Tests of stochastic dominance for multivariate distributions

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September 2010
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- Instead it may be necessary to perform tests on functions of the joint distribution functions!
- Whether we need to check the joint distributions or not depends on the cross derivatives of the class of welfare functions for which we are trying to establish partial orderings.
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- In multivariate settings comparing the marginal distributions of A and B for every variable may not be appropriate to ascertain stochastic dominance (and partial orderings).
- Instead it may be necessary to perform tests on functions of the joint distribution functions!
- Whether we need to check the joint distributions or not depends on the cross derivatives of the class of welfare functions for which we are trying to establish partial orderings.
- In other words, it depends on whether the class of functions is characterized by neutrality among the dimensions or not (i.e. they are either substitutes or complements, or both at times).
Two cases of multidimensional stochastic dominance: marginal distributions are sufficient

This is the case when the general class of welfare functions (e.g. poverty functions) have cross derivatives equal to zero.
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The test then requires:

1. Perform stochastic dominance test on each variable separately, using univariate tests
2. Interpret the results:
   - Declare $A \succeq B$ if and only if $A \succeq B$ or $A = B$ for all variables $(x_1, \ldots, x_V)$ and $\exists x_i | A \succeq B$
   - Similarly for $B \succeq A$
   - Declare $A = B$ if and only if $A = B$ for all variables $(x_1, \ldots, x_V)$
   - Declare indeterminacy (i.e. no dominance and no homogeneity) if and only if $\exists x_i | A \succeq B$ and $\exists x_j \neq i | B \succeq A$
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   - Declare $A \succeq B$ if and only if $A \succeq B$ or $A = B$ for all variables $(x_1, ..., x_V)$ and $\exists x_i | ADB$
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2. Interpret the results:
   - Declare $A \not\succ B \iff A \not\succ B$ or $A = B$ for all variables $(x_1, \ldots, x_V)$ and $\exists x_i | A \not\succ B$
   - Similarly for $B \not\succ A$
   - Declare $A = B$ if and only if $A = B$ for all variables $(x_1, \ldots, x_V)$
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Two cases of multidimensional stochastic dominance: joint distributions matter

When the class of welfare functions is sensitive to the way the dimensions are associated across the populations (e.g. rich people have a lot of everything versus some people have a lot of one thing but not the others) then joint distributions matter for ascertaining dominance.
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There are several conditions but just a few tests (or possible extensions of univariate ones) for these situations.
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Some tests are exclusive for continuous variables, others for discrete variables, and others for combinations.

We are going to study two of these for poverty functions: one for continuous variables and one for a combination of continuous and discrete variables (both by Duclos, Sahn and Younger, 2006).
A stochastic dominance condition is a relationship between functions of the distribution functions of a pair of samples, A and B, which in turn establishes a consistent ranking of A versus B across a range of evaluation functions belong to a specific class.
A stochastic dominance condition is a relationship between functions of the distribution functions of a pair of samples, A and B, which in turn establishes a consistent ranking of A versus B across a range of evaluation functions belong to a specific class. This is the form of a typical stochastic dominance condition (the left hand side is by definition the relationship between the functions):

$$R[G_A, G_B] \iff W_A(x) \geq W_B(x) \forall x \land \exists x | W_A(x) > W_B(x) \forall W \in W^D$$
The test of Duclos et al. (2006): Introduction

- This test is quite sui generis in that the dominance conditions are not based on functions of distribution functions, but rather on multiplicative FGT functions!
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- The conditions provided allow for poverty comparisons robust not just to dimension weights but chiefly to different poverty lines representing different identification criteria (e.g. union, intersection, intermediate)
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- It is a fantastic result.
- It allows for a wide range of conditions.
- The conditions provided allow for poverty comparisons robust not just to dimension weights but chiefly to different poverty lines representing different identification criteria (e.g. union, intersection, intermediate)
- Main drawback: it only considers classes of welfare functions characterized by substitutability between dimensions
The test of Duclos et al. (2006): preliminary notation

The conditions work over additive poverty functions:

\[ P(\lambda) = \int \int_{\Lambda(\lambda)} \pi(x, y; \lambda) dF(x, y) \]

The individual poverty function is characterized by:
\[ \pi(x, y; \lambda) \geq 0 \text{ if } \lambda(x, y) \leq 0 \text{ and } \pi(x, y; \lambda) = 0 \text{ otherwise} \]
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The multiplicative FGT surface (upon which the conditions are derived) is:

\[ P^{\alpha_x, \alpha_y}(z_x, z_y) = \int_0^{z_y} \int_0^{z_x} (z_x - x)^{\alpha_x} (z_y - y)^{\alpha_y} dF(x, y) \]

where \( \alpha_x \) and \( \alpha_y \) are positive integers
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The test of Duclos, Sahn and Younger (2006) for two continuous variables

