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Overview of Multidimensional Poverty Measures

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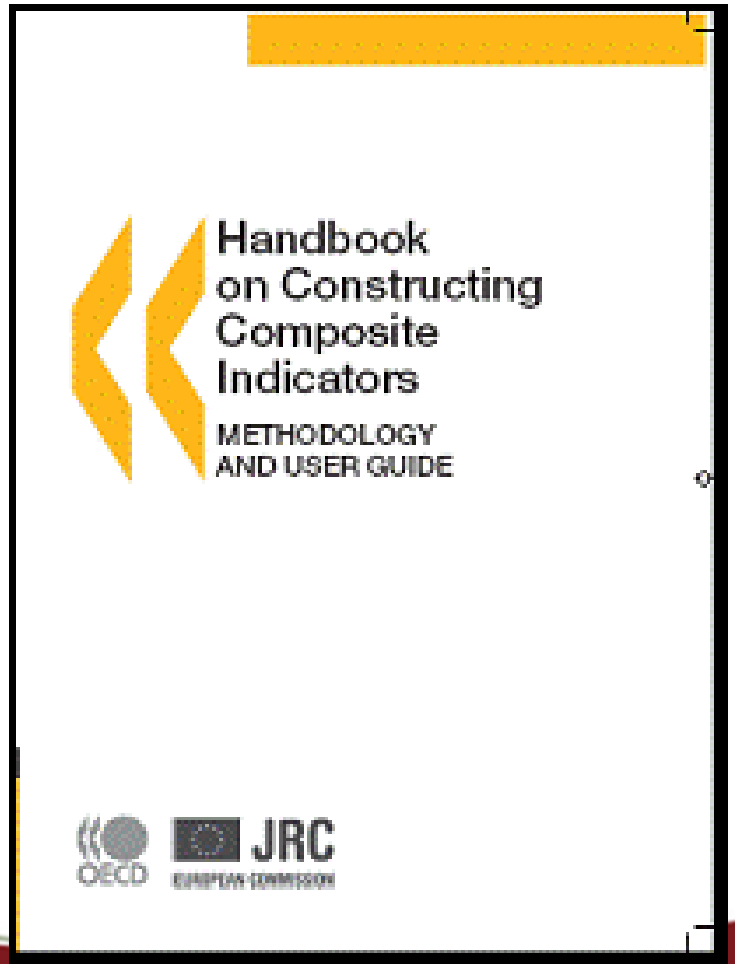
Multidimensional Measures are exploding

- Bandura (2006) found that over 50% of composite (multidimensional) indices related to many topics had been developed within the past five years.
- In the area of poverty/well-being the proportion appears to be even higher.

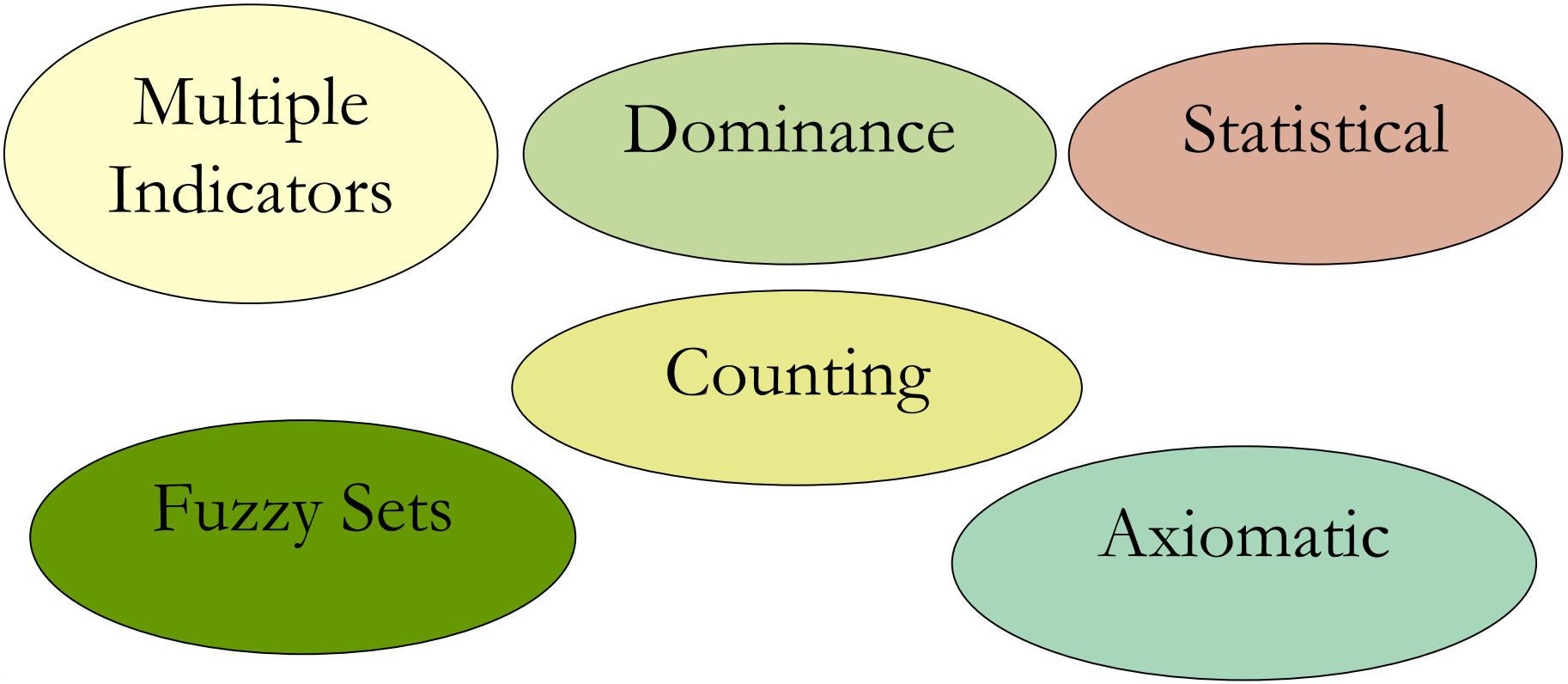
Various groups are synthesising different aspects of measurement methodology – e.g. this 2008 Handbook



Their use is ‘exploding’ but they are not new.

Different approaches have been used over time to address the multidimensionality of poverty.

Approaches to MD Poverty Measurement



Not mutually exclusive. There are overlaps.

Basic Notation - Typical Dataset

- Where x_{ij} is the achievement of individual *i* of attribute or dimension *j* .
- zj is the deprivation cutoff of attribute or dimension j.

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1d} \\ x_{21} & \cdots & x_{2d} \\ \cdots & & \\ & & \cdots \\ x_{n1} & \cdots & x_{nd} \end{bmatrix}$$

Dimensions

People

$$z = (z_1, z_2, \ldots\ldots, z_d)$$

Basic Notation

- Let matrix X=[xij], of size nxd the multidimensional distribution of d attributes among n individuals, with non-negative elements. And let $\mathcal{X}$ be the set of all possible achievement matrices.

- Let vector $z \in R^{d}_{++}$ be the cutoff vector containing the deprivation cutoff for each dimension.

Notation-Steps to measurement

- **Selection of the space**
- **Identification:** Who is multidimensionally poor? We need an 'identification function', a criterion that decides who is considered multidimensionally poor.

$$\rho : R_{+}^{d} \times R_{++}^{d} \rightarrow \{0,1\}$$

$$\rho(x_i, z) = 1 \quad \text{if} \quad i \quad \text{multi.poor}$$

$$\rho(x_i, z) = 0 \quad \text{if} \quad i \quad \text{not} \quad \text{multi.poor}$$

- Applying the identification function ϱ, we get the set of the multidimensionally poor, name it Z (different from the vector of poverty lines z)

Notation-Steps to measurement

Aggregation:

Given the identification method, a *poverty index* summarizes the information of the achievements among the poor into a real number.

A poverty index is defined as $P: \mathcal{X} \times z \rightarrow R$

We denote a poverty index based on an achievement matrix X and a deprivation cut-off vector z by P(X;z). It is implicitly assumed that the identification function is given.

1.1 Multiple Indicators Approach: Dashboards

- A set of indicators, ie. applying a "standard unidimensional measure to *each* dimension" (Alkire, Foster, Santos, 2011).
- Let $n_j \in N$ denote the population size covered by indicator j for all $j=1, \ldots d$.
- We summarize the achievements of all n_j people by vector $X_j \in R_+^{nj}$ and denote the corresponding deprivation cut-off by $z_j \in R_{++}$.

1.1 Multiple Indicators Approach: Dashboards

- We define a *deprivation index* P_j for dimension j by

  $P_j : R_+^{nj} \times R_{++} \rightarrow R$,which assesses the deprivation profile of people in dimension j . The dashboard of indicators, denoted by DI, is a d-dimensional vector that contains the deprivation indices of all d dimensions. Hence, technically, $DI=(P_1, \ldots, P_d)$.

1.1 Dashboards - Examples

- **Basic Needs Approach.**

"As a first step, it might be useful to define the best indicator for each basic need. At present the essential BN are considered to cover six areas: basic education, health, sanitation, water supply and housing and related infrastructure. This list is not exhaustive, nor do all needs listed have the same status. A limited set of core indicators covering these areas would be a useful device for concentrating efforts" (Hicks & Streeten, 1979).

1.1 Dashboards - Examples

- "...what seems to be called for is a genuinely multi-dimensional approach in which expenditure on market goods sit side-by-side with 'non-income' indicators of access to non-market goods and indicators of intra-household distribution" (Ravallion, 1996).
- "...multiple indices are required [for multidimensional poverty measurement], each measuring different things using the best data available for that task –presenting us with a large and eclectic dashboard" Ravallion (2011).

1.1 Dashboards - Examples

- Millenium Development Goals (UN, 2000): 48 indicators to monitor 18 targets to achieve the 8 goals.

- Sarkozy Comission also evaluates the pros and cons of using dashboards to monitor *sustainable* development.

1.1 Dashboards Examples: MDGs

Proportion of population below $1 (PPP)/day

Literacy rate of 15-24 years-old

Share of women in wage employment in the non-agricultural sector

Maternal mortality ratio

Under five mortality rate

Proportion of births attended by skilled personnel

Net enrolment ratio in primary education

Prevalence of underweight children under 5 years of age

Proportion of tuberculosis cases detected and cured under DOTS

Proportion of seats held by women in national parliament

Prevalence of deaths associated with malaria

1.1 Dashboards – Pros

- They broaden the space from one to many dimensions of poverty.
- They potentially allow *using the best data available for that task* (in practice most of the MDG indicators come from the same surveys).
- They offer a rich amount of information.

