Has the world moved forward? A robust multidimensional evaluation

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Abstract

This paper presents a multidimensional normative framework to evaluate whether the world moved forward between 1980 and 2008. We focus on the robustness of the results when attributing different weights to the different dimensions. Population-weighted international welfare has not worsened over this period. However, the trend in the underlying inequality is ambiguous, once sufficient weight is given to longevity and/or income.

1. Introduction

To answer the hazardous question whether the world has moved forward or not, one must specify first what is meant by ‘moving forward’. More specifically one has to answer two important questions. The first question is about the appropriate metric of progress. Is it income? Or should other, so-called non-income factors be taken into account as well? Second, one has to specify whose progress is of interest. Is it the progress of ‘Mr. average’? Or, should we also take distributional considerations into account?

In terms of average income, the world undeniably has moved forward between 1980 and 2008. One of the more enduring questions is whether countries have come closer together or not. This question has been addressed both in the macro-economic growth literature and in the literature on income inequality. One of the concepts used in the macro-economic growth literature, is $\sigma$-convergence: the world is said to $\sigma$-converge when the variance of the logarithm of GDP per capita decreases over time (Barro and Sala-i-Martin, 2004). Milanovic (2005, chapter 5) finds $\sigma$-divergence (or, increasing inequality) in the second half of the 20th century. Moreover, he shows that, when countries are given equal weights and per capita GDP is the income concept, the (unweighted international) inequality increases for other dispersion measures as well. This finding, however, is shown to be sensitive to several methodological choices. Firebaugh (2003), amongst others, weighs countries by their population size and finds a decrease in the so-called population-weighted international inequality. Milanovic (2005) and Sala-i-Martin (2006) incorporate the inequality within countries in their computations of the global income inequality by using primary survey data, and a combination of GDP per capita and dispersion data, respectively. The authors find substantially different results; see Anand and Segal (2008) for a recent overview and discussion of the methodological questions involved. Capéau and Decoster (2004) illustrate the sensitivity of the trend in population-weighted international inequality to the choice of the inequality aversion level. Dowrick and Akmal (2005) analyze the effect of using different conversion rates (to compare incomes between countries) on inequality. They show that inequality increases when using market exchange rates, while it decreases when purchasing power parity rates are used. Finally, Atkinson and Brandolini (2009) and Bosmans et al. (2010) show that changing the inequality concept from a relative to an absolute one leads to a more pessimistic view on population-weighted international inequality.

Income is not the only possible metric of well-being, and a sole focus on income may lead to a too narrow-sighted answer to the question whether the world moved forward or not. Therefore, we incorporate also non-income dimensions, longevity and education, in our analysis. Until recently, this multidimensional perspective...
received relatively little attention in the empirical literature on world inequality. Bourguignon and Morrisson (2002) and Becker et al. (2005) combine income and longevity to analyze the distribution of lifetime income. Morrisson and Murtin (2005) use a weighted sum of standardized income, longevity and education as a metric of well-being. Decancq et al. (2009) use the same three dimensions and calculate the trend in inequality using a flexible well-being indicator, which contains the Human Development Index and the lifetime income used by Becker et al. (2005) as special cases. Noorbakhsh which contains the Human Development Index and the lifetime expectancy at birth and the third dimension education differences in purchasing power.

The state of the world can be summarized by a distribution matrix \( X \) in \( \mathbb{R}^{n \times m} \), containing an \( m \)-dimensional outcome vector \( x_i \) for each individual \( i \); since population size is variable, \( n(X) \) refers to the number of individuals in distribution \( X \) with \( n(X) \geq 3 \). Despite the general set-up of the normative framework on the basis of individuals, recall that we use population-weighted country data in the next section. This boils down to attributing the country averages to each individual in that country.

We assume that the social welfare level of a distribution matrix \( X \) can be measured by a continuous social welfare function \( W \). We informally discuss six reasonable properties for \( W \) and derive the implied functional form; see Weymark (2006) for a formal treatment. The resulting methodology will then be applied in Section 3 to population-weighted country data for three dimensions of well-being: income, longevity and education. Section 4 concludes.

2. A normative framework

The first five properties only consider distributions with the same fixed population size; the sixth property is used to deal with distributions with different population sizes.

