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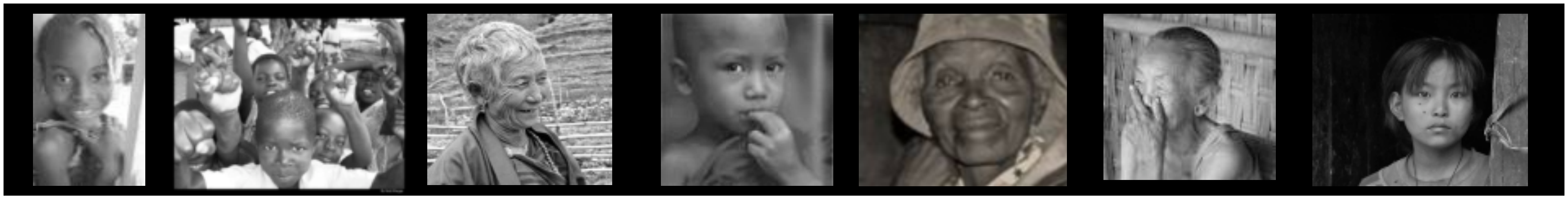
UNIVERSITY OF OXFORD

Solving exercises on AF Methodology




Tabita, Kenya
Rabiya, India
Stephanie, Madagascar
Agatha, Madagascar
Dalma, Kenya
Ann-Sophia, Kenya
Valérie, Madagascar

Exercise 1

Multidimensional Data

$$
\begin{array}{cc}
 & \text{Domains} \\
X = & \begin{bmatrix} 6 & 3 & 18 & 1 \\ 8 & 4 & 20 & 1 \\ 12 & 6 & 17 & 0 \\ 20 & 8 & 16 & 1 \\ 5 & 3 & 16 & 0 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

$$
z = \begin{bmatrix} 10 & 6 & 18.5 & 1 \end{bmatrix} \quad \text{Cut-offs}
$$

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

$$X = \begin{bmatrix} \underline{6} & \underline{3} & \underline{18} & 1 \\ \underline{8} & \underline{4} & 20 & 1 \\ 12 & 6 & \underline{17} & \underline{0} \\ 20 & 8 & \underline{16} & 1 \\ \underline{5} & \underline{3} & \underline{16} & \underline{0} \end{bmatrix} \qquad g^0 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$z = \begin{bmatrix} 10 & 6 & 18.5 & 1 \end{bmatrix}$$

Raw Headcounts

Compute average of each column of g^0

$$g^0 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$0.6 \quad 0.6 \quad 0.8 \quad 0.4 \qquad \text{Raw Headcounts}$$

Deprivations Count Vector

Sum individual's deprivations

$$g^0 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad ci = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ 4 \end{bmatrix}$$

Union Approach

Poor if deprived in any dimension, k = 1

Deprivation matrix

Censored deprivation matrix

$$g^0 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad ci = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ 4 \end{bmatrix} \qquad g^0(1) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad ci(1) = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ 4 \end{bmatrix}$$

Censor data of non-poor

Union Approach

Share deprivations of poor

$$g^0(1) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad ci(1) = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ 4 \end{bmatrix} \qquad ci(1)/d = \begin{bmatrix} 3/4 \\ 2/4 \\ 2/4 \\ 1/4 \\ 4/4 \end{bmatrix}$$

$$H = \frac{5}{5} = 1 \qquad \text{A} = \left(\frac{3}{4} + \frac{2}{4} + \frac{2}{4} + \frac{1}{4} + \frac{4}{4}\right)/5 = 0.6$$

$$M0 = H \times A = 1 \times 0.6 = 0.6$$

$$M0 = \mu(g^0(1)) = \frac{12}{20} = 0.6$$

Intersection Approach

Poor if deprived in all dimensions, k = d = 4

Deprivation matrix

$$g^0 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad ci = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ 4 \end{bmatrix}$$

Censored deprivation matrix

$$g^0(4) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad ci(4) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