The test of Duclos et al. (2006): examples of classes for two continuous variables

One class of functions is labeled $\prod_1^{1,1}(\lambda^*)$ and its aggregate poverty functions, $P(\lambda)$, are characterized by:

1. $\Lambda(\lambda) \subseteq \Lambda(\lambda^*)$
2. $\pi(x, y; \lambda) = 0$, whenever $\lambda(x, y) = 0$
3. $\pi_x \leq 0$ and $\pi_y \leq 0\ \forall x, y$
4. $\pi_{xy} \geq 0 \ \forall x, y$
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Another class of functions is labeled $\prod^{2,1}(\lambda^*)$ and its aggregate poverty functions are characterized by:

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Another class of functions is labeled $\mathcal{P}^{2,1}(\lambda^*)$ and its aggregate poverty functions are characterized by:

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The test of Duclos et al. (2006): dominance conditions for the examples

Let’s define $Q^A - Q^B \equiv \Delta Q$
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In their paper Duclos et al. (2006) show, for their examples, that:

$$\Delta P(\lambda) > 0 \forall P(\lambda) \in \prod_{1,1}^{1,1} (\lambda^*) \iff \Delta P^{0,0}(x, y) > 0 \forall x, y \in \Lambda(\lambda^*)$$
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These conditions can be extended into higher orders of dominance by dimension (the $\alpha$'s) and more dimensions
Duclos et al. (2006) show that the multiplicative FGT’s are asymptotically normally distributed
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Testing strategy

Duclos et al. (2006) show that the multiplicative FGT’s are asymptotically normally distributed
This is a very useful result (an extension of Davidson and Duclos, 2000) since:

\[ H_0: P_{\alpha_x,\alpha_y} A \leq P_{\alpha_x,\alpha_y} B \text{ against } H_a: P_{\alpha_x,\alpha_y} A > P_{\alpha_x,\alpha_y} B \]

Rejection of that null hypothesis in favor of that alternative would require that
\[ Z_{\Delta P} \geq Z_{\text{critical}} \forall x, y \text{ and } \exists x, y | Z_{\Delta P} > Z_{\text{critical}} \]
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3. For instance we can test $H_0 : P_A^{\alpha_x, \alpha_y} \leq P_B^{\alpha_x, \alpha_y}$ against $H_a : P_A^{\alpha_x, \alpha_y} > P_B^{\alpha_x, \alpha_y}$
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   $H_a : P_A^{\alpha_x, \alpha_y} > P_B^{\alpha_x, \alpha_y}$
4. Rejection of that null hypothesis in favor of that alternative would require that $Z_{\Delta P} \geq Z_{critical}$ $\forall x, y$ and
   $\exists x, y | Z_{\Delta P} > Z_{critical}$
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**Testing strategy: the covariance matrices**

As in the univariate case we have the case of independent samples (e.g. two countries) and dependent samples (e.g. panel data)
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- Now we need to account for more dimensions
- We are estimating the standard errors of differences of FGT as opposed to differences in functions of probabilities (e.g. the factorial is not considered)
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\[
\text{cov}\left[\hat{P}_{A,t}^{\alpha_x,\alpha_y}(z_x, z_y), \hat{P}_{A,t+1}^{\alpha_x,\alpha_y}(z_x, z_y)\right] = \\
\left(\frac{1}{N}\right)^2 \sum_{i=1}^{N} (z_y - y_{i,t})_{+}^{\alpha_y} (z_x - x_{i,t})_{+}^{\alpha_x} (z_y - y_{i,t+1})_{+}^{\alpha_y} (z_x - x_{i,t+1})_{+}^{\alpha_x} \\
- \left(\frac{1}{N}\right) \frac{1}{N} \sum_{i=1}^{N} (z_y - y_{i,t})_{+}^{\alpha_y} (z_x - x_{i,t})_{+}^{\alpha_x} \frac{1}{N} \sum_{i=1}^{N} (z_y - y_{i,t+1})_{+}^{\alpha_y} (z_x - x_{i,t+1})_{+}^{\alpha_x}
\]
The test of Duclos et al. (2006) for one continuous and several discrete variables

The basic setting (for one continuous and two discrete variables) is the following:
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- One continuous variable, $x$, and two discrete variables taking natural values from 1 to $K$ and 1 to $K^*$ respectively (higher values denoting better-off situations)
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- For the dominance conditions we use the following conditional FGT function: $P_\alpha(k, k^*; z) = \int_0^z (z - x)^\alpha f(x|k, k^*)dx$
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The basic setting (for one continuous and two discrete variables) is the following:

