The Inequality Adjusted HDI

Further insights

Maria Emma Santos
Session aims at answering these questions

1. Why an IHDI? What does it add to the *old* HDI and to the *new* HDI?
2. Why does it use the Atkinson Inequality Measure and not some other inequality measure?
3. What’s the intuition & interpretation of the IHDI?
4. How is it policy-relevant?
Why an Inequality-Adjusted HDI?

When measuring HD there are two types of inequality that may matter:

• Inequality *within* dimensions
• Inequality *between* dimensions

We will look at them sequentially.
Why an Inequality-Adjusted HDI?

• “Human development is the expansion of people’s freedoms to live long, healthy and creative lives; to advance other goals they have reason to value; and to engage actively in shaping development equitably and sustainably on a shared planet” (HDR, 2010, emphasis added).
Why an Inequality-Adjusted HDI?

• Yet, the old Human Development Index measured average attainments in three indicators:
  – income
  – life expectancy
  – education (using literacy and gross enrolment ratios).
Why an Inequality-Adjusted HDI?

Example:

• Mexico’s GDP per capita in 2000 was PPP $ 9,023
• Does it mean that every Mexican enjoyed PPP $ 9,023?
• Obviously not! In fact in 2002:
• 15.5% of people in Mexico live with less than US$1.25/day (now it is 4%) and 37.7 with less than US$2/day (now 8.2%)
• The richest 10% earned 32 times the poorest 10% (now 21 times)
• The Gini Coefficient was 53% (now 47%)

Source: HDR, 2002 & 2010
Why an Inequality-Adjusted HDI?

Example ‘cntd

• Mexico’s life expectancy in 2002 was 72 years (now it is 76).
• Does it mean that every Mexican could expect to live 72 years in 2002?
• Obviously not! In fact:
• In the Distrito Federal life expectancy was 77.2 years whereas in Chiapas it was 72.4 years.

Source: HDR Mexico, 2002
Why an Inequality-Adjusted HDI?

Example ‘cntd

• Mexico’s adult literacy rate was 91.4% in 2002 (now it is 92.8%)

• Does it mean that no matter where you are born you had a 91% chance of being literate?

• Obviously not! In fact:

• In the Distrito Federal literacy rate was 97% whereas in Chiapas it was 77%.

• Mexico’s combined Gross Enrolment ratio was 71%, in DF it is 75.3%; in Chiapas 63.28%

Source: HDR Mexico, 2002
Why an Inequality-Adjusted HDI?

• Thus, as we know, averages only tell us what the achievement would be for each person in a country if there was a perfectly equal distribution of achievements.

• A “toy example”. Imagine these income or years of education distributions:

• Distribution A=(5,5,5) vs. Distribution B=(2,5,8) Both have the same average or mean achievement=5, but which one do you prefer?
Why an Inequality-Adjusted HDI?

• So each component of the HDI was an average, a ‘mean’.
• Thus it completely ignores the actual distribution of each outcome across people...
• That is inequality within dimensions.
Why an Inequality-Adjusted HDI?

• However, the commitment of Human Development is the expansion of people’s freedoms… not the expansion of certain people’s freedoms, but all of them.

• Thus inequality matters to Human Development. *Both average achievements and the distribution* of those achievements are fundamental aspects of human development.
Why an Inequality-Adjusted HDI?

• Thus, the IHDI captures inequality within (ie. in each) dimensions by using the Atkinson measure of inequality.
Why does it use Atkinson’s Inequality Measure?

• Three main reasons:
  • It satisfies four basic properties that an inequality measure should have together with one particular property that other measures such as the Gini coefficient do not satisfy: **subgroup consistency**.
  • It has an intuitive and meaningful **interpretation**.
  • It has a neat **connection with the general means**, such as the Geo. Mean which is used to penalise inequality **between** dimensions.
Atkinson’s Measures of Inequality

• Core Concept:

**Equally Distributed Equivalent (EDE):** Given the distribution of an achievement (for example income), the equally distributed equivalent achievement is the level of achievement which if assigned to all individuals, would produce the same social welfare than the observed distribution.
Atkinson’s EDE Graphically

- **OJ**: Total given income
- **JK**: Set of all possible distributions of OJ.
- **I₁, I₂, I₃**: Social Welfare Levels
- **A**: Actual Distribution (1: OF, 2: AF)
- **CE**: Mean Income
- **BD**: Equally Distributed Equivalent Income
Atkinson’s Inequality Measure

\[ A_x = 1 - \frac{x_{EDE}}{\mu_x} \]