Censor data of non-poor

Intersection Approach

Share deprivations of poor

$$g^0(4) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad ci(4) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} \qquad ci(4)/d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4/4 \end{bmatrix}$$

$$H = \frac{1}{5} = 0.2 \qquad \mathrm{A} = \left(\frac{4}{4}\right)/1 = 1$$

$$M0 = H \times A = 0.2 \times 1 = 0.2$$

$$M0 = \mu(g^0(4)) = \frac{4}{20} = 0.2$$

Poverty Cut-off: k=2

Poor if deprived in two dimensions, k = 2

Deprivation matrix

Censored deprivation matrix

$$g^0 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad ci = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ 4 \end{bmatrix} \qquad g^0(2) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad ci(2) = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 0 \\ 4 \end{bmatrix}$$

Censor data of non-poor

Poverty Cut-off: k=2

Share deprivations of poor

$$g^0(2) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad ci(2) = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 0 \\ 4 \end{bmatrix} \qquad ci(2)/4 = \begin{bmatrix} 3/4 \\ 2/4 \\ 2/4 \\ 0 \\ 4/4 \end{bmatrix}$$

$$H = \frac{4}{5} = 0.80 \qquad \text{A} = \left(\frac{3}{4} + \frac{2}{4} + \frac{2}{4} + \frac{4}{4}\right)/4 = 11/16$$

$$M0 = \mu(g^0(2)) = \frac{11}{20} = 0.55$$

Poverty Cut-off: k=2

H = 4/5 - This means 80% of the population is poor (deprived in 2 or more dimensions)

A = 11/16 - On average the poor (those deprived in 2 or more dimensions) are deprived in approximately 69% of the indicators.

M0 = 11/20 - The poor in this society experience 55% of the total possible deprivations the society could experience.

Raw vs. Censored Headcounts

Deprivation Matrix

$$g^0 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

0.6 0.6 0.8 0.4

Raw Headcounts

Censored Deprivation Matrix

$$g^0(2) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

0.6 0.6 0.6 0.4

Censored Headcounts

Censored Headcounts

- 60% of the population is poor and deprived in income, education and BMI

- 40% is poor and deprived in water.

- The raw and censored headcounts differ in the case of the BMI indicator.

Break-down by Dimensions

M0 can be written as follows:

$$M0 = \frac{w_1}{d} CH_1 + \frac{w_2}{d} CH_2 + \frac{w_3}{d} CH_3 + \frac{w_4}{d} CH_4$$

Therefore,

$$\text{Contrib. of dimension i} = \frac{((w_i/d)CH_i)}{M0}$$

Break-down by Dimensions

$$g^0(2) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\left[\left(\frac{3}{5}\times\frac{1}{4}\right)+\left(\frac{3}{5}\times\frac{1}{4}\right)+\left(\frac{3}{5}\times\frac{1}{4}\right)+\left(\frac{2}{5}\times\frac{1}{4}\right)\right] = \frac{11}{20} = M0$$

$$\text{Contrib. of income} = \left(\frac{3}{5}\times\frac{1}{4}\right) \Big/ \left(\frac{11}{20}\right) = \frac{3}{11} \cong 0.27$$

$$\text{Contrib. of water} = \left(\frac{2}{5}\times\frac{1}{4}\right) \Big/ \left(\frac{11}{20}\right) = \frac{2}{11} \cong 0.18$$

Decomposition by Groups

M0 can be written as follows:

$$M0 = \frac{n_x}{n} M0_x + \frac{n_y}{n} M0_y$$

Therefore,

$$\text{Contrib. of group x} = \left(\frac{n_x}{n} M0_x\right) / M0$$

Decomposition by groups

Women's censored deprivation matrix

$$g^0{}_x(2) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Men's censored deprivation matrix

$$g^0{}_y(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$H_x = \frac{3}{3} = 1$$

$$H_y = \frac{1}{2} = 0.5$$

$$A_x = \left(\frac{3}{4} + \frac{2}{4} + \frac{2}{4}\right)/3 = \frac{7}{12}$$

$$A_y = 1 = 1$$

$$M0_x = \mu(g^0{}_x(2)) = \frac{7}{12} \cong 0.58$$

$$M0_y = \mu(g^0{}_x(2)) = \frac{4}{8} = 0.5$$

Decomposition by groups

Women's censored deprivation matrix

$$g^0{}_x(2) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Men's censored deprivation matrix

$$g^0{}_y(2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

So, what is the contribution of each gender for overall poverty?