1.1 Dashboards –Note:

- Base population differs by indicator, thus there may be or may not be overlapping subgroups of people.

1.1 Dashboards

BP: Total Population

BP: Total Employed People agricultural sector

BP: Number of children of official

BP: Number of tuberculosis cases

BP: Total occupied seats in parliament

BP: Total number of people 15-24 years

BP: Total recorded or estimated live births of under five children born alive

BP: Total number of births

BP: Children under 5 years of age

BP: Total number of deaths

1.1 Dashboards - Cons

- Lack of hierarchies amongst the indicators used (SSF, 2009)
- Lack of a single outline figure as GDP (SSF, 2009)
- Leave the questions about tradeoffs completely open (AFS, 2011)

1.1 Dashboards –Cons

- "Dashboards nevertheless suffer because of their heterogeneity, at least in the case of very large and eclectic ones, and most lack indications about causal links [...] or hierarchies amongst the indicators used. Further, as communications instruments, one frequent cricism is that they lack what has made GDP a success: the powerful attraction of a single headline figure allowing simple comparisons of socio-economic performance over time or across countries" (Stiglitz, Sen, Fitoussi, 2009)

1.1 Dashboards - Cons

- When the base population coincides in part or in total, overlaps in deprivations are overlooked, ie. the ***joint distribution*** is ignored. Blind to joint deprivations (Ferreira, 2011; AFS, 2011)

1.1 Dashboards - Cons

	Undernourished	Uneducated	No safe water	No electricity
Peter	1	0	0	0
Ana	0	1	0	0
John	0	0	1	0
Paula	0	0	0	1

	Undernourished	Uneducated	No safe water	No electricity
Peter	1	1	1	1
Ana	0	0	0	0
John	0	0	0	0
Paula	0	0	0	0

The marginal headcount ratios for each dimension are the same (25%), but the actual situation of people is not.

1.1 Dashboards - Cons

- Silent about **a fundamental question in poverty measurement: Who is poor overall? (IDENTIFICATION!)**
- How many poor people are? How poor are they? (AFS, 2011)

1.2 Multiple Indicators Approach: Composite Indices

- A function of variables and weights which maps attainments in a variety of attributes into a single real number, which may have cardinal meaning or be merely ordinal (Santos & Santos, 2013).
- Given a set of dimensional deprivation indices P_j (as in the dashboard), these are aggregated to obtain the composite index as:

$$CI: P_1 \times P_2 \times ..x\ P_d \rightarrow R.$$

1.2 Multiple Indicators Approach: Composite Indices

- Composite indices are one way to circumvent the problem raised by the richness of dashboards and to synthesize the abundant and purportedly relevant information into a single number (SSF, 2009).
- While assessing quality-of-life requires a plurality of indicators, there are strong demands to develop a single summary measure (SSF, 2009).

1.2 Composite Indices: Example The Human Poverty Index (HPI)

Dimension	Indicator
Survival deprivation	•P_1: Probability at birth of not surviving to age 40.
Education deprivation	•P_2: Adult illiteracy rate
Economic deprivation	P_3: Equally weighted avg of: •% of population without access to an improved water source •% of children under weight for age

1.2 Composite Index Example HPI-I

$$HPI\text{-}I=\{(1/3)[(P_1)^{\alpha}+(P_2)^{\alpha}+(P_3)^{\alpha}]\}^{1/\alpha}$$

- This is the ***general means*** expression.
- With $\alpha=1$, HPI is the arithmetic mean, all dimensions are equally weighted.
- For $\alpha>1$, higher weight is given to 'higher entries', ie: the dimensions in which there is most deprivation.
- For $\alpha<1$, higher weight is given to 'lower entries', ie: the dimensions in which there is least deprivation.
- Value used by the HDRO: $\alpha=3$.

1.2 Composite Indices - Pros

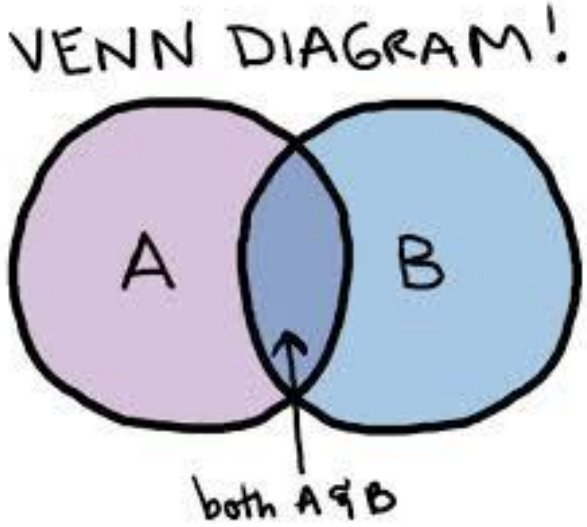
- Uses aggregate deprivation indices, thus
  - Can reflect deprivations of different population subgroups
  - Can combine distinct data sources
- They offer a summary measure.
- They offer a hierarchy and make **trade offs** explicit (which enables debate, see for example "Troubling tradeoffs of the Human Development Index", Ravallion 2011).
- Note: tradeoffs indicate how much of one desired component of the CI must be given up for an extra unit of another component, keeping the overall index constant.

1.2 Composite Indices – Cons

- The joint distribution is ignored (the indicators may have different base populations and may come from different data sources).

1.3 Multiple Indicators Approach: Venn Diagrams

- A Venn diagram is a collection of circles showing all possible logical relation between a finite numbers of **dimensions with binary outcomes**, such as – deprived and non-deprived.



1.3 Multiple Indicators Approach: Venn Diagrams

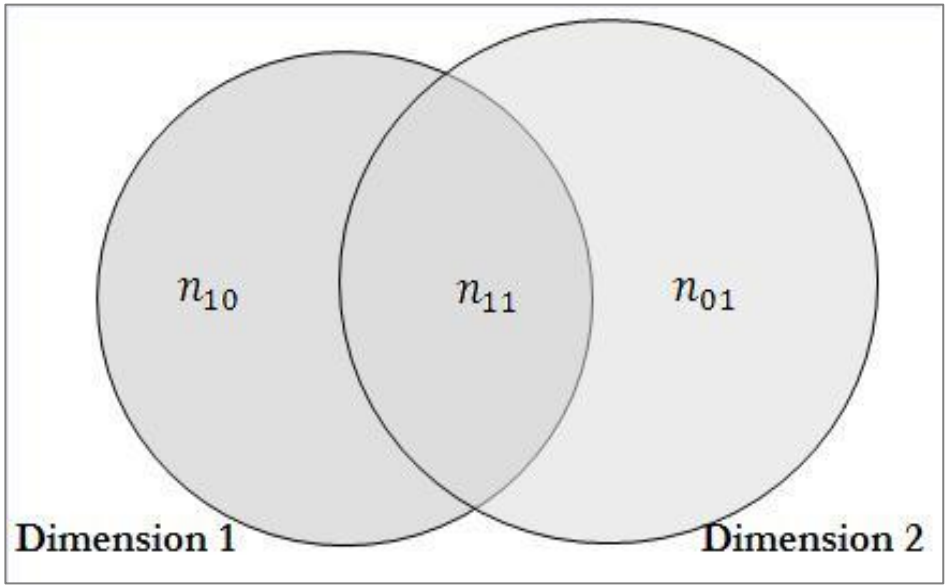
- When there are only two indicators, a Venn diagram provides diagrammatic represe of a 2x2 contingency table

		Dimension 2		To
		Non-Deprived	Deprived	
Dimension 1	Non-Deprived	n_{00}	n_{01}	n_{0+}
	Deprived	n_{10}	n_{11}	n_{1+}
Total		n_{+0}	n_{+1}	n

The interesting part! Those who are jointly deprived

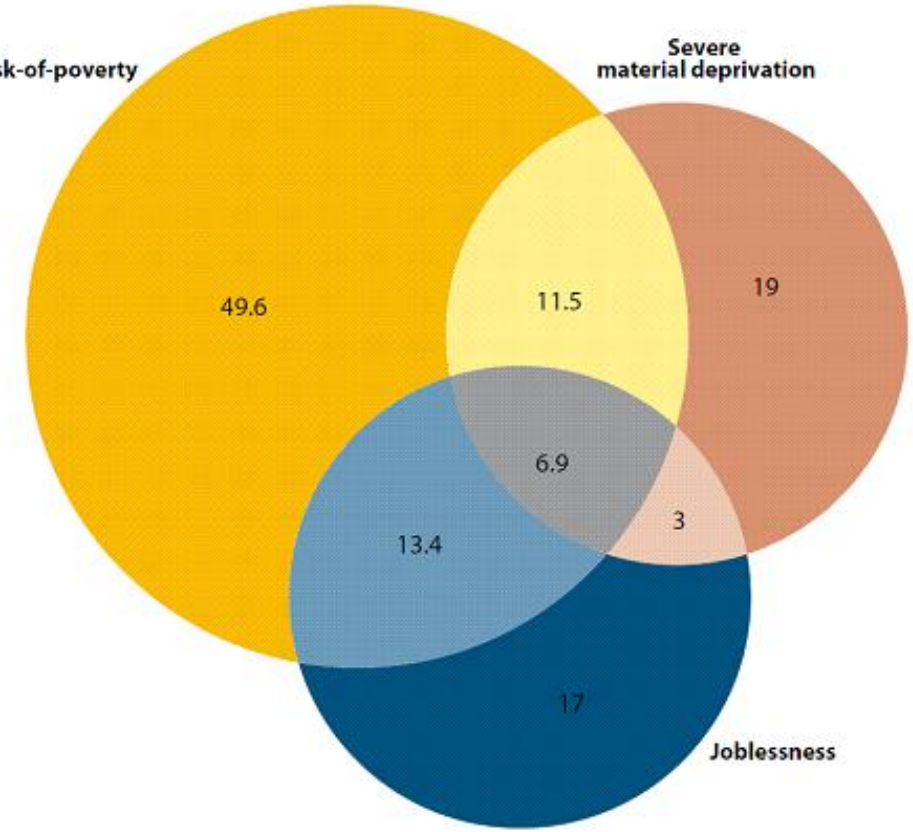
1.3 Multiple Indicators Approach: Venn Diagrams

- The extent of overlap between circles shows the extent to which deprivations in different dimensions overlap or the extent to which people are jointly deprived in a particular society.