- If there is perfect equality in the distribution of achievement \( x \), then \[ x_{EDE} = \mu \rightarrow A_x = 0 \]

- If there is at least some inequality in the distribution of achievement \( x \), then \[ x_{EDE} < \mu \rightarrow A_x > 0 \]
Atkinson’s Inequality Measure

\[ A_x = 1 - \frac{x_{EDE}}{\mu_x} \]

• Interpretation: It is the share of per capita achievement wasted as a result of inequalities in the distribution of achievement.
Atkinson’s Inequality Measure

• What’s the formula for $x_{EDE}$?
• Atkinson presents a parametric family, so there are several possible formulas depending on the aversion to inequality one wants to use.
• HDR 2010 used one particular case which is... The geometric mean!

$$x_{EDE} = g_x = (x_1)^{1/n} \ast (x_2)^{1/n} \ast ... \ast (x_n)^{1/n}$$
Atkinson’s Inequality Measure

\[ A_x = 1 - \frac{g_x}{\mu_x} \]

- This measure of inequality (as well as others of its family) satisfy 4 basic properties any inequality measure should satisfy:

1. The amount of inequality does not depend on the population size (Population Invariance).
2. That the amount of inequality does not depend on who has each achievement (Symmetry or Anonymity.)
Atkinson’s Inequality Measure

\[ A_x = 1 - \frac{g_x}{\mu_x} \]

• This measure of inequality (as well as others of its family) satisfy 4 basic properties any inequality measure should satisfy:

3. The amount of inequality does not depend on the total achievement (Scale invariance)

4. If there is a regressive transfer, inequality increases. (Pigou-Dalton Principle)
Atkinson’s Inequality Measure

\[ A_x = 1 - \frac{g_x}{\mu_x} \]

- This measure of inequality (as well as others of its family) satisfy 4 basic properties any inequality measure should satisfy.

- Many other inequality measures do not satisfy one of the properties. Examples:
  - The ratio of richest 10%/poorest 10%
  - The variance, the range, the variance of logs.
Atkinson’s Inequality Measure

• Additionally, Atkinson’s measure satisfies a policy-relevant property...

1. Imagine you have two regions in a country: North and South.

2. Suppose inequality in the South decreases whereas inequality in the North stays the same.

3. What would you like your overall measure of inequality (the one computed considering both regions, North and South) to do?
Atkinson’s Inequality Measure

• That’s what subgroup consistency requires! That if inequality in one population subgroup decreases (increases), and inequality in the other population subgroup remains unchanged, overall inequality should decrease (increase).

• The Gini coefficient does not satisfy this property. Why? Because it is rank-based.
So... Here is IHDI

\[ IHDI = \left[ H(1-A_H) \times E(1-A_{Edu}) \times I_L(1-A_{INL}) \right]^{1/3} \]

- Each dimension-index is adjusted by the loss due to inequality.
## IHDH

- **Example**

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Mozam.</td>
<td>0.45</td>
<td>45.7</td>
<td>0.20</td>
<td>28.2</td>
<td>0.25</td>
<td>58.1</td>
<td>0.244</td>
<td>0.144</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.90</td>
<td>12.3</td>
<td>0.69</td>
<td>17.9</td>
<td>0.68</td>
<td>31.6</td>
<td>0.787</td>
<td>0.564</td>
</tr>
</tbody>
</table>

$H \text{ In Adj} = 0.45 \times (1 - 0.457)$
IHDl

• ‘Overall’ Loss can be approximated as

\[
\frac{A_H + A_{edu} + A_{INL}}{3}
\]

• Contribution of each loss:

\[
\frac{A_H}{A_H + A_{edu} + A_{INL}}
\]
IHDII

Example

<table>
<thead>
<tr>
<th></th>
<th>$A_H$ % Loss in $H$</th>
<th>$A_E$ % Loss in Educ.</th>
<th>$A_I$ % Loss in Income</th>
<th>Overall Loss</th>
<th>Contrib Loss in Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mozam.</td>
<td>45.7</td>
<td>28.2</td>
<td>58.1</td>
<td>44</td>
<td>44%</td>
</tr>
<tr>
<td>Mexico</td>
<td>12.3</td>
<td>17.9</td>
<td>31.6</td>
<td>20.6</td>
<td>51%</td>
</tr>
</tbody>
</table>
Why an Inequality-Adjusted HDI?

