AF Method

Sabina Alkire, 2 October 2013, IDB
The job of a ‘measure’ or an ‘index’ is to distill what is particularly relevant for our purpose, and then to focus specifically on that. … The central issues in devising an index relate to systematic assessment of importance. Measurement has to be integrated with evaluation. This is not an easy task.

—Amartya Sen (1989)
Sources


• See also Alkire, S., Foster, J.E., 2011. “Understandings and Misunderstandings of Multidimensional Poverty Measurement,” *Journal of Economic Inequality*
Outline

• Motivation
• Multidimensional Data
• Identification
• Aggregation
• Examples
Challenge

• A government would like to create an official multidimensional poverty indicator

• **Desiderata**
  
  – It must understandable and easy to describe
  – It must conform to “common sense” notions of poverty
  – It must be able to target the poor, track changes, and guide policy.
  – It must be technically solid
  – It must be operationally viable
  – It must be easily replicable

  • What would you advise?
Practical Steps

• Select
  – Purpose of the index (monitor, target, etc)
  – Unit of Analysis (indy, hh, cty)
  – Dimensions
  – Specific variables or indicators for each dimension
  – Whether variables or dimensions should be aggregated with others or left independent
  – Cutoff for each independent variable/dimension
  – Value of deprivation for each variable/dimension
  – Identification method
  – Aggregation method
This morning’s focus:

- **Identification** — Dual cutoffs
- **Aggregation** — Adjusted FGT

- Purpose, Variables, Dimensional Cutoffs, Weights and all other steps — Assume given

- Sen (1976)
Review: Unidimensional Poverty

Variable – income
Identification – poverty line
Aggregation – Foster-Greer-Thorbecke ’84

Example: \( \text{Incomes} = (7,3,4,8) \quad \text{Poverty line} \ z = 5 \)

Deprivation vector \( g^0 = (0,1,1,0) \)
Headcount ratio \( = P_0 = \mu(g^0) = 2/4 \)
Normalized gap vector \( g^1 = (0, 2/5, 1/5, 0) \)
Poverty gap \( = P_1 = \mu(g^1) = 3/20 \)
Squared gap vector \( g^2 = (0, 4/25, 1/25, 0) \)
FGT Measure \( = P_2 = \mu(g^2) = 5/100 \)
Multidimensional Data

Matrix of well-being scores for $n$ persons in $d$ domains

$$y = \begin{bmatrix}
13.1 & 14 & 4 & 1 \\
15.2 & 7 & 5 & 0 \\
12.5 & 10 & 1 & 0 \\
20 & 11 & 3 & 1 \\
\end{bmatrix}$$
Multidimensional Data

Matrix of well-being scores for $n$ persons in $d$ domains

\[ y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix} \]

\[ z = (13 \quad 12 \quad 3 \quad 1) \]
z vector = Deprivation Cutoffs

- **Schooling**: “How many years of schooling have you completed?”
  - 6 or more (bold is non-poor)
  - 1-5 years (non-bold is poor)

- **Drinking Water**: “What is the main water source for drinking for this household?”
  - 9. Piped Water
  - 8. Well/Pump (electric, hand)
  - 7. Well Water
  - 6. Spring Water / Rain Water / River/Creek Water / Pond/Fishpond
  - 5. Other

- **Sanitation**: “Where do the majority of householders go to the toilet?”
  - 11. Own toilet with septic tank
  - 10. Own toilet without septic tank
  - 9. Shared toilet
  - 8. Public toilet
  - 7. Creek/river/ditch (without toilet)
  - 6. Yard/field (without toilet)
  - 5. Sewer
  - 4. Pond/fishpond
  - 3. Animal stable
  - 2. Sea/lake
  - 1. Other
Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

\[
y = \begin{bmatrix}
13.1 & 14 & 4 & 1 \\
15.2 & 7 & 5 & 0 \\
12.5 & 10 & 1 & 0 \\
20 & 11 & 3 & 1
\end{bmatrix}
\]

\[
z = (13 \ 12 \ 3 \ 1)
\]

(Oxford Poverty & Human Development Initiative)
Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

\[
g^0 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

\( z \quad (13 \quad 12 \quad 3 \quad 1) \)  

Cutoffs
Identification

Matrix of deprivations

\[ g^0 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 \\
\end{bmatrix} \]
### Identification – Counting Deprivations

Let's denote the matrix $g^0$ as follows:

$$
g^0 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0
\end{bmatrix}
$$

Here, the first column represents the Domains, and the last four elements correspond to Persons, with 0 indicating absence and 1 indicating presence of a deprivation in that domain. The values in the second to fourth rows correspond to the number of such cases.