1.3 Venn Diagrams - Example

Multiple indicators from the Europe 2020 target. Figures for EU-27 in million of persons

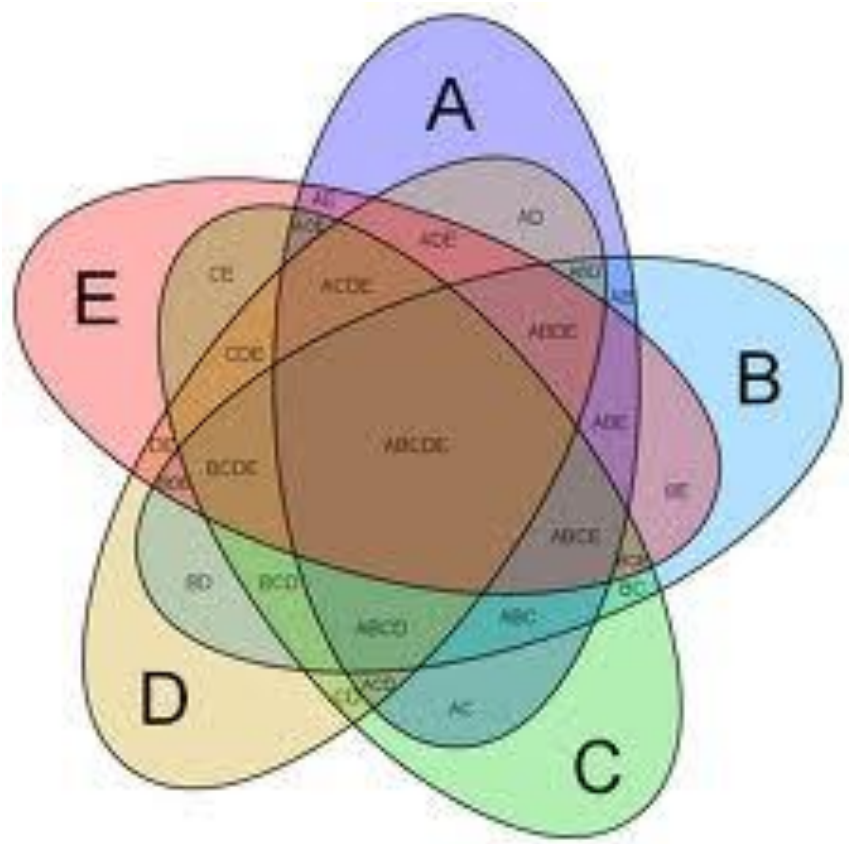


Easier to see than contingency tables for 3-4 variables

Source: EU-SILC, Eurostat-CEPS/INSTEAD calculations. Reprinted from Atkinson, A. B., E. Marlier, F. Monatigne, and A. Reinstadler (2010) `Income poverty and income inequality', in *Income and Living Conditions in Europe,* Atkinson and Marlier (eds), Eurostat, page 127.

1.3 Multiple Indicators Approach: Venn Diagrams

- Venn Diagram becomes hard to read for 5+ variables



1.3 Venn Diagrams : Pros

- A visual tool to identify overlapping and non-overlapping deprivations in the population.
- Consider the joint distribution.
- (Note: Usually go together with headcount and headcount ratio as aggregation too)
- Examples of application: Atkinson et al (2010), Ferreira and Lugo (2012).

1.3 Venn Diagrams : Cons

- No summary measure. Thus, no complete ordering.
- No hierarchies, no tradeoffs.
- Do not offer a ranking.

2. Dominance Approach

Motivation:

- To be able to ascertain whether poverty is unambiguously lower in A than in B ***regardless*** of:

  (a) the poverty line (identification) and/or

  (b) the poverty measure (aggregation).

- Such a claim certainly has strong political power!

(Clearly one wants to avoid the possibility of contradictory rankings at different parameters of identification or aggregation)

2. Dominance Approach

Origins:

- **Unidimensional inequality:** Atkinson 1970
- **Unidimensional poverty:** Atkinson 1987, Foster and Shorrocks 1988a,b, Jenkins and Lambert 1998 .

Extensions:

- **Multidimensional welfare**: Atkinson & Bourguignon 1982, 1987; Bourguignon 1989
- **Multidimensional poverty:** Duclos, Sahn & Younger (2006a,b).

2. Dominance Approach

A**P**B if and only if

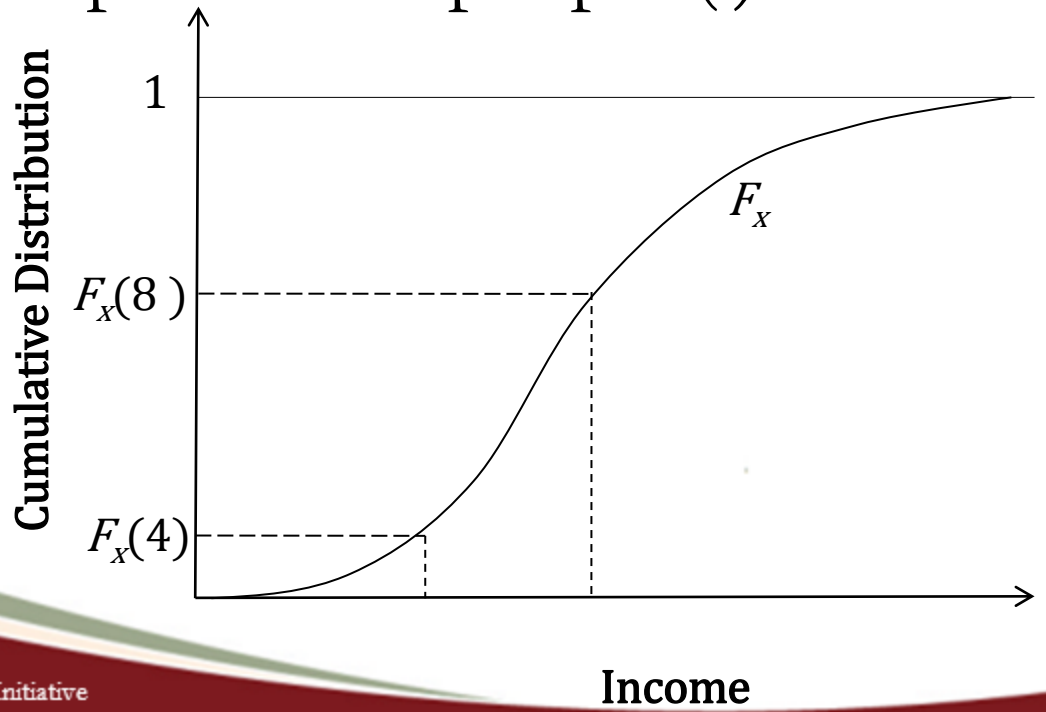
$P(x_A;z) - P(x_B;z) \leq 0$ for all z in R_{++} and

$P(x_A;z) - P(x_B;z) < 0$ for some z in R_{++}

A**P**B means that A has *unambiguously* **less poverty than** B with respect to poverty index P for all poverty lines.

2. Dominance Approach

- Key tool: **Cumulative distribution functions** (cdf) of the variable/s of interest.
- One dimension: given variable x, the cdf $F_x(s)$ gives the proportion of people (i) such that $x_i \leq s$



2. Dominance Approach

- Two dimensions: given variables x and y, the cdf of the **joint probability** distribution gives the proportion of people (i) whose variable values are below the different possible combinations of values for the two variables ($x_i \leq s$ and $y_i < l$)

- Note that the cdf is now cumulative in **both** dimensions. Thus, it is a **surface**.

A two-dimensional cdf

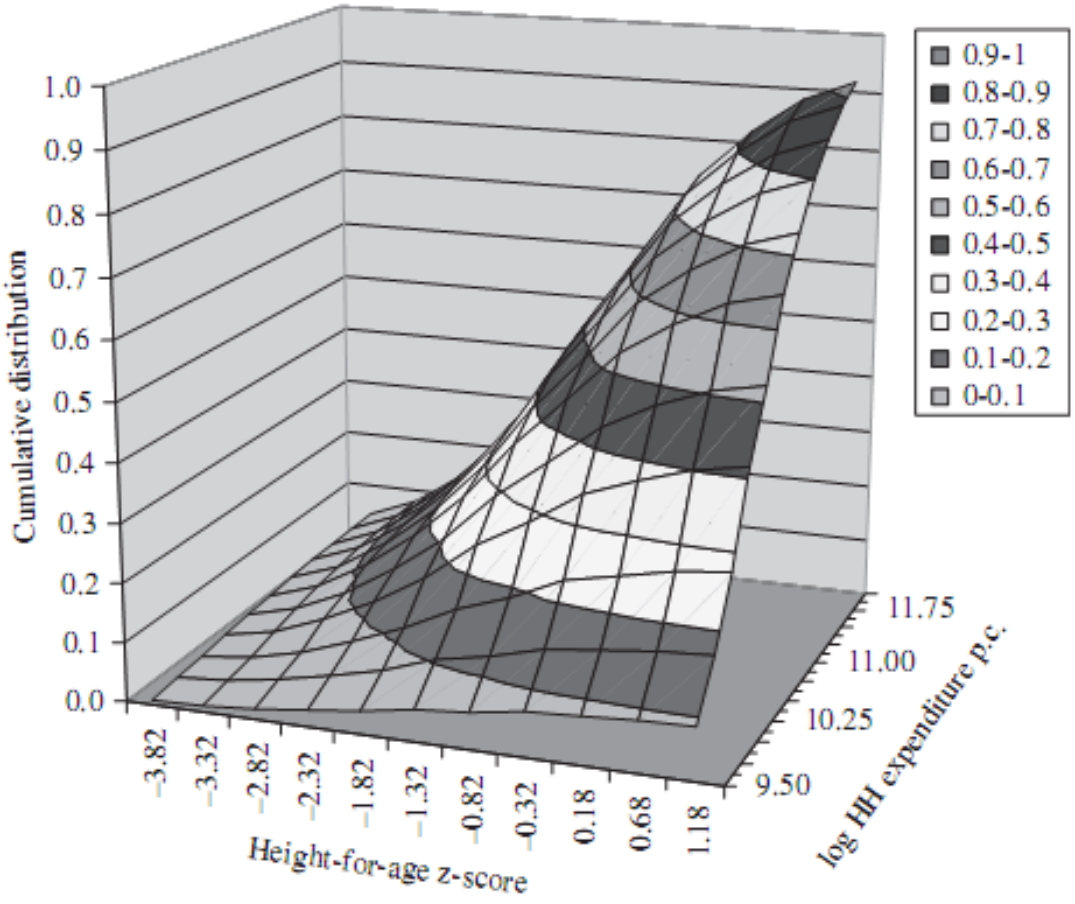


Fig. 3. *Dominance Surface for Ghanaian Children, 1989*

Source: Duclos, Sahn and Younger, (2006).

