Multidimensional Poverty: How much substitution is allowed?

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Based on a paper with E. Maasoumi
Measuring poverty in 3 steps

1. Choose a well-being indicator
   \[ Y_i \]  \text{ with distribution } F(Y_i) \]

2. Choose poverty line
   \[ z \quad \text{poor;} \quad \text{non-poor} \]

3. Aggregate individuals
   \[ P(Y_i ; 0) \]
Monetary approach to poverty

1. **Well-being indicator**: consumption or income

\[ Y_i = \sum_{j=1}^{J} p_j x_{ij} \]

individual \( i = 1, 2, \ldots, n \)
good \( j = 1, 2, \ldots, J \)
prices \( p_1, p_2, \ldots, p_J \)

2. **Poverty line**: value of “basic goods”

\[ z = \sum_{j=1}^{J_0} p_j z_{j0} \]

where \( z_{j0} \) is the minimum quantity of

good \( j_0 \in J_0 \) and \( J_0 \subseteq J \)

3. **Poverty measure**:

\[ P(Y; z) = FGT_\alpha = \frac{1}{n} \sum_{i=1}^{n} \left( \max \left( \frac{z - y_i}{z}; 0 \right) \right)^\alpha \]

where each poor contributes to total poverty
according to his/her relative shortfall from threshold \( \left( \frac{z - y_i}{z} \right)^\alpha \)
Objections

1. There are no markets (and hence prices) for all relevant ‘goods’ in $J_0$

2. Even if markets exist, prices in the market are not necessarily the best weights (Tsui, 2001)

3. “Essentiality”. Ass. perfect substitutability between attributes
   Unless … each attribute is seen as “essential” so that distance to $z_j$ still matters
Poverty Approaches

Perfect Substitution (monetary)
Poverty Approaches

$\text{Essentiality} \ (\text{Union})$

$Z_h$

$Z_e$

health

education
Poverty Approaches
Framework for poverty measure

- Perfect Substitution (monetary)
- Intermediate
- Union

Diagram showing health and education with references to $Z_h$, $Z_e$. 
B. Non-monetary approach

1. **Well-being indicator:**
   (from IT)
   \[ s_{ij} = f(x_{ij}) \]

2. **Poverty line:**
   I. Aggregate Poverty Line \( z(s) \) … *shortfall of well-being* (Tsui 2002)
   II. Composite Poverty Line \( z_j \) … ‘*well-being*’ of *shortfalls* (BC 2003)

3. **Poverty measure:** \( FGT \)
B. Non-monetary approach

\[ s_{ij} = f(x_{ij}) \] will depend on two factors:

1. Poverty line (aggregate or composite)
2. Poverty focus

- **Strong**: if a person is poor in attribute \( j \) but rich in \( q \), receiving more of \( q \) does not affect the level of poverty
  
  \[ \Rightarrow \text{Union} \text{ a person is poor if he falls short of at least one attribute (no substitution between attributes above and below PL)} \]

- **Weak**: allows for substitution between attributes above and below poverty line.

<table>
<thead>
<tr>
<th></th>
<th>Aggregate Poverty line</th>
<th>Composite Poverty Line</th>
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<tbody>
<tr>
<td><strong>Strong</strong></td>
<td>[ s_{ij} = \min {x_{ij}, z_j} ] (I)</td>
<td>[ s_{ij} = \frac{(z_j - x_{ij})}{z_j} ] (II)</td>
</tr>
<tr>
<td><strong>Weak</strong></td>
<td>[ s_{ij} = x_{ij} ] (III)</td>
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</tbody>
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B. Non-monetary approach

\[
p_i = \max \left( \frac{S_z - S_i}{S_z}; 0 \right) = \max \left( 1 - \frac{\left( \sum_{j=1}^{m} w_j x_{ij}^\beta \right)^{1/\beta}}{\left( \sum_{j=1}^{m} w_j z_j^\beta \right)^{1/\beta}}; 0 \right)
\]

Aggregate PL shortfall in well-being

\[
p_i = \left( \sum_{j=1}^{m} w_j \max \left( 1 - \frac{x_{ij}}{z_j}; 0 \right) \right)^{1/\beta}
\]

Component PL 'well-being' of shortfall
B. Non-monetary approach

\[
\begin{align*}
(\text{I}) - (\text{III}) & \quad \text{Aggregate PL shortfall in well-being} \\
(\text{II}) & \quad \text{Component PL 'well-being' of shortfall}
\end{align*}
\]

\[
p_i = \max \left( \frac{S_i - S_z}{S_z} ; 0 \right) = \max \left( 1 - \left( \frac{\sum_{j=1}^{m} w_j x_{ij}^\beta}{\sum_{j=1}^{m} w_j z_j^\beta} \right)^{1/\beta} ; 0 \right)
\]

\[
p_i = \left( \sum_{j=1}^{m} w_j \max \left( 1 - \frac{x_{ij}}{z_j} ; 0 \right) \right)^{1/\beta}
\]
Properties

- **Continuity**: to avoid jumps in the function – technical requirement
- **Symmetry** w.r.t. individuals: the identity of the individuals is not relevant for the evaluation of poverty (characteristics outside X do not matter)
- **Replication Invariance**: the size of the population does not affect the measurement. Necessary for cross-population comparisons
- **Monotonicity**: poverty index does not increase when the condition of the poor improve
- **Subgroup consistency**: if poverty decreases for a subgroup, total poverty cannot increase / **Subgroup Decomposability**
- **Scale Invariance**: poverty index is not sensitive to the unit of measurement
- **Poverty Criteria Invariance**: no dramatic change in the evaluation of poverty for changes in the poverty line z not affecting the number of the poor
- **Poverty Non-increasing Minimal Transfer**: Variation of Pigou-Dalton principle, among poor
- **Poverty-Nondecreasing Rearrangement**: more correlation, higher poverty, lower welfare.

- Poverty focus (strong or weak)