Inequality-adjusted HDI: more advanced

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We start from a general "generalized mean of generalized means" 

\[
M(\sigma, \delta) = \left[ \frac{1}{D} \sum_{d=1}^{D} \left( \frac{1}{N} \sum_{n=1}^{N} x_{nd}^\sigma \right)^{\frac{1}{\sigma}} \right]^{\frac{1}{\delta}}
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Now if \( \sigma = \delta = \epsilon \):

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FLS(\epsilon) = \left[ \frac{1}{ND} \sum_{d=1}^{D} \sum_{n=1}^{N} x_{nd}^{\epsilon} \right]^{\frac{1}{\epsilon}} \forall \epsilon \neq 0
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An illustration of the FLS measures
Data issues

- When information from some dimension is aggregated at an intermediate level between the individual and the country (e.g. municipalities) the IHDI, and its associated loss function, need to be considered as upper and lower bounds, respectively.
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- With the Atkinson measures, for each dimension, it turns out that if the imputed value is above (below) the geometric mean of the unaffected observations then the total geometric mean will also be above (below) the restricted geometric mean.
Illustration of the imputation problem
Inequality-adjusted HDI: more advanced

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- Stochastic dominance for the FLS and IHDI

Stochastic dominance for the FLS family, including the IHDI

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▶ Say we want to adjust the HDI for inequality using other inequality aversion parameters. Could that affect the robustness of our country rankings? In principle, yes.
▶ Say we want to tinker with weights for the aggregation across dimensions. That could also affect country rankings!
▶ Are there conditions under which a country is better-off in terms of inequality-adjusted human development regardless of the aforementioned choices?
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First order: (sufficient but not necessary)

\[ \Delta FLS(\epsilon, w) \geq 0 \iff \Delta F(x_i) \leq 0 \forall i \in [1, D] \]
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Several other robustness methods are available. We will discuss some in the lectures on robustness.
A note on sub-group consistency relevant to the IHDI

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However this does not mean that a reduction in inequality in all regions should lead to a reduction in inequality nationally. Why?

Because population composition could change as well! Changes in population composition are not ”covered” by the property of sub-group consistency.