First, only the outcome vector of an individual matters (anonymity) and thus individuals with the same outcome vector are treated in the same way. Second, a higher outcome in one of the dimensions of an individual improves, ceteris paribus, social welfare in the world (increasingness). Third, when comparing two distributions, two individuals with exactly the same outcome in either distribution do not influence the social welfare comparison of both distributions (separability). Fourth, rescaling the well-being dimensions, e.g., expressing longevity in months rather than years, does not affect the social welfare evaluation (ratio-scale invariance). Fifth, a lower correlation between individual outcomes improves, ceteris paribus, social welfare (correlation increasing majorization). Sixth, replicating a distribution an arbitrary number of times \( k \), i.e., replacing each individual by \( k \) copies, does not change the welfare level (replication invariance). Imposing these six properties together leads to the following result. The proof, which heavily hinges on a result by Tsui (1995), may be found in Appendix A.

Proposition 1. A continuous social welfare function \( W \) defined over distribution matrices \( X \) in \( \mathbb{R}^{n \times m} \) satisfies anonymity, increasingness, separability, ratio-scale invariance, correlation increasing majorization and replication invariance if and only if \( W(X) \) is ordinally equivalent to

\[
\sum_{i=1}^{n} \left( \prod_{j=1}^{m} w_j (x_{ij}) \right) \left( 1 - \varepsilon \right)^{2(\mu(X) - \mu(\mu(X)))} \left( 1 - \frac{1}{1 - \varepsilon} \right)^{1-\varepsilon},
\]

with \( \varepsilon > 0 \) and \( i \) and \( \varepsilon > 1 \).

Note three things. First, note that the parameter \( \varepsilon \) measures the inequality aversion in the well-being space: the higher is \( \varepsilon \), the more sensitive the social welfare measure becomes to the well-being of individuals at the bottom of the well-being distribution. Second, the weighting scheme \( W = (w_1, w_2, \ldots, w_m) \) measures the relative importance of the different well-being dimensions and will be the focus of our analysis in the next section. Third, once a social welfare function \( W \) is derived, Kolm (1977) proposes the following decomposition:

\[
W(X) = W(\mu(X)) \times \prod_{i=1}^{n} \left( 1 - \frac{W(X_i)}{W(\mu(X))} \right),
\]

with \( \mu(X) = (\mu_1(X), \mu_2(X), \ldots, \mu_m(X)) \) the vector of average outcomes in each dimension. The first term \( W(\mu(X)) \) is a measure of social welfare in a ‘hypothetical’ world in which each individual has the average world outcome in each dimension. In this world, inequality between individuals has been averaged out. The term between square brackets, \( 1 - W(X_i)/W(\mu(X)) \), can then be interpreted as the level of inequality of the distribution, denoted \( I(X) \).

3. Results

In this section the social welfare function obtained in expression (1) is applied to international data from the World Development Indicators (2009). We focus on three dimensions of well-being. The first dimension, income, is measured by GDP per capita, corrected for differences in purchasing power. Longevity is measured by life expectancy at birth and the third dimension education, is captured by an education index, which linearly combines the gross secondary enrolment rate (with weight 1/3) and the literacy rate (with weight 2/3). These dimensions are very similar to the main components of the Human Development Index. We focus on 4 points in time: 1980, 1990, 2000, and 2008 (the most recent data) to limit the number of time comparisons. For these dimensions, only average data at the country level are available. Therefore, we attribute the outcome vector of each country to all its citizens. Table 1 provides summary statistics for the three indicators.

At least three caveats apply. Some observations are missing and cannot be imputed in a reasonable way, mainly for some post-communist and Sub-Saharan African countries. These countries have been removed from the analysis in all periods, leaving us with