- Domain 1: 0 cases
- Domain 2: 2 cases
- Domain 3: 4 cases
- Domain 4: 1 case
Identification – Counting Deprivations

Q/ Who is poor?

\[ g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \]

\[ c \]

\[ \begin{array}{cccc} \hline & 0 & 2 & 4 & 1 \\ \hline \text{Persons} & 0 & 2 & 4 & 1 \\
\end{array} \]
Identification – Union Approach

Q/ Who is poor?
A1/ Poor if deprived in any dimension $c_i \geq 1$

<table>
<thead>
<tr>
<th>Domains</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0</td>
<td>0</td>
</tr>
<tr>
<td>0 1 0 1</td>
<td>2</td>
</tr>
<tr>
<td>1 1 1 1 1</td>
<td>4</td>
</tr>
<tr>
<td>0 1 0 0</td>
<td>1</td>
</tr>
</tbody>
</table>

$g^0 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0
\end{bmatrix}$

Persons
Identification – Union Approach

Q/ Who is poor?
A1/ Poor if deprived in any dimension $c_i \geq 1$

<table>
<thead>
<tr>
<th>Domains</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0</td>
<td>0</td>
</tr>
<tr>
<td>0 1 0 1</td>
<td>2</td>
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<tr>
<td>1 1 1 1</td>
<td>4</td>
</tr>
<tr>
<td>0 1 0 0</td>
<td>1</td>
</tr>
</tbody>
</table>

$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Observations

Union approach often predicts high numbers.
Charavarty et al ’98, Tsui ‘02, Bourguignon & Chakravarty 2003 etc use the union approach
**Identification – Intersection Approach**

Q/ Who is poor?
A2/ Poor if deprived in all dimensions $c_i = d$

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

<table>
<thead>
<tr>
<th>Domains</th>
<th>$c$</th>
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<tbody>
<tr>
<td>Persons</td>
<td>0</td>
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<tr>
<td></td>
<td>2</td>
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<td></td>
<td>4</td>
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<td></td>
<td>1</td>
</tr>
</tbody>
</table>
Identification – Intersection Approach

Q/ Who is poor?
A2/ Poor if deprived in all dimensions $c_i = d$

Observations
Demanding requirement (especially if $d$ large)
Often identifies a very narrow slice of population
Atkinson 2003 first to apply these terms.

\[ g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \]

Domains \quad c

Persons

0
2
4
1
Identification – Dual Cutoff Approach

Q/ Who is poor?
A/ Fix cutoff \( k \), identify as poor if \( c_i \geq k \)

\[
g^0 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 2 \\
1 & 1 & 1 & 1 & 4 \\
0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1
\end{bmatrix}
\]

Domains \( c \) Persons

0 0 0 0 0
0 1 0 1 2
1 1 1 1 4
0 1 0 0 1
0 1 0 0 1

\[g_0 = \begin{bmatrix}
0 \\
2 \\
4 \\
1 \\
1
\end{bmatrix}
\]

\[g_1 = \begin{bmatrix}
0 \\
2 \\
4 \\
1 \\
1
\end{bmatrix}
\]

\[g_2 = \begin{bmatrix}
0 \\
2 \\
4 \\
1 \\
1
\end{bmatrix}
\]
Identification – Dual Cutoff Approach

Q/ Who is poor?
A/ Fix cutoff $k$, identify as poor if $c_i \geq k$ (Ex: $k = 2$)

$$g^0 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}$$

$$c = \begin{bmatrix}
0 \\
2 \\
4 \\
1
\end{bmatrix}$$
Identification – Dual Cutoff Approach

Q/ Who is poor?
A/ Fix cutoff $k$, identify as poor if $c_i \geq k$ (Ex: $k = 2$)

$$g^0 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
0 \\
2 \\
4 \\
1
\end{bmatrix}$$

Note
Includes both union ($k = 1$) and intersection ($k = d$)
Identification – The problem empirically

<table>
<thead>
<tr>
<th>$k = \text{H}$</th>
<th>91.2%</th>
<th>75.5%</th>
<th>54.4%</th>
<th>33.3%</th>
<th>16.5%</th>
<th>6.3%</th>
<th>1.5%</th>
<th>0.2%</th>
<th>0.0%</th>
<th>0.0%</th>
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<td>Union 1</td>
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<td>Inters. 10</td>
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</tr>
</tbody>
</table>