• In the OLD HDI the three ‘averages’ were averaged:

\[
\text{HDI} = \frac{1}{3} [H_1 + E_2 + I_3]
\]

• That’s why it is referred as a ‘mean of means’

• What does the averaging across dimensions mean?

• It means that a low achievement in one dimension can be compensated by a high achievement in some other dimension.
Why an Inequality-Adjusted HDI?

• Example of ‘compensations’ across dimensions:

<table>
<thead>
<tr>
<th>Country</th>
<th>HDI (old)</th>
<th>Life Exp. Index</th>
<th>Educ. Index</th>
<th>Income Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>South Africa</td>
<td>0.683</td>
<td>0.442</td>
<td>0.843</td>
<td>0.765</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(51.5 years)</td>
<td>(88% lit; 76.8% GER)</td>
<td>(PPP 9,757)</td>
</tr>
<tr>
<td>Morocco</td>
<td>0.654</td>
<td>0.767</td>
<td>0.574</td>
<td>0.620</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(71 years)</td>
<td>(55.6% lit; 61% GER)</td>
<td>(PPP 4108)</td>
</tr>
</tbody>
</table>

• South Africa ~ 49.2 mill. people
• Morrocco ~ 31.2 mill. people
Why an Inequality-Adjusted HDI?

• “Human development is the expansion of people’s freedoms to live long, healthy and creative lives…”

• It does not say: Human development is the expansion of some of people’s freedoms to live long or healthy or creative lives
Why an Inequality-Adjusted HDI?

• Each dimension of development is intrinsically important and different. Thus, they are not substitutes, or at least they are ‘imperfectly’ substitutable.

• Although income is the one dimension in the HDI that is merely a means, it is acting as a surrogate for other intrinsically important dimensions.
Why an Inequality-Adjusted HDI?

- Thus, they decided to penalize for inequality between dimensions in the HDI: “Poor performance in any direction is now directly reflected in the HDI, and there is no longer perfect substitutability across dimensions. This method captures how well rounded a country is across the three dimensions” (HDR 2010, p. 15)
- That’s why they now use the geometric mean to aggregate the 3 indices in the new HDI, and this is also used in the IHDI.
The New Aggregation Formula for the HDI

\[ \text{HDI} = [H_1 * E_2 * I_3]^{(1/3)} \]

• (Using the new indicators in both cases):

\[ \text{HDI}^* \leq \text{HDI} \]

• If there is no inequality between dimensions, then HDI* = HDI

• If there are uneven achievements across dimensions, then HDI* < HDI. The greater the inequality, the lower HDI* w.r.t. HDI
The New Aggregation Formula for the HDI

• Example

<table>
<thead>
<tr>
<th>Country</th>
<th>Life Exp. Index</th>
<th>Educ. Index</th>
<th>Income Index</th>
<th>HDI*</th>
<th>HDI Arith. Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gabon</td>
<td>0.65</td>
<td>0.62</td>
<td>0.67</td>
<td>0.649</td>
<td>0.649</td>
</tr>
<tr>
<td>Mozambique</td>
<td>0.45</td>
<td>0.20</td>
<td>0.25</td>
<td>0.284</td>
<td>0.301</td>
</tr>
</tbody>
</table>
What does the IHDI add to the *old* HDI and to the *new* HDI?

- To the OLD HDI, it adds sensitivity to *both* inequality within and between dimensions.

- To the NEW HDI, it adds sensitivity to inequality *within* dimensions.
So...again... Here is IHDI

\[ \text{IHDI} = \left[ H(1-A_H) \times E(1-A_{Edu}) \times I_L(1-A_{INL}) \right]^{1/3} \]

- Each dimension-index is adjusted by the loss due to inequality \textit{within} dimensions.
- The overall index is penalized for inequality \textit{between} dimensions.
IHDİ

• The ‘overall’ loss can be obtained more exactly as:

Overall % Loss = (HDI - IHDİ)/HDI
What these countries could potentially have in HD if there was no inequality within dimensions.