$$\text{Contrib. of women} = \left(\frac{n_x}{n} \times M0_x\right) / M0 = \left(\frac{3}{5} \times \frac{7}{12}\right) / \frac{11}{20} \cong 0.636$$

$$\text{Contrib. of men} = \left(\frac{n_y}{n} \times M0_x\right) / M0 = \left(\frac{2}{5} \times \frac{4}{8}\right) / \frac{11}{20} \cong 0.364$$

Change in Deprivation Matrix (1)

$$X = \begin{bmatrix} \underline{6} & \underline{3} & \underline{18} & 1 \\ \underline{8} & \underline{4} & 20 & 1 \\ 12 & 6 & \underline{17} & \underline{0} \\ 20 & 8 & \underline{16} & 1 \\ \underline{5} & \underline{3} & \underline{16} & \underline{0} \end{bmatrix} \qquad X' = \begin{bmatrix} \underline{6} & \underline{3} & \underline{18} & \boxed{\underline{0}} \\ \underline{8} & \underline{4} & 20 & 1 \\ 12 & 6 & \underline{17} & \underline{0} \\ 20 & 8 & \underline{16} & 1 \\ \underline{5} & \underline{3} & \underline{16} & \underline{0} \end{bmatrix} \qquad g^0 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$z = \begin{bmatrix} 10 & 6 & 18.5 & 1 \end{bmatrix}$$

How does this change affect H and M0?

H does not change. But M0 will increase, because it satisfies ***dimensional monotonicity***.

Change in Deprivation Matrix (1)

Censor data of non-poor

$$g^0 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad g^0(2) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad ci(2) = \begin{bmatrix} 4 \\ 2 \\ 2 \\ 0 \\ 4 \end{bmatrix} \qquad ci(2)/4 = \begin{bmatrix} 4/4 \\ 2/4 \\ 2/4 \\ 0 \\ 4/4 \end{bmatrix}$$

$$H = \frac{4}{5} = 0.80 \qquad A = \left(\frac{4}{4} + \frac{2}{4} + \frac{2}{4} + \frac{4}{4}\right)/4 = 0.75$$

$$M0 = \mu(g^0(k)) = \frac{12}{20} = 0.60$$

Change in Deprivation Matrix? (2)

$$X = \begin{bmatrix} \underline{6} & \underline{3} & \underline{18} & 1 \\ \underline{8} & \underline{4} & 20 & 1 \\ 12 & 6 & \underline{17} & \underline{0} \\ 20 & 8 & \underline{16} & 1 \\ \underline{5} & \underline{3} & \underline{16} & \underline{0} \end{bmatrix} \qquad X' = \begin{bmatrix} \underline{\boxed{4}} & \underline{3} & \underline{18} & 1 \\ \underline{8} & \underline{4} & 20 & 1 \\ 12 & 6 & \underline{17} & \underline{0} \\ 20 & 8 & \underline{16} & 1 \\ \underline{5} & \underline{3} & \underline{16} & \underline{0} \end{bmatrix} \qquad g^0 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$z = \begin{bmatrix} 10 & 6 & 18.5 & 1 \end{bmatrix}$$

How does this change affect H and M0?

H and M0 do not change, because they do not satisfy monotonicity.

Weighted Deprivation Matrix

Apply the weights to the Deprivation Matrix

$$g^0 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad g^0 = \begin{bmatrix} 1.5 & 1.5 & 0.5 & 0 \\ 1.5 & 1.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0 \\ 1.5 & 1.5 & 0.5 & 0.5 \end{bmatrix} \qquad ci = \begin{bmatrix} 3.5 \\ 3 \\ 1 \\ 0.5 \\ 4 \end{bmatrix}$$

Vector of weights $w = \begin{bmatrix} 1.5 & 1.5 & 0.5 & 0.5 \end{bmatrix}$

Identification

Who is poor when k=2?

$$g^0 = \begin{bmatrix} 1.5 & 1.5 & 0.5 & 0 \\ 1.5 & 1.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0 \\ 1.5 & 1.5 & 0.5 & 0.5 \end{bmatrix} \quad ci = \begin{bmatrix} 3.5 \\ 3 \\ 1 \\ 0.5 \\ 4 \end{bmatrix} \quad g^0(2) = \begin{bmatrix} 1.5 & 1.5 & 0.5 & 0 \\ 1.5 & 1.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.5 & 1.5 & 0.5 & 0.5 \end{bmatrix} \quad ci(2) = \begin{bmatrix} 3.5 \\ 3 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