Poverty in India for 10 dimensions:
91% of population would be targeted using union,
0% using intersection
Need something in the middle.
(Alkire and Seth 2009)
Create Censored Deprivation Matrix

Censor data of nonpoor

\[ g^0 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0
\end{bmatrix} \]

Domains \[c\]

1

Persons

0

2

4

1
Censored Deprivation Matrix

Censor data of nonpoor

\[
g^0(k) = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
c(k) = \begin{bmatrix}
0 \\
2 \\
4 \\
0
\end{bmatrix}
\]

Domains

Persons
Aggregation – Headcount Ratio

\[ g^0(k) = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 2 \\
1 & 1 & 1 & 1 & 4 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

<table>
<thead>
<tr>
<th>Domains</th>
<th>$c(k)$</th>
<th>Persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0 1 0 1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1 1 1 1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>0 0 0 0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
### Aggregation – Headcount Ratio

**$g^0(k)$**

<table>
<thead>
<tr>
<th>Domains</th>
<th>$c(k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0</td>
<td>0</td>
</tr>
<tr>
<td>0 1 0 1</td>
<td>2</td>
</tr>
<tr>
<td>1 1 1 1</td>
<td>4</td>
</tr>
<tr>
<td>0 0 0 0</td>
<td>0</td>
</tr>
</tbody>
</table>

Two poor persons out of four: **$H = 1/2$**
Aggregation – Intensity (A)

Need to augment information
depredation shares among poor

<table>
<thead>
<tr>
<th>Domains</th>
<th>$c(k)$</th>
<th>$c(k)/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0 1 0 1 0</td>
<td>2</td>
<td>2/4</td>
</tr>
<tr>
<td>1 1 1 1 1</td>
<td>4</td>
<td>4/4</td>
</tr>
<tr>
<td>0 0 0 0 0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
### Aggregation – Intensity (A)

Need to augment information

deprivation shares among poor

<table>
<thead>
<tr>
<th>Domains</th>
<th>c(k)</th>
<th>c(k)/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0 1 0 1</td>
<td>2</td>
<td>2/4</td>
</tr>
<tr>
<td>1 1 1 1</td>
<td>4</td>
<td>4/4</td>
</tr>
<tr>
<td>0 0 0 0 0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

\[ g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

A = average deprivation share among poor = 3/4
**Aggregation – Adjusted Headcount Ratio**

Adjusted Headcount Ratio = $M_0 = HA$

\[
g^0(k) = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>Domains</th>
<th>$c(k)$</th>
<th>$c(k)/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0 1 0 1 0</td>
<td>2</td>
<td>2/4</td>
</tr>
<tr>
<td>1 1 1 1 1</td>
<td>4</td>
<td>4/4</td>
</tr>
<tr>
<td>0 0 0 0 0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

A = average deprivation share among poor = 3/4
Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = $M_0 = HA = \mu(g^0(k))$

<table>
<thead>
<tr>
<th>Domains</th>
<th>$c(k)$</th>
<th>$c(k)/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0 1 0 1 1</td>
<td>2</td>
<td>2 / 4</td>
</tr>
<tr>
<td>1 1 1 1 1</td>
<td>4</td>
<td>4 / 4</td>
</tr>
<tr>
<td>0 0 0 0 0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

A = average deprivation share among poor = 3/4
Adjusted Headcount Ratio \( M_0 = HA = \mu(g^0(k)) = \frac{6}{16} = .375 \)

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\( g^0(k) \) = Domains \( c(k) \) \( c(k)/d \)

\( 0 \) \( 0 \)
\( 2 \) \( 2/4 \)
\( 4 \) \( 4/4 \)

A = average deprivation share among poor = \( \frac{3}{4} \)
**Aggregation – Adjusted Headcount Ratio**

Adjusted Headcount Ratio = \( M_0 = HA = \mu(g^0(k)) = \frac{7}{16} = 0.44 \)

<table>
<thead>
<tr>
<th>Domains</th>
<th>( c(k) )</th>
<th>( c(k)/d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \begin{array}{cccc} 0 &amp; 0 &amp; 0 &amp; 0 \ 1 &amp; 1 &amp; 0 &amp; 1 \ 1 &amp; 1 &amp; 1 &amp; 1 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{array} ]</td>
<td>0</td>
<td>( 3 \ldots \frac{3}{4} )</td>
</tr>
<tr>
<td>[ \begin{array}{cccc} 1 &amp; 1 &amp; 0 &amp; 1 \ 1 &amp; 1 &amp; 1 &amp; 1 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{array} ]</td>
<td>( 4 \ldots \frac{4}{4} )</td>
<td></td>
</tr>
</tbody>
</table>