- One continuous variable, $x$, and two discrete variables taking natural values from 1 to $K$ and 1 to $K^*$ respectively (higher values denoting better-off situations)
- For the dominance conditions we use the following conditional FGT function: $P^\alpha(k, k^*; z) = \int_0^z (z - x)^\alpha f(x|k, k^*) dx$
- The generic aggregate poverty index is defined over several poverty lines: $P(z(1, 1); z(1, 2); \ldots; z(1, K^*); \ldots; z(K, K^*))$
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The poverty functions under comparison are considered to be evaluated over poverty lines for x which depend on the combinations of the discrete variables’ values (KK*)
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This test seems to be even more relevant in empirical applications
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This test seems to be even more relevant in empirical applications

It assumes ordinality in all discrete variables (sensible)
The test of Duclos et al. (2006) for one continuous and several discrete variables: the example they provide

They find a dominance condition for a general class named \( \Pi^1(z(1, 1); z(1, 2); \ldots; z(1, K^*); \ldots; z(K, K*)) \)
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They find a dominance condition for a general class named $\Pi^1(z(1, 1); z(1, 2); \ldots; z(1, K*); \ldots; z(K, K*))$.

This class has the following characteristics:

1. $z(k, k*) \geq z(l, k*)$ if $k < l$
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The test of Duclos, Sahn and Younger (2006) for one continuous and several discrete variables

They find a dominance condition for a general class named $\mathcal{D}^1(\lambda(1, 1); \lambda(1, 2); \ldots; \lambda(1, K\ast); \ldots; \lambda(K, K\ast))$. This class has the following characteristics:

1. $\lambda(k, k\ast) \geq \lambda(l, k\ast)$ if $k < l$

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This class has the following characteristics:

1. \[ z(k, k^*) \geq z(l, k^*) \text{ if } k < l \]
2. \[ z(k, k^*) \geq z(k, l^*) \text{ if } k^* < l^* \]
3. \[ \pi^{(1)}_{k, k^*} \leq \pi^{(1)}_{k+1, k^*} \leq 0 \quad \forall x, k, k^* \]
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3. $\pi_{k,k*}^{(1)} \leq \pi_{k+1,k*}^{(1)} \leq 0 \quad \forall x, k, k*$

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4. \( \pi^{(1)}_{k,k^*} \leq \pi^{(1)}_{k,k^*+1} \leq 0 \ \forall x, k, k^* \)

5. \( \pi_{k,k^*}(z(k, k^*)) = 0 \ \forall k = 1, \ldots, K; k^* = 1, \ldots, K^* \)
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The condition for this class
\[ \Pi^1(z(1, 1); z(1, 2); \ldots; z(1, K*); \ldots; z(K, K*)) \] is:
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The condition for this class

\[ \Pi^1(z(1, 1); z(1, 2); ..., z(1, K*); ..., z(K, K*)) \]

is:

\[ \Delta P(\zeta(1, 1), ..., \zeta(K, K*)) > 0 \forall P(\zeta(1, 1), ..., \zeta(K, K*)) \in \Pi^1 \]

\[ \wedge \forall \zeta(k, k*) \in [0, z(k, k*)], k = 1, ..., K; k* = 1, ..., K* \]

\[ \sum_{i=1}^{i} \sum_{k=1}^{j} \Delta P^0(k, k*; \zeta) > 0, \forall \zeta \in [0, z(i, j)] \]

\[ \wedge i = 1, ..., K; j = 2, ..., K* \]
The test of Duclos et al. (2006) for one continuous and several discrete variables: estimations of statistics

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$$\sum_{k=1}^{i} \sum_{k^* = 1}^{j} \hat{P}^\alpha(k, k^*; \zeta) = \frac{1}{N} \sum_{m=1}^{N} (\zeta(k_m, k^*_m) - x_m)^\alpha_+ l(k_m \leq i \land k^*_m \leq j)$$
The test of Duclos et al. (2006) for one continuous and several discrete variables: estimations of statistics

Another example, the covariance of dependent samples with one continuous variable and one discrete variable:

$$\text{cov}(\sum_{k=1}^{j} \hat{P}_t^{\alpha}(k; \zeta), \sum_{k=1}^{j} \hat{P}_{t+1}^{\alpha}(k; \zeta)) =$$

$$\frac{1}{N^2} \sum_{m=1}^{N} (\zeta(k_m,t) - x_{m,t})^{\alpha} + (\zeta(k_{m,t+1}) - x_{m,t+1})^{\alpha} I(k_{m,t} \leq j \land k_{m,t+1} \leq j)$$

$$- \left(\frac{1}{N}\right) \sum_{k=1}^{j} \hat{P}_t^{\alpha}(k; \zeta) \sum_{k=1}^{j} \hat{P}_{t+1}^{\alpha}(k; \zeta)$$