Censor non-poor data

Aggregation

Poor if deprived in two dimensions, k = 2

$$g^0(2) = \begin{bmatrix} 1.5 & 1.5 & 0.5 & 0 \\ 1.5 & 1.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.5 & 1.5 & 0.5 & 0.5 \end{bmatrix} \quad ci(2) = \begin{bmatrix} 3.5 \\ 3 \\ 0 \\ 0 \\ 4 \end{bmatrix} \quad ci(2)/4 = \begin{bmatrix} 3.5/4 \\ 3/4 \\ 0 \\ 0 \\ 4/4 \end{bmatrix}$$

$$H = \frac{3}{5} = 0.60 \qquad \text{A} = \left(\frac{3.5}{4} + \frac{3}{4} + \frac{4}{4}\right)/3 = 0.875$$

$$M0 = \mu(g^0(2)) = \frac{10.5}{20} = 0.525$$

Tabita, Kenya | Rabiya, India | Stéphanie, Madagascar | Agathe, Madagascar | Dalma, Kenya | Ann-Sophia, Kenya | Valérie, Madagascar

Exercise 2

Multidimensional Data

$$\begin{array}{cc} & \text{Domains} \\ X = & \begin{bmatrix} 4 & 1 & 5 \\ 8 & 4 & 6 \\ 12 & 1 & 11 \\ 3 & 4 & 6 \\ 15 & 1 & 9 \\ 12 & 5 & 12 \end{bmatrix} \end{array} \quad \text{Persons}$$

$$z = \begin{bmatrix} 10 & 3 & 8 \end{bmatrix} \quad \text{Cut-offs}$$

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

$$X = \begin{bmatrix} \underline{4} & \underline{1} & \underline{5} \\ \underline{8} & 4 & \underline{6} \\ 12 & \underline{1} & 11 \\ \underline{3} & 4 & \underline{6} \\ 15 & \underline{1} & 9 \\ 12 & 5 & 12 \end{bmatrix} \qquad g^{0} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$z = \begin{bmatrix} 10 & 3 & 8 \end{bmatrix}$$

Poverty Cut-off: k=2

Poor if deprived in two dimensions, k = 2

Censored deprivation matrix

$$g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ci = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \quad g^0(2) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ci(2) = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

Censor data of non-poor

Adjusted Headcount, M0

$$g^0(2) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad ci(2) = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} \qquad ci(2)/3 = \begin{bmatrix} 1 \\ 2/3 \\ 0 \\ 2/3 \\ 0 \\ 0 \end{bmatrix}$$

$$H = \frac{3}{6} = 0.50 \qquad \text{A} = \left(1 + \frac{2}{3} + \frac{2}{3}\right)/3 = \frac{7}{9} \cong 0.778$$

$$M0 = \mu(g^0(2)) = \frac{7}{18} \cong 0.389$$

Adjusted Poverty Gap, M1

Censored deprivation matrix | Matrix of normalized gaps

$$X=\begin{bmatrix} \underline{4} & \underline{1} & \underline{5} \\ \underline{8} & 4 & \underline{6} \\ 12 & \underline{1} & 11 \\ \underline{3} & 4 & \underline{6} \\ 15 & \underline{1} & 9 \\ 12 & 5 & 12 \end{bmatrix} \qquad g^0(2)=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad g^1(2)=\begin{bmatrix} \frac{(10-4)}{10} & \frac{(3-1)}{3} & \frac{(8-5)}{8} \\ \frac{(10-8)}{10} & 0 & \frac{(8-6)}{8} \\ 0 & 0 & 0 \\ \frac{(10-3)}{10} & 0 & \frac{(8-6)}{8} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$z=\begin{bmatrix} 10 & 3 & 8 \end{bmatrix}$$

Adjusted Poverty Gap, M1

Matrix of normalized gaps

$$g^1(2) = \begin{bmatrix} 0.6 & 0.667 & 0.375 \\ 0.2 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0.7 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M1 = \mu(g^1(2)) \cong \frac{0.6 + 0.2 + 0.7 + 0.667 + 0.375 + 0.25 + 0.25}{18} = 0.169$$