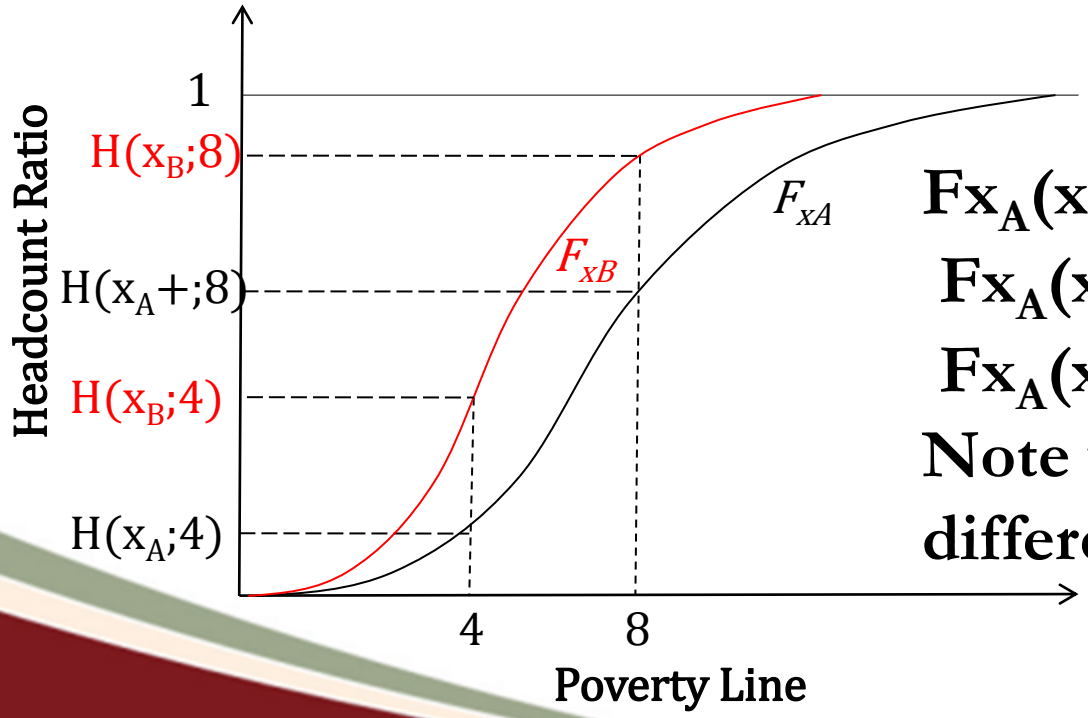
2. Dominance Approach

- **Key procedure**: To look whether one cdf lies completely below the other or not.
- That is **first order dominance**, which is associated to the head count ratio. Why? Because by looking at the cdfs we are comparing the proportion of people who are below a certain threshold in each variable in each country or distribution. (The cdf graph offers precisely the information on headcount ratios!)

Poverty Ordering Based on H

Given the cdf of two distributions A and B, if Fx_B lies no where to the right of the cdf of A, then B has no lower headcount ratio than A for each and every poverty line.

First order Stochastic Dominance (FSD) . A FSD B



$Fx_A(x)$ FSD $Gx_B(x)$ iff
$Fx_A(x) - Gx_B(x) \leq 0$ for all
$Fx_A(x) - Gx_B(x) < 0$ for some
Note we are taking the difference btw the two cdfs

2. Dominance Approach in the MD case

In the 2 dimensional case, one needs to see the difference between two surfaces like this one(rather than two lines) and check whether the difference always has the same sign.

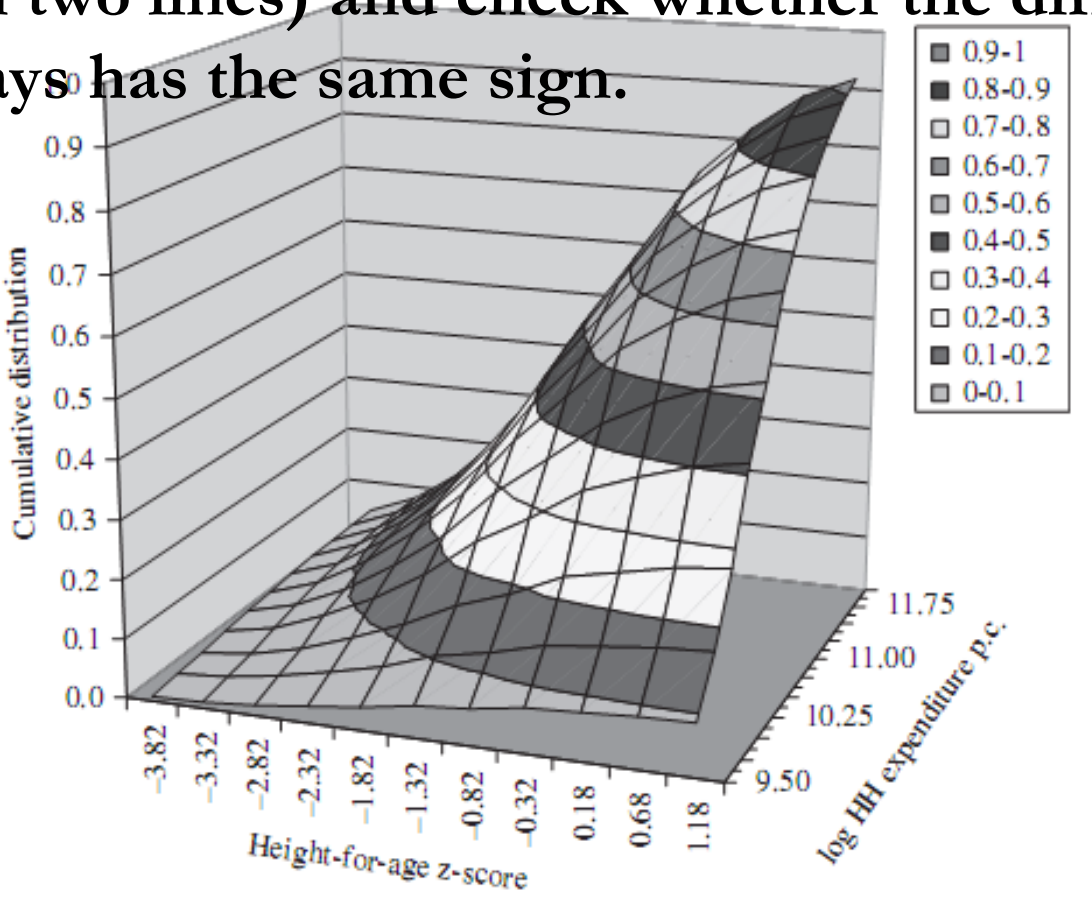


Fig. 3. *Dominance Surface for Ghanaian Children, 1989*

Source: Duclos, Sahn and Younger, (2006).

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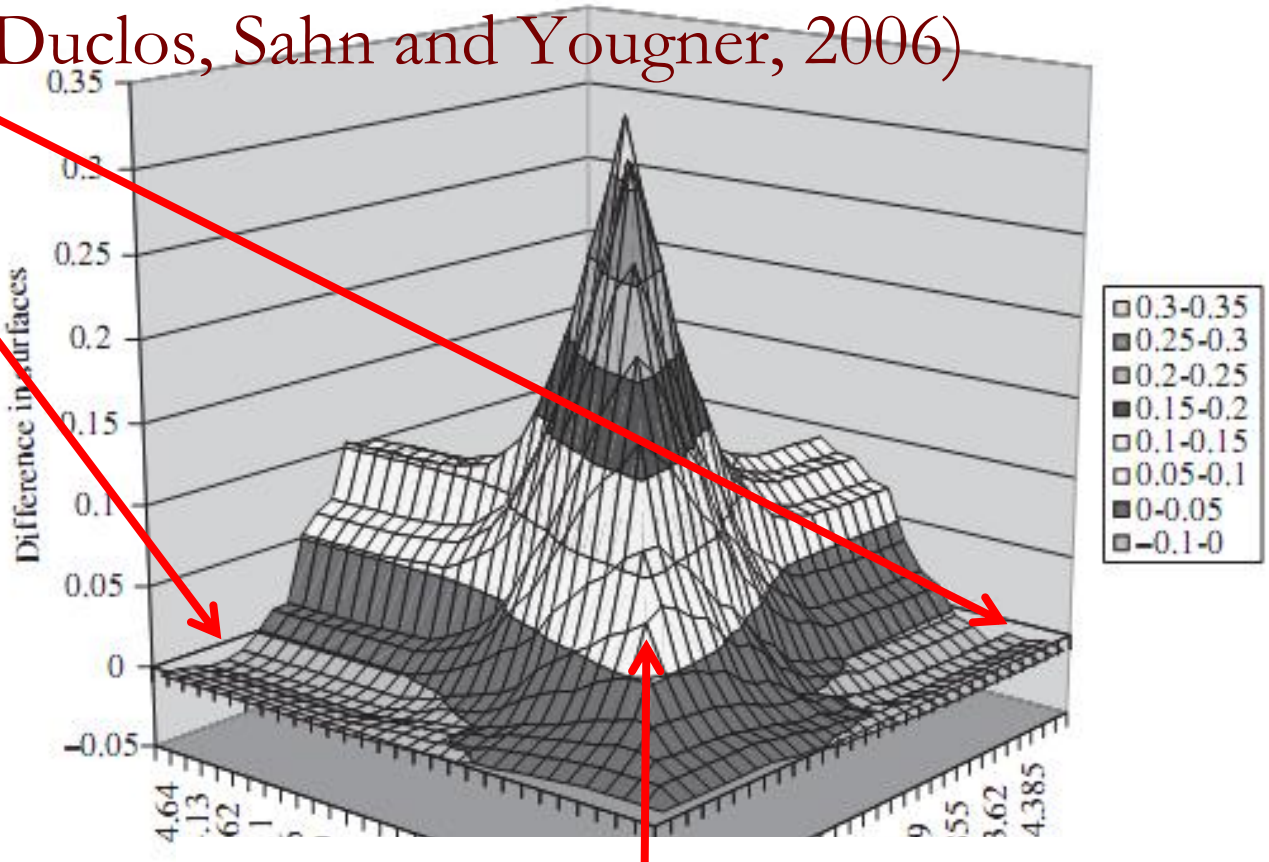
The difference between 2-dimensional cdfs

(hypothetical distributions)

(from Duclos, Sahn and Yougner, 2006)

Positive <u>and</u> negative differences in the marginal distributions (ie. No dominance!)