A = average deprivation share among poor = \( \frac{3}{4} \)

Note: if person 2 has an additional deprivation, \( M_0 \) rises
Satisfies dimensional monotonicity
Aggregation: Adjusted FGT Family

Adjusted FGT is $M_\alpha = \mu(g^\alpha(\tau))$ for $\alpha \geq 0$

Domains

$$g^\alpha(k) = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0.42^\alpha & 0 & 1^\alpha \\
0.04^\alpha & 0.17^\alpha & 0.67^\alpha & 1^\alpha \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Persons

**Theorem 1** For any given weighting vector and cutoffs, the methodology $M_{ka} = (\rho_k, M_\alpha)$ satisfies: decomposability, replication invariance, symmetry, poverty and deprivation focus, weak and dimensional monotonicity, nontriviality, normalisation, and weak rearrangement for $\alpha \geq 0$; monotonicity for $\alpha > 0$; and weak transfer for $\alpha \geq 1$. 
Extension – General Weights

Modifying for weights: identification and aggregation (technically weights need not be the same, but conceptually probably should be)

• Use the $g_0$ or $g_1$ matrix
• Choose relative weights for each dimension $w_d$
• Apply the weights (sum = $d$) to the matrix
• $c_k$ now reflects the weighted sum of the dimensions.
• Set cutoff $k$ across the weighted sum.
• Censor data as before to create $g_0 (k)$ or $g_1 (k)$
• Measures are still the mean of the matrix.
Example: Weights

Matrix of deprivations

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Weighting vector $\omega = (0.5, 2, 1, 0.5)$
Example: Weights

Matrix of deprivations

Weighting vector \( \omega = (0.5 \ 2 \ 1 \ 0.5) \)

\[
g^0 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0.5 \\
0.5 & 2 & 1 & 0.5 \\
0 & 2 & 0 & 0
\end{bmatrix}
\]

Domains

Persons
Weighted Deprivation Matrix

Note that we use the same notation as for the deprivation matrix on purpose.

Where

- \( g_{ij}^0 = w_j \) if \( x_{ij} < z_j \) (deprived)
- \( g_{ij}^0 = 0 \) if \( x_{ij} \geq z_j \) (non-deprived)

- Or equivalently:

\[
\begin{bmatrix}
0 & \cdots & 0 \\
0 & \cdots & 0 \\
0 & \cdots & 0 \\
\end{bmatrix}
\]

\[
g^0 =
\begin{bmatrix}
g_{11}^0 & \cdots & g_{1d}^0 \\
g_{21}^0 & \cdots & g_{2d}^0 \\
0 & \cdots & 0 \\
g_{n1}^0 & \cdots & g_{nd}^0 \\
\end{bmatrix}
\]

\[
z = (z_1, z_2, \ldots, z_d)
\]

\[
w = (w_1, w_2, \ldots, w_d)
\]
AF Method: Decompositions

By Population Subgroup

\[ M_0 \quad \text{Poverty} \]
\[ H \quad \text{Headcount} \]
\[ A \quad \text{Intensity} \]

Post-identification: By Dimension

Censored Headcount (column vector)
Percentage Contribution (weighted)

All draw on censored matrix
Deprivation: if \( y_{id} < z \) person \( i \) is deprived in \( y_d \)

Poverty: if \( c_i < k \) person \( i \) is poor.

Deprivation cutoffs: the \( z \) cutoffs for each dimension

Poverty cutoff: the overall cutoff \( k \)

Dimension: for AF – a column in the matrix having its own deprivation cutoff (sometimes called an ‘indicator’)

Joint distribution: showing the simultaneous or coupled deprivations a person/hh has
Deprivation Count Vector

Where the ‘deprivation count’ or score for each person is the sum of her weighted deprivations

- \( c_i = g_{i1} + \ldots + g_{id} \)

\[
\begin{bmatrix}
  c_1 \\
  c_2 \\
  \vdots \\
  c_n
\end{bmatrix}
\]
Identify the poor

Given a poverty cut-off $k$, we compare the deprivation count with the $k$ cutoff and then censor the deprivations of those who were not identified as poor.

\[
\rho_k(x_i; z) = \begin{cases} 
1 & \text{if } c_i \geq k \quad \text{poor} \\
0 & \text{if } c_i < k \quad \text{non-poor}
\end{cases}
\]

\[
c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}
\]
Key point: Deprivation and Censored Matrix