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<td>0.107</td>
<td>0.155</td>
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<td>Mexico</td>
<td>0.90</td>
<td>0.69</td>
<td>0.68</td>
<td>0.751</td>
<td>0.787</td>
<td>0.564</td>
<td>0.469</td>
<td>0.593</td>
</tr>
</tbody>
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Overall loss in Mozambique = 45.3%
In Mexico = 21%
Let’s look at some results...

- Average loss in HDI = 22%
- It ranges from 6% (Czech Republic) to 45% (Mozambique)
- More than 80% of countries lose more than 10% and almost 40% lose more than 25%.
- Countries with less human development have more multidimensional inequality, thus larger loses in HD.
Largest and Smallest loses among HDI groups.
Let’s look at some results...

• Regional patterns:
  – Sub-Saharan Africa has the greatest loss, followed by South Asia and the Arab States.
  – South Asia: high inequality in health and education.
  – Arab States: mostly in education.

• In more than 1/3 countries inequality in health, education or both exceeds that in income.
Loss in the HDI and its components due to inequality, by region

- Living standards
- Education
- Health

Example:
- Arab States: 19% Living standards, 57% Education, 24% Health
- East Asia and the Pacific: 43% Living standards, 33% Education, 24% Health
Is the IHDI policy relevant? How?

• Yes! It draws the attention to *distribution* of outcomes within and between dimensions.

• Thus it will favour policies aimed at not just increasing ‘average’ achievement but rather guaranteeing that less advantaged sectors can also achieve such outcomes.

• Please feedback on your own country experiences that have failed and succeeded in promoting equality of achievements.
Clarification on the IHDI formula

• Last week, the following formulation was used (also appears in the technical note of the HDR 2010).

\[
\text{IHDI} = (\text{IHDI}^*/\text{HDI}^*) \times \text{HDI}
\]

• Now \( \text{IHDI}^* = [I_{NL}(1-A_{INL}) \times E(1-A_{Edu}) \times H(1-A_{H})]^{1/3} \)
  – Where \( I_{NL} \) is the normalized non-logged GNI index

• And \( \text{HDI}^* = [I_{NL} \times E \times H]^{1/3} \)
• So, \( \text{IHDI}^*/\text{HDI}^* = [(1-A_{INL}) \times (1-A_{Edu}) \times (1-A_{H})]^{1/3} \)
Clarification on the IHDI formula

• However, $\text{HDI} = [I_L \times E \times H]^{1/3}$
  – Where $I_L$ is the normalized logged GNI index
• Thus,

\[
\text{IHDI} = \left[ (1-A_{\text{INL}}) \times (1-A_{\text{Edu}}) \times (1-A_H) \right]^{1/3} \times [I_L \times E \times H]^{1/3}
\]

\[
= [I_L(1-A_{\text{INL}}) \times E(1-A_{\text{Edu}}) \times H(1-A_H)]^{1/3}
\]

*Note that $I_L$ is used instead of $I_{NL}$ for the IHDI calculation*
Deriving Atkinson’s EDE

• Assuming and additive welfare:

$$W(x) = \frac{1}{n} \sum_{i=1}^{n} u(x_i)$$

• And this utility function (Constant Relative Risk Aversion)

$$u(x_i) = \begin{cases} 
A + B \frac{x_i^\beta}{\beta} & \beta < 1, \beta \neq 0 \\
\ln x_i & \beta = 0
\end{cases}$$
Deriving Atkinson’s EDE

• The income level which, if assigned to all individuals produces the same social welfare than the observed distribution

\[
A + B \frac{x_{EDE}^\beta}{\beta} = \frac{1}{n} \sum_{i=1}^{n} \left( A + B \frac{x_i^\beta}{\beta} \right) \quad \beta \leq 1; \beta \neq 0
\]

\[
\ln x_{EDE} = \frac{1}{n} \sum_{i=1}^{n} \ln x_i = \ln \left( \prod x_i^{1/n} \right) \quad \beta = 0
\]
Deriving Atkinson’s EDE

• Thus, Atkinson’s EDE is given by

\[
x_{EDE} = \begin{cases} 
\left[ \frac{1}{n} \sum_{i=1}^{n} x_i^\beta \right]^{1/\beta} & \beta \leq 1, \beta \neq 0 \\
\prod_{i=1}^{n} x_i^{1/n} & \beta = 0 
\end{cases}
\]

• These are also known as the General Means of Order Beta (in this case for \(\beta \leq 1\))