Adjusted Poverty Gap, M1

Matrix of normalized gaps

$$g^1(2) = \begin{bmatrix} 0.6 & 0.667 & 0.375 \\ 0.2 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0.7 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G \cong \frac{0.6+0.2+0.7+0.667+0.375+0.25+0.25}{7} = 0.435$$

$$M1 = HAG \cong 0.50 \times 0.778 \times 0.435 = 0.169$$

Adjusted FGT Measure, M2

Matrix of normalized gaps

$$g^1(2) = \begin{bmatrix} 0.6 & 0.667 & 0.375 \\ 0.2 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0.7 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Matrix of squared normalized gaps

$$g^2(2) = \begin{bmatrix} 0.36 & 0.445 & 0.141 \\ 0.04 & 0 & 0.0625 \\ 0 & 0 & 0 \\ 0.49 & 0 & 0.0625 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M2 = \mu(g^2(2)) \cong 0.089$$

Adjusted FGT Measure, M2

Matrix of squared normalized gaps

$$g^2(2) = \begin{bmatrix} 0.36 & 0.445 & 0.141 \\ 0.04 & 0 & 0.0625 \\ 0 & 0 & 0 \\ 0.49 & 0 & 0.0625 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$S = \frac{0.36+.04+0.49+0.445+0.141+0.0625+0.0625}{7} \cong 0.229$$

$$M2 = HAS \cong 0.50 \times 0.778 \times 0.229 = 0.089$$

Interpretation

M0 = 0.389 – The poor in this society experience 38.9% of the total possible deprivations the society could experience.

M1 = 0.169 – The poor in this society experience 16.9% of the highest possible sum of normalised gaps that the society could experience.

M2 = 0.089 – The poor in this society experience 8.9% of the highest possible sum of squared normalised gaps that the society could experience.

Contribution of each Dimension to M0

$$g^0(2) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Contrib. of income} = \left(\frac{3}{6} \times \frac{1}{3}\right) \Big/ \left(\frac{7}{18}\right) = \frac{3}{7} \cong 0.429$$

$$\text{Contrib. of health} = \left(\frac{1}{6} \times \frac{1}{3}\right) \Big/ \left(\frac{7}{18}\right) = \frac{1}{7} \cong 0.143$$

$$\text{Contrib. of education} = \left(\frac{3}{6} \times \frac{1}{3}\right) \Big/ \left(\frac{7}{18}\right) = \frac{1}{7} \cong 0.429$$

Decomposition of M1 by groups

Matrix of normalized gaps

$$g^1(2) = \begin{bmatrix} 0.6 & 0.667 & 0.375 \\ 0.2 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0.7 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Sub-matrices of normalised gaps:

$$g^1{}_x(2) = \begin{bmatrix} 0.6 & 0.667 & 0.375 \\ 0.2 & 0 & 0.25 \\ 0 & 0 & 0 \end{bmatrix}$$

$$g^1{}_y(2) = \begin{bmatrix} 0.7 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Decomposition by groups

Group x

$$g^{1}{}_{x}(2)=\begin{bmatrix}0.6 & 0.667 & 0.375\\ 0.2 & 0 & 0.25\\ 0 & 0 & 0\end{bmatrix}$$

$$H_x=\frac{2}{3}\cong 0.667$$

$$A_x=\left(\frac{3}{3}+\frac{2}{3}\right)/2=\frac{5}{6}$$

$$G_x=\frac{2.092}{5}=0.418$$

$$M1_x=\frac{2.092}{9}\cong 0.232$$

Group y

$$g^{1}{}_{y}(2)=\begin{bmatrix}0.7 & 0 & 0.25\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$H_y=\frac{1}{3}=0.333$$

$$A_y=\frac{2}{3}=0.667$$

$$G_y=\frac{0.95}{2}=0.475$$

$$M1_y=\frac{0.95}{9}\cong 0.106$$

Decomposition by groups

Group x

$$g^1{}_x(2) = \begin{bmatrix} 0.6 & 0.667 & 0.375 \\ 0.2 & 0 & 0.25 \\ 0 & 0 & 0 \end{bmatrix}$$

Group y

$$g^1{}_y(2) = \begin{bmatrix} 0.7 & 0 & 0.25 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, what is the contribution of each group for the overall poverty gap?