$g^0(k)$ needed for associated partial indices

Deprivation Matrix

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$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Censored Deprivation Matrix, $k=2$

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$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Headcount Ratio of MD Poverty

It is the proportion of people who have been identified as poor. Thus:

\[ H = \frac{\sum_{i=1}^{n} \rho_k(x_i; z)}{n} = \frac{q}{n} \]

Where \( q \) is the number of poor people. Headcount Ratio is sometimes called the \textit{incidence} of poverty, or the poverty rate.
Intensity (or breadth) of MD Poverty

• It is the average proportion of deprivations in which the poor are deprived.

\[ A = \frac{\sum_{i=1}^{n} c_i(k)}{dq} \]

Note that it is simple to compute:

1) You compute the proportion of total deprivations each poor person has \((c_i(k)/d)\). Note we need to use the censored deprivation count vector, ie: we ignore the deprivations of the non-poor.

2) You take the average of those proportions (that’s why we divide by \(q\), the number of the poor)
Multidimensional Poverty: $M_0$

(Adjusted Headcount Ratio)

• It is the product of incidence and intensity.

$$M_0 = H \ast A$$

• Or equivalently, it is the mean of the censored (weighted) deprivation matrix:

$$M_0 = \mu(g_0^0(k)) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{d} g_{ij}^0(k)}{nd}$$
How do we interpret $M_0$?

- **$M_0$ Interpretation**
  
  $M_0$ is the mean of the weighted censored deprivation matrix.

  Thus, **it gives the proportion of weighted deprivations that the poor experience in a society of all the total potential deprivations that the society could experience.**
How do we decompose $M_0$ by Indicators?

There are two useful but distinct indicators to look at:

1) Censored headcount ratios
2) Contributions by indicators and dimensions.
Censored Headcount Ratios

- Censored headcount ratios are the % of people who are poor and deprived in a certain indicator.
- Careful! Censored headcounts are not the % of the poor deprived in a certain indicator.
- Raw headcount ratios are the % of people who are deprived in a certain indicator.
Censored Headcount Ratios

• They are the mean of each column of the (weighted) censored deprivation matrix divided by the indicator's weight.

\[
H_j^C = \frac{\sum_{i=1}^{n} g_{ij}^0(k)}{w_j \cdot n}
\]

• \(M_0\) is the weighted sum of the censored headcount ratios.

\[
M_0 = \sum_{j=1}^{d} \left( \frac{w_j}{d} \right) H_j^C
\]
Raw Headcount Ratios

• These are the deprivation rates by dimension, i.e. the proportion of people who are deprived in that dimension.

• It is simply the mean of each column of the [uncensored] deprivation matrix:

\[ H_j = \left( g_{1j}^0 + g_{2j}^0 + \ldots + g_{nj}^0 \right) / n \]
Contribution by Indicator and Dimension

• It is the proportion of total poverty which arises from a particular deprivation.

• Recall from the previous slide:

$$M_0 = \sum_{j=1}^{d} \left( \frac{W_j}{d} \right) H_j^C$$

• Thus, the contribution of indicator $j$ to overall poverty is given by:

$$C_j = \frac{(w_j / d) H_j^C}{M_0}$$
Contribution by Indicator and Dimension

• Note: The sum of the contributions of all indicators needs to add up to 1 (or 100%).

• The percentage contribution includes the weight on each indicator, and is useful when the weights are not equal.

• If there are more than one indicators in a dimension, the dimensional contribution is simply the sum of the indicators’ contribution.
How do we decompose $M_0$ by population subgroups?

Visually...

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GROUP A

GROUP B
Decomposition by Population Subgroups

If the entire population $X$ (of size $n$) is divided into two subgroups $X_1$ (of size $n_1$) and $X_2$ (of size $n_2$), then overall $M_0$ is the weighted sum of $M_0$ in each subgroup:

$$M_0(X; z) = \left( \frac{n_1}{n} \right) M_0(X_1; z) + \left( \frac{n_2}{n} \right) M_0(X_2; z)$$

Thus, the contribution of subgroup $i$ to overall poverty is

$$C_{Gi} = \left( \frac{n_i}{n} \right) M_0(X_i; z)$$
Decomposition by Population Subgroups

• Note that the sum of the contributions of all groups needs to add up to 1 (or 100%).
• You take the mean of the matrices of each population subgroup individually.
• The population-weighted mean of subgroups is equal to the national $M_0$. 