Positive difference in the interior section (ie. dominance in the intersection approach!)

2. Dominance Approach in MD case

- The difference between the two bivariate cdfs can be considered for any choice of poverty definitions: union, intersection and intermediate criterions to identify the poor (a range of ***poverty frontiers***).

Intersection criterion frontier λ_1

Union criterion frontier λ_2

Intermediate criterion frontier λ_3

Source: Duclos, Sahn and Younger (2006)

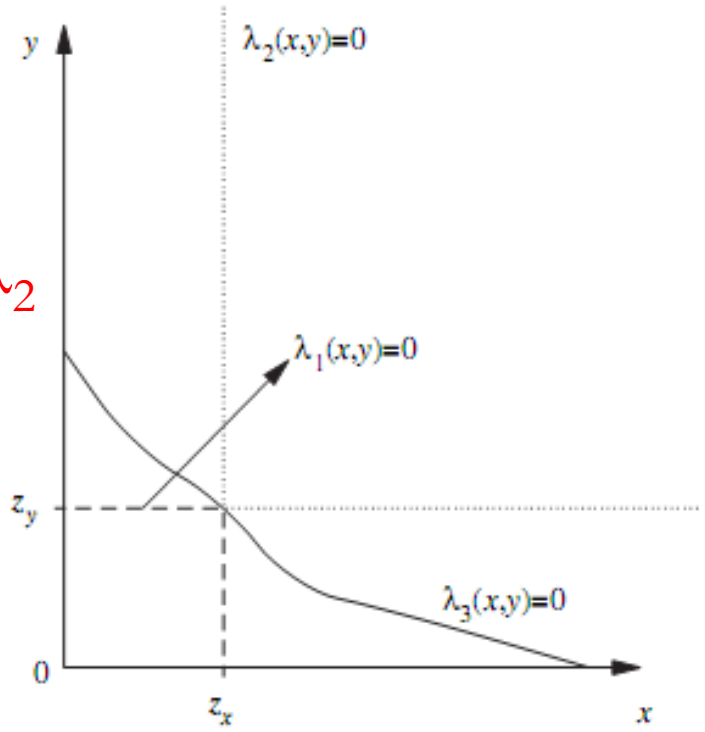


Fig. 1. *Union and Intersection Poverty Indices*

2. Dominance Approach in MD case Example

- Duclos, Sahn and Younger (2006).
- Stunting and child survival probability in Cameroon and Madagascar (using DHS, 1997).
- By univariate dominance , Madagascar is poorer than Cameroon, either measured by stunting or survival probability.
- BUT with the bivariate comparison they find there is no such dominance.

2. Dominance Approach in MD case Example

Negative difference for all points of the two surfaces between urban and rural areas of Viet Nam. Thus rural areas are poorer than urban ones.

Duclos, Sahn and Younger (2006).

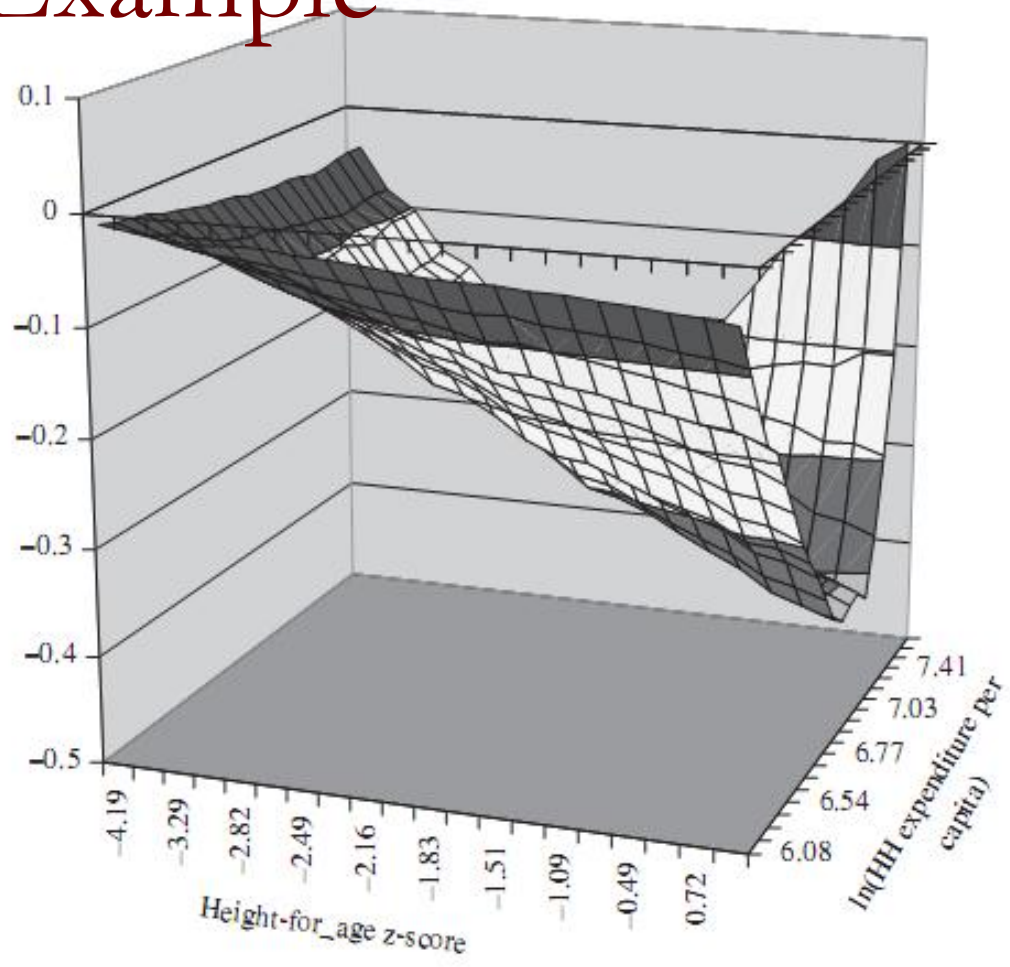


Fig. 5. *Urban Minus Rural Dominance Surface for Viet Nam*

2. Dominance Approach in MD

Negative difference for all points of the two surfaces between urban and rural areas of Viet Nam. Thus rural areas are poorer than urban ones.

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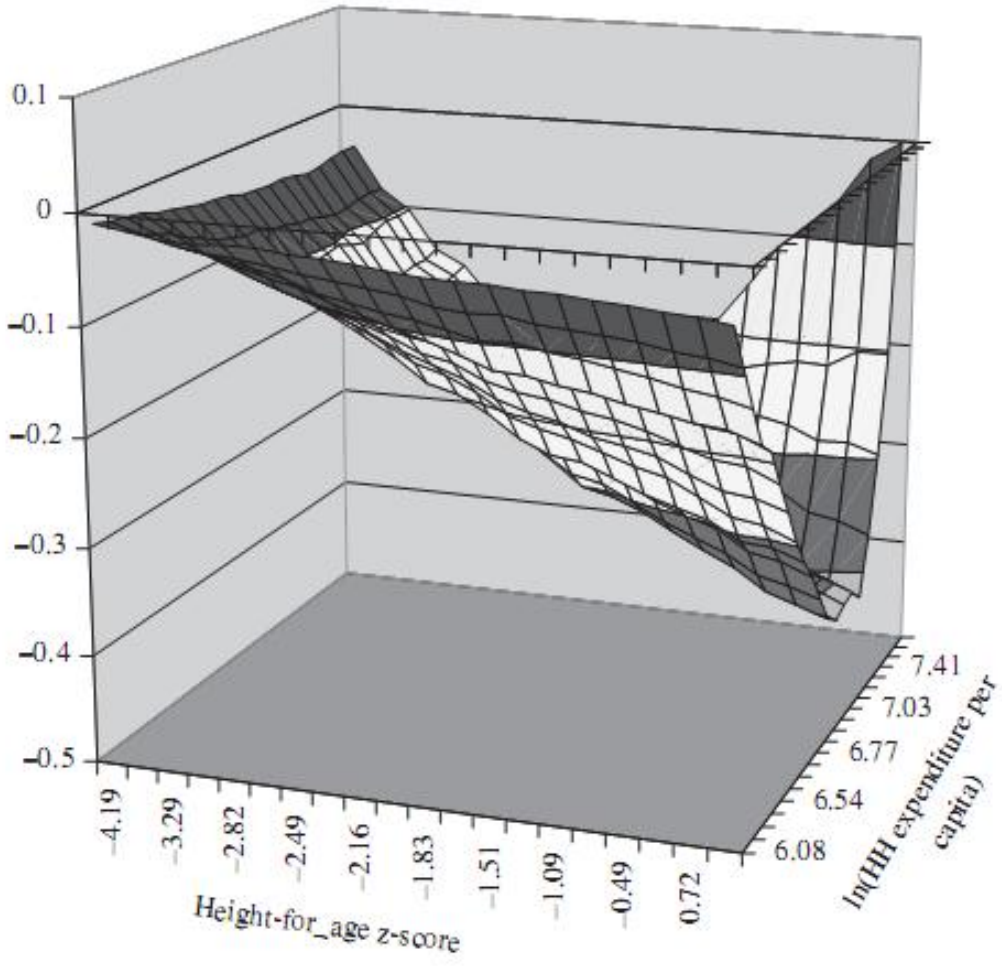


Fig. 5. *Urban Minus Rural Dominance Surface for Viet Nam*

2. Dominance Approach in MD

- Bourguinon and Chakravarty (2002) offer the mathematical conditions to evaluate MD dominance for different classes of MD poverty measures.
- Duclos, Sahn and Younger (2006) offer the mathematical conditions to evaluate MD dominance for a (broad) class of MD poverty measures and introduce the statistical tools to perform the required statistical tests.
- Some applications:
  - Batana and Duclos (2008), 2 dimensions, 6 countries 2008
  - Labar and Bresson (2011), China 1991-2006
  - Anaka and Kobus (2012), 2 dimensions, Polish Gminas

2. Dominance Approach in MD-Pros

- It offers a tool to produce strong empirical assertions about poverty comparisons.
- It considers the joint distribution.
- It can use discrete or continuous data.
- It avoids 'controversial' decisions on parameter values (... But for the same reason, it does not promote discussion or thought on them)

2. Dominance Approach in MD-Cons

- No summary measure, no hierarchy or tradeoff. No complete ordering.
- By offering ordinal ranks (pair-wise dominance), it does not show cardinally meaningful extent of differences.
  - E.g. cannot say if poverty fell faster in period 1 than 2

2. Dominance Approach in MD-Cons

- For 2+ dimensions, there is limited real applicability because of reduced size datasets:

  "In theory, extending our results to more than two dimensions is straightforward. In practice, though, most existing datasets in developing countries are probably not large enough to support tests on more than a few dimensions of well-being. This is because the curse of dimensionality (Bellman, 1961) affects our non-parametric estimators." (DSY, 2006)

3. Statistical Approaches

Motivation:

- To address multidimensionality but in a 'digestible' way. Thus, the main aim of these techniques is to reduce dimensionality.
- We will focus on the role that these techniques play at the **identification** and/or at the **aggregation** steps of poverty measurement.

