms:

$$\forall I, J \in Q, \forall c_I, c_J \in Z: |I| = |J| \Rightarrow c_I \sim c_J$$

(36)

And

$$\forall I, J \in Q, \forall c_I, c_J \in Z: |I| > |J| \Rightarrow c_I \succ c_J$$

(37)

Let us start with a proof of (36) by induction:

Axiom ANO gives us the basis of the induction for the case where $n = 1$, as it tells us that for any individuals $\{i\}, \{j\} \in N: c_i \sim c_j$. Our inductive hypothesis is that this holds for all n . In order to prove the inductive step, we need to show that this holds true also for $n + 1$. Consider the sets $I, J \in Q$, such that $c_I \neq \emptyset$ and $c_J \neq \emptyset$.

Let $|I| = |J| = n + 1$, with $1 \leq n \leq |N|$.

Further, let $\{i\} \in I$ and let $D \subset I$, such that $D = I \setminus \{i\}$.

Since $|I| = n + 1$ and since $|\{i\}| = 1$ by definition of a singleton, it follows that $|D| = n$.

Logically, $\{i\}$ must either be a member of J or not be a member of J . If $\{i\} \in J$, we know, by construction of c_J that $c_i \cap c_J \neq \emptyset$. However, if $\{i\} \notin J$ it can either be the case that $c_i \cap c_J \neq \emptyset$ or that $c_i \cap c_J = \emptyset$. This gives rise to 3 possible cases:

Case 1 ($\{i\} \in J$):

Let $E \subset J$ such that $E = J \setminus \{i\}$.

By construction of c_I and c_J we know that $c_D \neq \emptyset$ and $c_E \neq \emptyset$.

Since $|J| = n + 1$ and $|\{i\}| = 1$, it follows that $|E| = n$.

Since we know that $|D| = n = |E|$, we can apply the inductive hypothesis to get $c_D \sim c_E$.

By applying the IND axiom, we then get: $c_D \cap c_i \sim c_E \cap c_i$, which,

By construction of D and E is equivalent to $c_I \sim c_J$.

Case 2 ($\{i\} \notin J$ and $c_i \cap c_j \neq \emptyset$):

If $\{i\} \notin J$ and $|I| = |J| = n + 1$, then there must be a $\{j\} \in J$, such that $\{j\} \notin I$.

Let $F \subset J$ such that $F = J \setminus \{j\}$.

By construction of c_j we know that $c_F \neq \emptyset$.

Since $|J| = n + 1$ and $|\{j\}| = 1$ by definition of a singleton, it follows that $|F| = n$.

We can thus apply the inductive hypothesis to get: $c_D \sim c_F$, and then

Apply IND to obtain: $c_D \cap c_i \sim c_F \cap c_i$.

By construction of D , this is equivalent to $c_I \sim c_F \cap c_i$.

Let $\{f\} \in F$.

Since $F \subset J$ it follows from $\{i\} \notin J$ that $\{i\} \notin F$, and from $\{f\} \in F$ that $\{f\} \in J$.

Consequently, we have: $|(F \setminus \{f\}) \cup \{i\}| = n$ and $|J \setminus \{f\}| = n$.

By applying the induction hypothesis, we get: $c_{F \setminus \{f\}} \cap c_i \sim c_{J \setminus \{f\}}$.

We can then apply IND to obtain: $c_F \cap c_i \sim c_J$.

We have now shown that $c_I \sim c_F \cap c_i$ and that $c_F \cap c_i \sim c_J$.

Since \succsim is transitive over Z , this implies that: $c_I \sim c_J$.

Case 3 ($\{i\} \notin J$ and $c_i \cap c_j = \emptyset$):

This is a special case in which, by definition of a non-consensus, we have $n = 0$ for $c_i \cap c_j$. So we need to show that the indifference holds for all $n = 0$ and $n + 1 = 1$.

If it is the case that $c_i \cap c_j = \emptyset$, then there must be at least one element, $\{j\} \in J$, such that $c_i \cap c_j = \emptyset$.

By ING, we have: $c_i \cap c_j \sim c_i \cap c_j$

Since we know that $i \in I$, and that $c_i \cap c_j = \emptyset$ we can also use ING to expand I around i to obtain: $c_i \cap c_j \sim c_i \cap c_j$.

Since \succsim is transitive on Z , we have: $c_i \cap c_j \sim c_i \cap c_j$ and $c_i \cap c_j \sim c_i \cap c_j \Rightarrow c_i \cap c_j \sim c_i \cap c_j$

The choice sets of individuals i and j can be compared using the ANO axiom, which yields: $c_i \sim c_j$. By definition of a singleton consensus set, we have: $c_i \cap c_i \neq \emptyset$ and $c_j \cap c_j \neq \emptyset$, which covers the case when $n = 1$.

Proof for (37):

Let $|I| > |J|$.

Now, let $G \subset I$, such that $|G| = |J|$.

By construction of c_I , we know that $c_G \neq \emptyset$.

By (36), we have: $c_G \sim c_j$.

Further, by MON, we have: $c_I \succ c_G$

Finally, we can apply transitivity of \succsim to obtain: $c_I \succ c_j$.

QED.

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