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Mean</th>
<th>Min.</th>
<th>Max.</th>
<th>St.dev.</th>
</tr>
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<tr>
<td>1980 GDP/capita (in PPP US$)</td>
<td>5663</td>
<td>430</td>
<td>95434</td>
<td>1293</td>
</tr>
<tr>
<td>Longevity (in years)</td>
<td>63</td>
<td>43</td>
<td>76</td>
<td>0.93</td>
</tr>
<tr>
<td>Education index (in %)</td>
<td>57</td>
<td>11</td>
<td>130</td>
<td>2.60</td>
</tr>
<tr>
<td>1990 GDP/capita (in PPP US$)</td>
<td>6490</td>
<td>400</td>
<td>50309</td>
<td>1055</td>
</tr>
<tr>
<td>Longevity (in years)</td>
<td>65</td>
<td>32</td>
<td>79</td>
<td>0.93</td>
</tr>
<tr>
<td>Education index (in %)</td>
<td>70</td>
<td>11</td>
<td>106</td>
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</tr>
<tr>
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<td>7809</td>
<td>348</td>
<td>60177</td>
<td>1231</td>
</tr>
<tr>
<td>Longevity (in years)</td>
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<tr>
<td>Education index (in %)</td>
<td>81</td>
<td>15</td>
<td>120</td>
<td>2.20</td>
</tr>
<tr>
<td>2008 GDP/capita (in PPP US$)</td>
<td>9641</td>
<td>354</td>
<td>70981</td>
<td>1468</td>
</tr>
<tr>
<td>Longevity (in years)</td>
<td>71</td>
<td>42</td>
<td>83</td>
<td>1.00</td>
</tr>
<tr>
<td>Education index (in %)</td>
<td>87</td>
<td>27</td>
<td>115</td>
<td>1.90</td>
</tr>
</tbody>
</table>
individuals in 111 countries for which we have all the necessary data, covering 87% of the total world population in 2008. Moreover, neglecting the (considerable) within country income inequality is known to bias the results towards optimism; see, e.g., Anand and Segal (2008). Therefore, the optimistic results we obtain later on should be handled with caution. Finally, Deaton (2005) shows that GDP per capita can substantially differ from income measured via income surveys (in the global income inequality literature). Only if the relation between GDP per capita and true income would remain constant over time for each country, our results do not change due to the ratio-scale invariance of our measure.

We start by looking at the two terms at the right-hand side of Eq. (2). Afterwards we combine both terms to look at overall social welfare. The first term \( W(\mu(X)) \) captures the progress of a ‘hypothetical’ world in which each individual has the average world outcome in each dimension. Note that comparisons of distributions based on \( W(\mu(\cdot)) \) do not depend on the inequality aversion parameter \( \varepsilon \). From Table 1, it is clear that the four indicators separately made an unambiguous improvement between 1980 and 2008. Hence, any intermediate weighting scheme \( w \) must lead to an optimistic picture as well. However, this average picture may hide some individuals suffering from regress in some dimensions. What if we look more carefully at the entire distribution of the three dimensions?

To do so, the level of inequality \( l(X) = 1 - W(X)/W(\mu(X)) \) in the world is analyzed. The results are visualized in triangles (see Fig. 1). Every point of such a triangle represents a different weighting scheme \( w \). The weight of income, \( w_{gdp} \), can be read on the horizontal axis, the weight of longevity, \( w_{long} \), can be read on the vertical axis; the weight of education \( w_{edu} \) equals \( 1 - w_{gdp} - w_{long} \) and is the horizontal distance from the diagonal. Each corner of the triangle corresponds to an extreme case where all weight is assigned to one of the dimensions. For later use, the point \((1,0)\) represents the weighting scheme in which all weight is given to GDP per capita (the standard case analyzed in the income inequality literature), while the point \((1/3,1/3)\) corresponds with an equal weighting scheme (a focal point in the multidimensional inequality literature). Each figure compares two years: 1980–1990, 1990–2000, 2000–2008 and 1980–2008. Recall that the inequality level depends on the inequality aversion parameter \( \varepsilon \). Therefore, the color scale ranges from white to black. White corresponds with an improvement in inequality (lower inequality) for all levels of inequality aversion in the range \([0,10]\); Black stands for a worsening according to all levels of inequality aversion. The intermediate cases are depicted in a grayscale, such that lighter colors correspond to more agreement (among the different inequality aversion levels) that we deal with an improvement.\(^2\)

Fig. 1 shows that the inequality comparisons are not robust with respect to the weighting scheme; in particular, confining attention to the special cases \((1,0)\) and \((1/3,1/3)\) provides only a partial picture. Moreover, all figures have in common that sufficient weight on education, roughly \(1/3\) or more, leads to an optimistic view. The conclusion that the distribution matters when longevity and/or income get larger weights, is consistent with the findings of Becker et al. (2005) who refer to the devastating effects of AIDS on longevity in Sub-Saharan Africa and with Capéau and Decoster (2004) who report that *income* inequality is sensitive to the level of inequality aversion.