3. Statistical Approaches

Descriptive

Purpose: to summarise or describe well-being/deprivation status of a population.

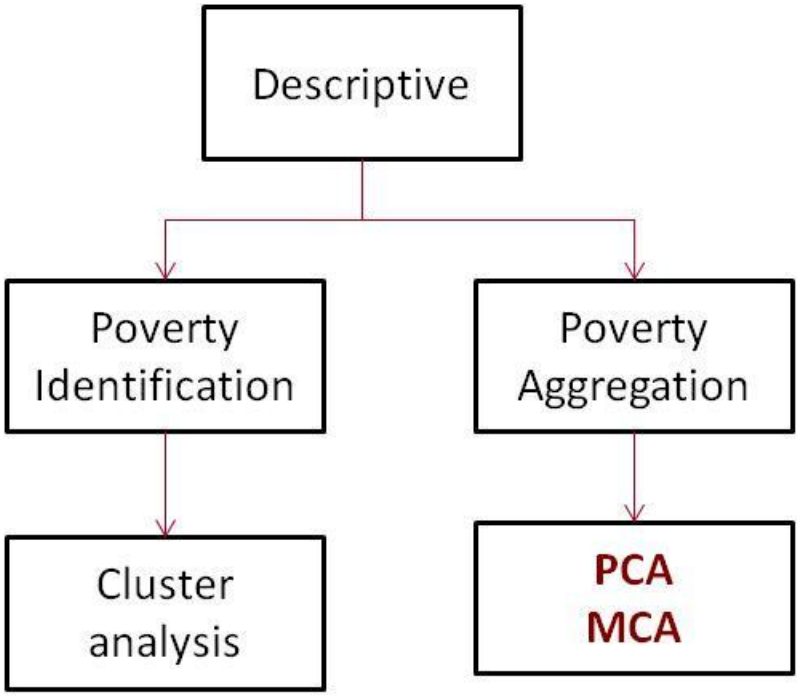
Techniques: Principal components (cardinal data), correspondence analysis (binary data), cluster analysis

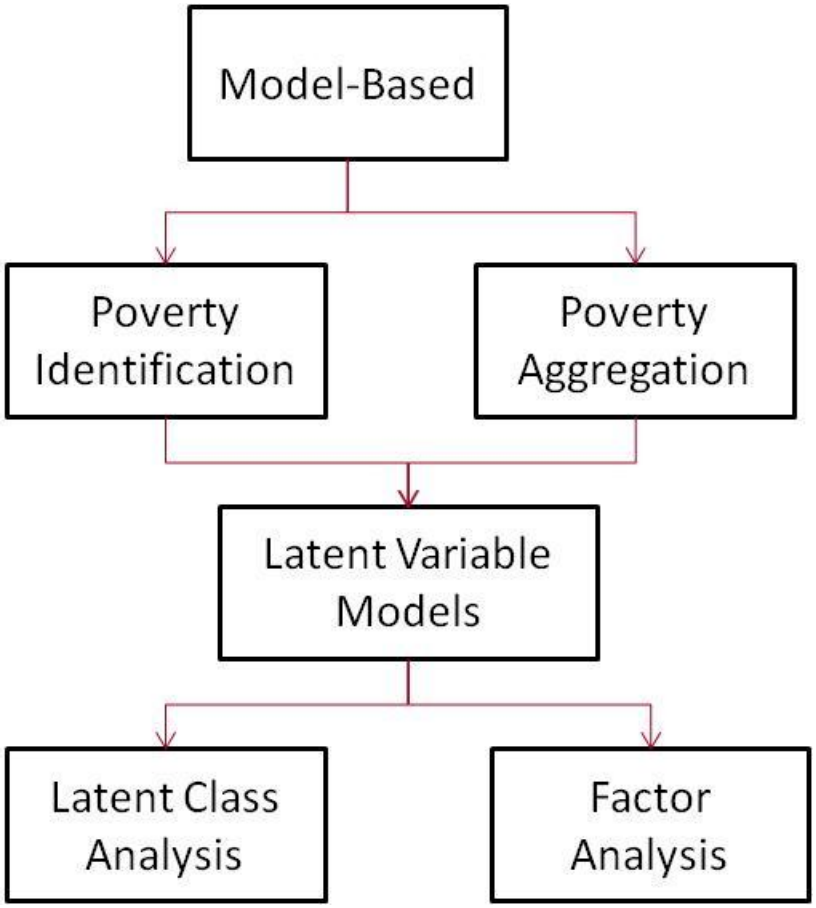
Model-based

Purpose: to make **inferences** about the well-being/deprivation status of a population(s). Thus make assumptions about the joint distribution.

Techniques: Latent variable models – Factor analysis is the most widely applied, others are latent class or structural equation models.

Multivariate Statistical Methods





Methods Used for Aggregation

A) What do they have in common?

Both descriptive and model-based methods like PCA, MCA or FA summarise the well-being/deprivation status of a population through a combination of the indicators into a 'composite measure', called component, axis or factor respectively.

This draws from the main principle or aim of these techniques which is to reduce dimensionality.

Methods Used for Aggregation

B) In what they differ?

They differ in the mathematical or statistical procedure used for computing these composites called often 'scores'.

Principal Component Analysis - Example

Filmer and Pritchett (1999, 2001) popularised an asset index approach, that proxies the welfare status of a population.

They developed their index in the context of analysing the associations between household economic status and schooling outcomes when the available datasets did not include information on households expenditures (DHS surveys).

Principal Component Analysis - Example

Their approach uses principal component analysis to compute this asset index.

Since then the asset index approach has been used for a diversity of purposes, including the analysis of inequalities, changes in poverty (Sahn and Stifel 2000, Stifel and Christiaensen 2007, Mckenzie 2005).

PCA – The Method

- Is a descriptive multivariate method that transforms a set of correlated variables into a 'new' set of uncorrelated components. These retain as much variability as possible of the original variables.

- The aim of the technique is to reduce the dimensionality of a set of variables with a 'minor' loss of information.

- The information is represented by the correlation (covariance) matrix of the indicators involved in the analysis.

How does it work?

- PCA includes 3 successive steps:

a) Computation of the principal components

b) Extraction or selection of the number of components

c) Rotation of retained components to facilitate interpretation

An example

The asset index proposed by Filmer and Pritchet (2000) is then:

$$A_i = a_1 x_{1i} + a_2 x_{2i} + \ldots + a_k x_{ki}$$

where A_i is the asset index for household i,

x_s are indicators or asset ownership and housing quality variables

a_s are the weights, obtained from the first PC, used to aggregate the indicators into an index.

Note: Filmer and Pritchet applied PCA to binary data. A more convenient technique would be to use MCA. Although there is an equivalence between PCA and MCA, but with different cardinal values.

Statistical Methods - Pros

- Simple to use and compute (even with binary data – MCA)
- Addresses multidimensionality.
- Allows to consider the joint distribution.
- Allows using ordinal and cardinal data.
- Allows identification and aggregation.

Statistical Methods - Cons

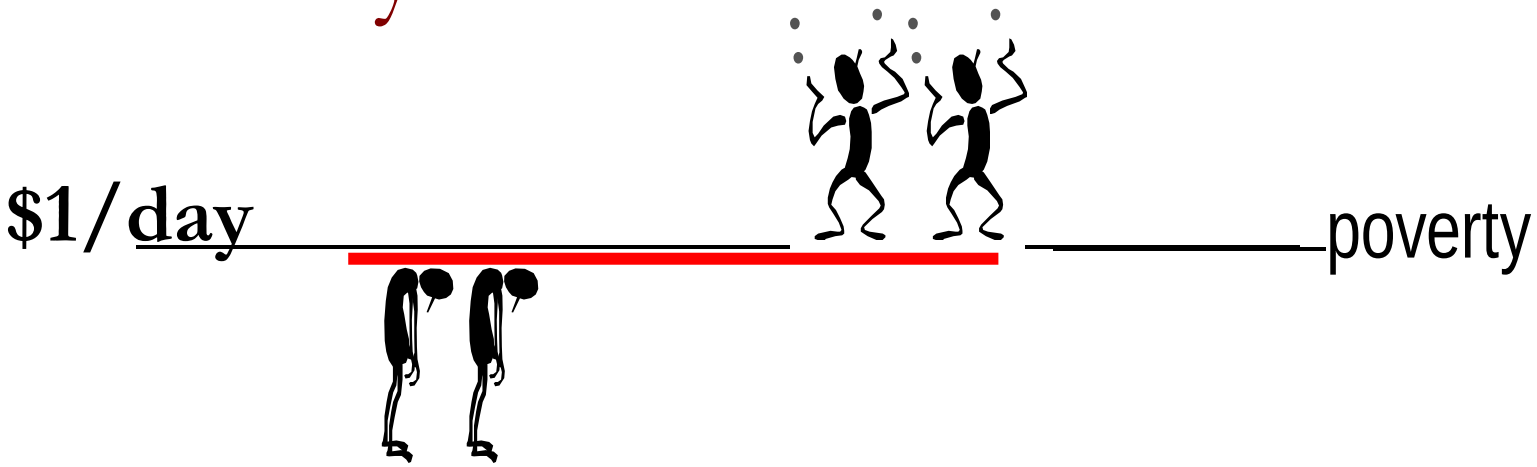
- Tradeoffs are determined by the correlation matrix, thus not explicit.
- Results are sensitive to the selected components or factors.
- Comparisons across different datasets (countries, time periods) requires pooled data.