Contrib. of group x $= \left(\frac{n_x}{n} \times M1_x\right)/M1 = (0.5 \times 0.232)/0.169 \cong 0.686$

Contrib. of group y $= \left(\frac{n_y}{n} \times M1_y\right)/M1 = (0.5 \times 0.106)/0.169 \cong 0.314$

Change in Deprivation Matrix (1)

$$X = \begin{bmatrix} \underline{4} & \underline{1} & \underline{5} \\ \underline{8} & 4 & \underline{6} \\ 12 & \underline{1} & 11 \\ \underline{3} & 4 & \underline{6} \\ 15 & \underline{1} & 9 \\ 12 & 5 & 12 \end{bmatrix} \qquad X' = \begin{bmatrix} \underline{4} & \underline{1} & \underline{5} \\ \underline{8} & \boxed{\underline{2}} & \underline{6} \\ 12 & \underline{1} & 11 \\ \underline{3} & 4 & \underline{6} \\ 15 & \underline{1} & 9 \\ 12 & 5 & 12 \end{bmatrix} \qquad g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boxed{1} & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$z = \begin{bmatrix} 10 & 3 & 8 \end{bmatrix}$$

H does not change. But M0, M1 and M2 increase, because they satisfy dimensional monotonicity.

Change in Deprivation Matrix (2)

$$X = \begin{bmatrix} \underline{4} & \underline{1} & \underline{5} \\ \underline{8} & 4 & \underline{6} \\ 12 & \underline{1} & 11 \\ \underline{3} & 4 & \underline{6} \\ 15 & \underline{1} & 9 \\ 12 & 5 & 12 \end{bmatrix} \qquad X' = \begin{bmatrix} \underline{4} & \underline{1} & \underline{5} \\ \underline{\boxed{4}} & 4 & \underline{6} \\ 12 & \underline{1} & 11 \\ \underline{3} & 4 & \underline{6} \\ 15 & \underline{1} & 9 \\ 12 & 5 & 12 \end{bmatrix} \qquad g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$z = \begin{bmatrix} 10 & 3 & 8 \end{bmatrix}$$

H and M0 do not change. But M1 and M2 increase, because they satisfy monotonicity.

New Weights - Identification

Apply the weights

Censor non-poor

$$g^{0\prime} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad g^{0} = \begin{bmatrix} 2 & 0.5 & 0.5 \\ 2 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 2 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad ci = \begin{bmatrix} 3 \\ 2.5 \\ 0.5 \\ 2.5 \\ 0.5 \\ 0 \end{bmatrix} \qquad g^{0}(2) = \begin{bmatrix} 2 & 0.5 & 0.5 \\ 2 & 0 & 0.5 \\ 0 & 0 & 0 \\ 2 & 0 & 0.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$H = \frac{3}{6} = 0.50$$

Adjusted Headcount, M0

Censored deprivation matrix

$$g^{0}(2)=\begin{bmatrix} 2 & 0.5 & 0.5 \\ 2 & 0 & 0.5 \\ 0 & 0 & 0 \\ 2 & 0 & 0.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M0 = \mu(g^0(2)) = \frac{8}{18} \cong 0.444$$

Adjusted Poverty Gap, M1

Apply weights to the matrix of normalized gaps

$$g^1(2) = \begin{bmatrix} 2(0.6) & 0.5(0.667) & 0.5(0.375) \\ 2(0.2) & 0 & 0.5(0.25) \\ 0 & 0 & 0 \\ 2(0.7) & 0 & 0.5(0.25) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M1 = \mu(g^1(2)) \cong 0.21$$

Adjusted FGT Measure, M2

Apply weights to the matrix of squared normalized gaps

$$g^2(2) = \begin{bmatrix} 2(0.36) & 0.5(0.445) & 0.5(0.141) \\ 2(0.04) & 0 & 0.5(0.0625) \\ 0 & 0 & 0 \\ 2(0.49) & 0 & 0.5(0.0625) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M2 = \mu(g^2(2)) \cong 0.119$$