\(^2\) In practice, we use a very fine grid for the inequality aversion levels in \([0,10]\). The grayscale is calculated on the basis of the percentage of inequality aversion levels in the grid for which inequality increases.
Next, we focus on social welfare defined in (1), which trades off average performance and inequality. We do not report these figures, since we obtain a welfare progress for virtually all periods. Only a light grey zone arises for the period 1990–2000, in the case when almost all weight is given to GDP per capita.

4. Conclusion

We introduce a normative framework to address the question whether the world has moved forward or not on the basis of income and non-income data, more precisely, GDP per capita, longevity and education. We focus on the robustness of the results for alternative weighting schemes that reflect different value judgements on the relative importance of these well-being dimensions.

Our results can be summarized as follows. Using multidimensional population-weighted international welfare – i.e., multidimensional welfare between countries weighted by population size – the world improved between 1980 and 2008, irrespective of the weighting scheme and the inequality aversion level. The underlying inequality picture, however, is more ambiguous. If sufficient weight is given to education – roughly speaking, more than 1/3 of the total weight – then inequality decreases. But, if longevity and/or income get relatively more weight, the inequality trend crucially depends on the normative choice of the inequality aversion level.

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Appendix A. Proof

It can easily be checked that the continuous social welfare function $W$ defined in expression (1) satisfies all six properties. To prove the opposite direction, we start from a result by Tsui (1995, theorem 1) for distribution matrices with the same fixed population size, say $n$. A continuous multidimensional social welfare function $W^m$ (to compare distribution matrices with population size equal to $n$) satisfies anonymity, increasingness, separability and ratio-scale invariance if and only if it is ordinally equivalent to $\sum_{i=1}^n U(x^i)$, with

$$U(x) = \frac{1}{1 - \epsilon} \left( \prod_{j=1}^{m} x^j \right)^{1-\epsilon} \text{or} \; U(x) = \log \prod_{j=1}^{m} x^j,$$

(3)

with $w_j > 0$ for all $j=1,2,...,m$, $\sum_{j=1}^{m} w_j = 1$ and $\epsilon \neq 1$. Suppose we want to compare two distributions $X$ and $Y$ which might have a different population size, say $n$ and $n'$, respectively. Replicating the first distribution $n'$ times and the second distribution $n$ times leads to distributions $X'$ and $Y'$ with the same population size $n \times n'$. Replication invariance requires that $W^m(X) = W^{m \times n'}(X')$ and $W^m(Y) = W^{m \times n'}(Y')$ and thus $W^m(X) \geq W^m(Y)$ is true if and only if $W^{m \times n'}(X') \geq W^{m \times n'}(Y')$ holds. But, by construction, the latter statement reduces to the comparison.

$$n \sum_{i=1}^n U(x^i) \geq n \sum_{i=1}^n U(y^i) \text{ or } \frac{1}{n} \sum_{i=1}^n U(x^i) \geq \frac{1}{n} \sum_{i=1}^n U(y^i),$$

with $U$ as defined above.

Finally, all properties together show that $W(X)$ is ordinally equivalent with

$$\frac{1}{1 - \epsilon} \prod_{j=1}^{m} x^j \left( \prod_{j=1}^{m} x^j \right)^{1-\epsilon} = \frac{1}{1 - \epsilon} \prod_{j=1}^{m} x^j,$$

(4)

with $w_j > 0$, $j=1,2,...,m$, $\sum_{j=1}^{m} w_j = 1$ and $\epsilon > 1$. Expression (4) is in turn ordinally equivalent to

$$\frac{\left( \frac{1}{n} \sum_{i=1}^n x_i \right)^{1-\epsilon}}{n^{1-\epsilon}}.$$

References


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3 These expressions can be obtained from the original theorem by Tsui (1995) by simple reworking. In his theorem, Tsui uses the additional requirement of quasi-concavity which further restricts the admissible parameter choices.