4. Fuzzy Sets Approach

Motivation:

- The concept of a threshold that unambiguously dichotomizes the population into poor and non-poor, or deprived and non-deprived, assumes complete certainty about what is enough.

4. Fuzzy Sets – Motivation



Is living with $0.99/day really different from living with 1.01/day?

Is being -2.1 sd from the mean of height-for-age really different from being -1.9 sd?

In social investigation and measurement, it is undoubtedly more important to be vaguely right than to be precisely wrong. (Sen, 1991: 45)

4. Fuzzy Sets – Proposal

- Thus, the idea is to allow for fuzziness in the identification of the poor or deprived using the theory of fuzzy sets (Zadeh, 1965) used in computer science and mathematics.
- Key first papers Cerioli & Zani (1990), Cheli & Lemmi (1995), Chiapero-Martineti (1994, 1996, 2000)

4. Fuzzy Sets – Proposal

- Rather than identifying the "poor" and "non-poor"/ "deprived" and "non-deprived", the approach allows for varying "degrees of membership to the set of the poor or deprived.
- Thus, identification function

Traditional ID fc: ϱ: X x z $\rightarrow$ {0,1} (either 0 OR 1)

Fuzzy Sets' ID fc: ϱ: X x z $\rightarrow$ [0,1] (values btw 0 and 1)

4. Fuzzy Sets in steps (1)

1. Membership function $\mathbf{m}_j(x_{ij})$ to determine the **degree of membership to the group of deprived in each dimension.**

Note: Traditional *implicit* membership fc:

$\mathbf{m}_j(x_{ij})=1$ if $xij<zj$ (deprived)

$\mathbf{m}_j(x_{ij})=0$ if $xij\geq zj$ (non-deprived)

Example of a fuzzy set membership function:

$\mathbf{m}_j(x_{ij})=1$ if $xij=\min(x_{ij})$

$\mathbf{m}_j(x_{ij})=(\max(x_{ij})-x_{ij})/\max(xij)-\min(x_{ij}))$

$\mathbf{m}_j(x_{ij})=0$ if $xij\geq zj$ (non-deprived)

4. Fuzzy Sets in steps (2)

Note that the selection of the membership function $\mathbf{m}_j(x_{ij})$ is the key step.

2. Aggregating the dimensional 'degrees of membership' , or in other words, the probabilities of being deprived in each dimension, so as to determine **Mi**, the **degree of membership to the group of the poor**. Typically: a weighted mean:

$$M_i=(m_1(x_{i1})w_1+...+m_d(x_{id})w_d)/(w_1+...+w_d)$$

Note M_i (as m_i) also ranges from 0 to 1.

4. Fuzzy Sets in steps (3)

3. Aggregating the individual poverty levels (weighted deprivation probabilities, or degrees of membership) into an aggregate poverty level.

Example (an arithmetic mean):

$$P=M_1+...+M_n/n$$

4. Fuzzy Sets applications

- There is a set of interesting applications of the approach to different countries (Chiappero-Martineti (2000) to Italy, Lelli (2001) to Belgium, Deusch & Silber (2005) to Israel, Roche (2009) to Venezuela, Amarante et al (2010) to Uruguay, D'Ambrosio et al (2011) to different European countries, Bastos and Machado (2009) to Portugal.

4. Fuzzy Sets - Pros

- It places emphasis on the identification step, paradoxically overlooked in other approaches which aim to measure poverty.
- It also offers different aggregation methodologies.
- It offers a summary measure, a hierarchy among dimensions, explicit tradeoffs and a complete ranking.
- Some of the measures comply with desirable standard axioms of poverty measurement.
- It allows considering joint deprivation.
- It allows ordinality and cardinality of the variables.

4. Fuzzy Sets - Cons

- While the fuzziness at the identification step is attractive, it takes away the intuition of the 'counting approach' to identifying the poor (coming soon).

- The uncertainty with respect the divide between poor and non-poor/deprived and non-deprived can be addressed in other ways without foregoing intuition and transparency, via post-estimation robustness tests.

5. Axiomatic Approach

Motivation:

- To develop a poverty measure that complies with a number of desirable properties. That is, a poverty measure that **does not change** under certain transformations of people's achievements and that it **does change** in a particular direction under other transformations of people's achievements.

5. Axiomatic Approach

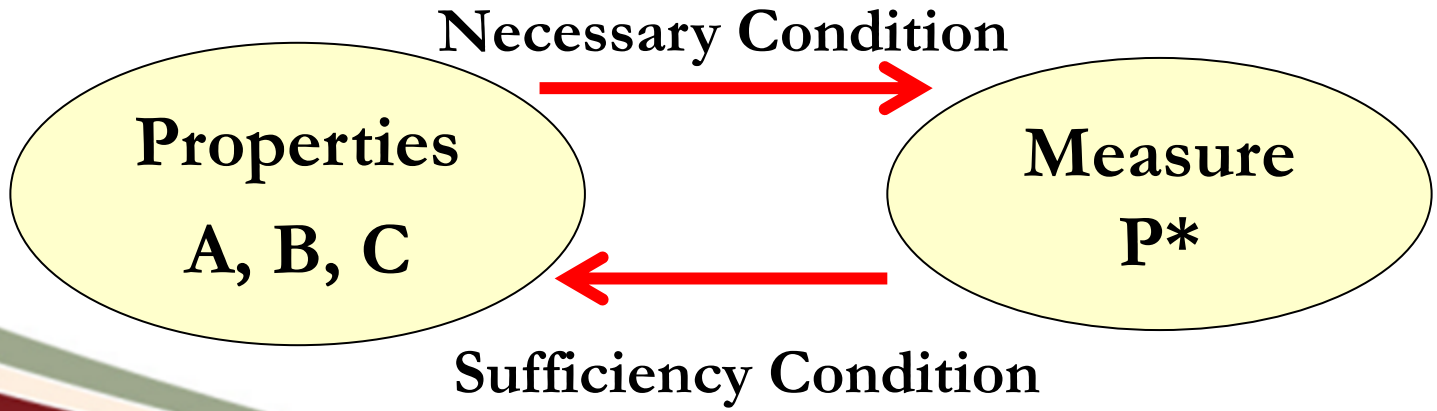
Origins:

- Unidimensional Poverty Measurement : Seminal paper: Sen, 1976, "Poverty: An ordinal approach to measurement". Introduced identification and aggregation, monotonicity and transfer axioms. (Precendent, Watts, 1969).
- Gave rise to a branch of the literature in poverty measurement. This was built upon unidimensional inequality measurement (also based on properties).
- Some key papers: Foster, Greer & Thorbecke (1984), Chakravarty (1983), Clark, Hemming and Ulph (1981), Atkinson (1987), among others.

5. Axiomatic Approach

Two possible procedures:

1. Introduce a number of principles considered desirable and then derive a measure satisfying these principles. This is called ***characterization*** in mathematics. Entails a *sufficiency condition:* the measure satisfies these principles, *and* a *necessity condition*: this is the *only* measure that satisfies the set of proposed principles.



5. Axiomatic Approach

Two possible procedures:

2. Introduce a number of properties that are considered desirable and then propose a measure or family of measures satisfying these properties, but without claiming it to be the *only* measure or family of measures to do so.

In any of the two the KEY question is…

which are the truly desirable and justifiable properties?

5. Axiomatic Approach

Extensions to the MD case:

- Some key papers: Chakravarty, Mukherjee and Ranade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Chakravarty and D'Ambrosio (2006), Alkire and Foster, (2007, 2011), Bossert, Chakravarty and D'Ambrosio (2009), Maasoumi & Lugo (2008).

5. Axiomatic Approach- MD case

A multidimensional poverty measure is defined as $P : X \times z \times \rho \rightarrow R$, where z is the vector of deprivation cutoffs for each dimensions and ρ identifies who is poor

5. Axiomatic Approach

Extensions to the MD case:

- Maasoumi and Lugo (2008) measures are also related to the so-called "**Information Theory Approach**" because of the aggregation formula they use: general means.
- General means are shown to **minimize the distance between two entropy functions** (where entropy is the sum of the information content of each event weighted by its probability). The two distributions under consideration can be associated to the 'ideal' non-poor distribution and the actual observed one.

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5. Axiomatic Approach- Identification

Use of a counting approach to identification

- A particular method of identifying the poor by "counting the number of dimensions in which people suffer deprivation, (…) people have scores corresponding to the number of dimensions on which they fall below the threshold" (Atkinson, 2003)
- Three possible criterions within the approach: union, intersection, intermediate. **KEY decision for addressing joint deprivations**.
- Most measures use a union approach. AF method allows for union, intersection or intermediate, with an emphasis on the relevance of the middle-ground.

5. Axiomatic Approach- Aggregation

- Most measures are extensions of unidimensional poverty measures, FGT (most prominently) or Watts, or Chakravarty (1983). But some also use some other formulas such as the general means.

5. Axiomatic Approach- Requirements

- Same datasource (to address joint deprivation).
- Most of the measures require cardinality of the variables used.
- Only Alkire and Foster (2011), Chakravarty and D'Ambrosio (2006) and Bossert, Chakravarty, and D'Ambrosio (2009) allow for meaningful treatment of ordinal variables.

5. Axiomatic Approach- Examples

Bourguignon & Chakravarty (2003)

$$P_{BC1}(X;Z)=\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{d}w_j g_{ij}^{\alpha_j}\ ;\text{with } \alpha_j \geq 1. \qquad P_{BC2}(X;Z)=\frac{1}{n}\sum_{i=1}^{n}\left[\sum_{j=1}^{d}w_j g_{ij}^{\theta}\right]^{\alpha/\theta};$$

Chakravarty & D'Ambrosio (2006)

$$P_{CD1}(X;Z)=\frac{1}{n}\sum_{i=1}^{n}\left[\sum_{j=1}^{d}w_j g_{ij}^{0}\right]^{\beta}$$

Bossert, Chakravarty & D'Ambrosio (2009)

$$P_{BCD}(X;Z)=\left(\frac{1}{n}\sum_{i=1}^{n}\left[\sum_{j=1}^{d}w_j g_{ij}^{0}\right]^{\beta}\right)^{1/\beta}$$

Alkire & Foster (2007, 2011)

$$M_{\alpha}(X,z)=\frac{1}{nd}\sum_{i=1}^{n}\sum_{j=1}^{d}w_j g_{ij}^{\alpha}(k) \text{ with } \alpha \geq 0$$

5. Axiomatic Approach- Pros

- Clarity and transparency about the behavior of the poverty measure. Ease of analysis.
- Summary measure
- Complete ranking, clear hierarchy and explicit tradeoffs.
- Address the joint distribution allowing to focus on the jointly deprived.

5. Axiomatic Approach- Note

- Axiomatic measures can be complemented and benefit from the other approaches at different stages (when inspecting the data, when selecting parameters and indicators, when performing robustness tests).

This course...

The focus of this course is the Alkire & Foster (2007, 2011) methodology, which belongs to the axiomatic approach. But we will link to many of the other methods (esp. dominance – robustness, statistical and counting) which can complement in various important ways the AF method.

6. Counting Approach

- Refers to a particular method for identifying the poor.

Motivation:

- Up to the ’70s the prevailing approach to measuring poverty was the poverty line approach, or income method:
- The poor were those below the poverty line, which represented the “minimum necessaries for the maintenance of merely physical efficiency” (nutritional requirements) in monetary terms + minimum sums for clothing, fuel and hh sundries according to family size (Townsend, 1954).

6. Counting Approach

Motivation:

- "Human beings have basic needs: food, shelter, clothing, health, education. (…) We are still in a stage where the most important concern of development is the level of satisfaction of basic needs for the poorest sections in each society(…)" (Cocoyoc Declaration, UNEP-UNCTAD, 1974). Endorsed by other institutions over the decade.

6. Counting Approach

Motivation:

- The Basic Needs Approach drew attention to the importance of looking at the *actual* satisfaction of basic needs (or at least access to key commodities)
- Thus, it fostered the so-called direct method to measure poverty (Sen, 1981).

6. Counting Approach

Motivation:

- Direct Method to measure poverty:

  A *list* of needs considered to be basic alongside minimum levels of satisfaction (cutoffs) would be specified. In such context is that *counting* the number of deprivations naturally emerged as a method to identifying the poor.

6. Counting Approach in Steps

1. Defining a list of relevant indicators
2. Assigning a weight to each considered indicator
3. Defining a threshold of satisfaction (*deprivation cutoff*) for each indicator
4. Creating binary deprivation scores for each person in each indicator: "1" =deprived, "0"= non-deprived
5. Producing a deprivation score by taking a weighted sum (or average) of deprivations.
6. Setting a threshold score of poverty (or *poverty cutoff score*) such that if the person has a deprivation score at or above the threshold, she is considered poor.

6. Counting Approach

Note:

1. Not all implementations of the counting approach are explicitly linked to the basic needs approach, but one can find an implicit link in most of them
2. The capability approach framework (Sen) builds in part upon the BN approach but offers more systematic, consistent and solid philosophical grounds (for example, it completely moves away from resources to focus on *functionings*). A counting approach is also a natural procedure to implement the CA approach.

6. Counting Approach
Salient Implementations

Europe:

- Townsend (1979): "Poverty in the UK".
- Defined 60 indicators covering 12 dimensions (diet, clothing, fuel and light, home amenities, housing conditions and facilities, the immediate environment of the home, conditions at work, family support, recreation, education, health and social relations).
- Equal weights to all indicators, although the number of indicators within each dimension varied greatly.
- For 'illustrative purposes', he then focused on a shorter list of 12 items covering major aspects of dietary, household, familial, recreational and social deprivation

6. Counting Approach
Salient Implementations

Townsend (1979):

- For 'illustrative purposes', he then focused on a shorter list of 12 items covering major aspects of dietary, household, familial, recreational and social deprivation.
- Discards union criterion because: "No single item by itself, or pair of items by themselves, can be regarded of symptomatic of general deprivation. People are idiosyncratic and will indulge in certain luxuries and apply certain prohibitions for religious, moral, educational or other reasons, whether they are rich or poor"

6. Counting App. Implementations

Townsend (1979):

- However, he did not use this counting approach to identify and count the poor. Rather, he explored the **correlation between censored deprivation scores and household income** (adjusted for household size) in order to **derive an income threshold below which people are "disproportionately deprived"** (p. 255).
- In other words, he used a *direct approach* to 'validate' the poverty line to be used in the *income approach* to poverty measurement.
- **Yet his work inspired a prominent body of subsequent work within and outside Europe.**

6. Counting App. Implementations

Mack and Lanslay (1985): "Poor Britain"

2 innovations:

1) 'socially perceived' necessities

- the list of items considered as necessities was constructed using a survey (1983 Breadline Britain), for the first time ever, about the public's perceptions of minimum needs.
- Method referred to as the **'consensual or perceived deprivation approach to measuring poverty'**.
- Of the original 35 considered items, they retained the 26 items that were considered to be a necessity by strictly more than 50% of the population (majority rule).

6. Counting App. Implementations

Mack and Lanslay (1985): "Poor Britain"

2 innovations:

2) Enforced lack

- It discriminated between people who did not have an item because they could not afford it, from those for whom it was a voluntary choice
- Who were the poor?
- Those who could not afford three or more items from a list of 22, each equally weighted (poverty cutoff selected after crossing the *enforced* lack of necessities with income levels and spending patterns).

6. Counting App. Implementations

Europe

- Callan, Nolan and Whelan (1993) proposed to identify the poor combining both resource *and* deprivation measures following Ringen (1987, 1988) in this respect.
- The authors grouped the 24 initial variables into 3 dimensions via factor analysis. (1) basic lifestyle (consisting of eight items such as food and clothes), (2) housing and durables (consisting of seven items related to housing quality and facilities) and (3) 'other' aspects of lifestyle (consisting of nine items such as social participation and leisure activities, having a car or telephone).

6. Counting App. Implementations

Europe

- These were then evaluated in terms of the proportion of people who regarded each item as a necessity,a nd thus they restricted their material deprivation index to the 8 items of the basic lifestyle dimension.
- They identified as poor, people who both lacked at least one of the eight basic items *and* fell below the relative income poverty line (60% of the average equivalent disposable income in the sample).
- **Consistent poverty approach** thus requires deprivation both in standard of living, measured by different deprivation indicators, and resources, measured by an income poverty threshold.

6. Counting App. Implementations

Europe & beyond

- There is a wide range of applications of the consensual approach, as well as the consistent approach to poverty measurement (both using counting), with variations:
- Gordon et al. (2000) for Britain, Muffels et al (1992), The Netherlands (used subjective weights for the indicators), Hallerod (1995) for Sweden, Eurostat (2002), among others.
- Outside Europe: Mayer and Jencks (1989) and Bauman (1998, 1999) in the US.

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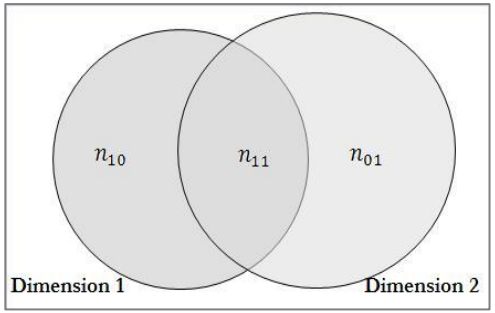
6. Counting App. Implementations

Europe & beyond – implementations of the consensual approach, and the consistent approach

- **UK:** Callan, Nolan and Whelan (1993) Callan et al. (1999), Whelan *et al.* (2001), Layte *et al.* (2000), Gorden *et al* 2000, Whelan *et al.* (2001), Whelan, Nolan, Maitre (2006),
- **Netherlands** Muffels *et al.* (1992) index of relative deprivation
- **Sweden:** Halleröd (1995) Proportional Deprivation Index (PDI). Halleröd et al (2006) applied PDI to Britain, Finland and Sweden.
- **Europe**: Layte et al (2001), Guio 2005, Guio and Maquet 2006, Guio 2009, Decanq et al 2013)
- **Outside Europe:** Mayer and Jencks (1989) and Bauman (1998, 1999) in the US.

6. Counting App. Implementations

- Many of these studies do:
- Cross-tabs between material deprivation and income deprivation.



- Often, they select the cutoff for poverty in material deprivation so that it 'matches' the income poverty headcount, and then explore the two groups overlap.
- Findings indicate that the degree of overlapping is reduced.

6. Counting App. Implementations

- **Unsatisfied Basic Needs** (UBN) Approach – Prominent in Latin America
- Seminal work in Chile (Kast and Molina, 1975) and Argentina by INDEC (1984), together with CEPAL.
- Used five **census** indicators:
  - Overcrowding (3+ people/room)
  - Housing
  - Sanitation
  - Children attending primary school
  - HH: (head has 2 years or less education) or high dependency (4+/worker).

6. Counting App. Implementations

- **Unsatisfied Basic Needs**
- Reported as **official statistics** in: Bolivia, Chile, Colombia, Ecuador, Guatemala, Honduras, Nicaragua, Paraguay, Peru, Uruguay, Venezuela.
- **Identification:** Reported headcount ratio with 1+, 2+ and 3+ deprivations
- Powerful in terms of **policy**: used for policy mapping at very disaggregated levels (Census information)

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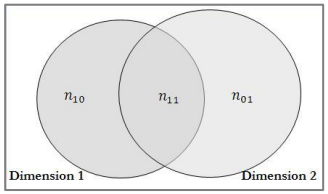
6. Counting App. Implementations

- **Further Developments:**
- 'Integrated method' identified hh that were income poor AND had Unsatisfied basic needs, or experienced only income poverty/ubn



- Non-counting proposal to cardinalize ordinal data and combine it with income poverty (Boltvinik)
- Targeting (NGO programmes – Kudumbashree)
- 'Graduation' from programmes (BRAC)
- Poverty Scorecards (NGOs, Schreiner)

6. Counting Approaches - Pros

- Clarity, Simplicity, Transparency, Intuition for identifying the multiply deprived.
- Allow to look at joint deprivations.
- When combined with an aggregation measure (which is usually the case), they offer a summary measure, a complete ranking.
- Tradeoffs explicit and clear.
- Allow for both cardinal and ordinal variables.

6. Counting Approaches - Cons

- Relies on the particular selection of indicators (appropriateness for the particular purpose)
- Relies on the weights assigned to the dimensions.
- Relies on dichotomies (deprived/non-deprived) so no sensitivity to the depth for identification.
- Sometimes a counting approach to identify the poor is combined with aggregations methodologies that take the intuition away, sometimes assigning cardinal meaning to ordinal values.