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UNIVERSITY OF OXFORD

Summer School on Multidimensional Poverty

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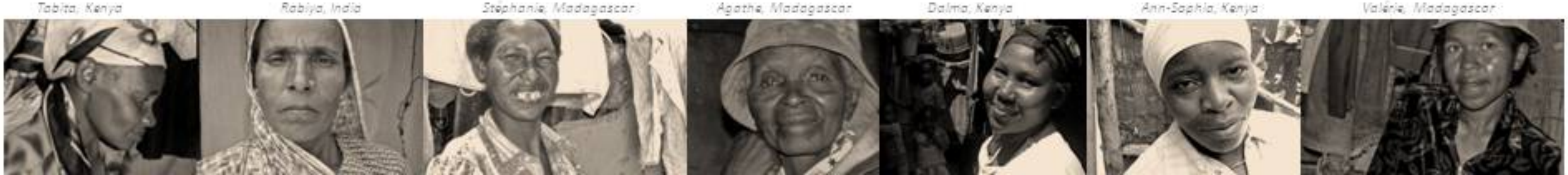

Sources

- Alkire, S., Foster, J.E., 2011. "Counting and Multidimensional Poverty Measurement," *Journal of Public Economics*

- See also Alkire, S., Foster, J.E., 2011. "Understandings and Misunderstandings of Multidimensional Poverty Measurement," *Journal of Economic Inequality*

Outline

- Motivation
- Multidimensional Data
- Identification
- Aggregation
- Examples

Challenge

- A government would like to create an official multidimensional poverty indicator
- **Desiderata**
  - It must understandable and easy to describe
  - It must conform to "common sense" notions of poverty
  - It must be able to target the poor, track changes, and guide policy.
  - It must be technically solid
  - It must be operationally viable
  - It must be easily replicable

• **What would you advise?**

Practical Steps

- **Select**
  - Purpose of the index (monitor, target, etc)
  - Unit of Analysis (indy, hh, cty)
  - Dimensions
  - Specific variables or indicators for each dimension
  - Whether variables or dimensions should be aggregated with others or left independent
  - Cutoff for each independent variable/dimension
  - Value of deprivation for each variable/dimension
  - **Identification method**
  - **Aggregation method**

This Presentation

- Assumes that the purpose, variables, dimensional cutoffs, values, etc. have been selected
- Focus here on the **methodology** for measuring poverty
  - **Identification**
  - **Aggregation**
- Note
  - Identification step is more challenging when there are many dimensions

AF Methodology: Overview

Identification of poor – Dual cutoffs

Deprivation cutoffs - each deprivation counts

Poverty cutoff - in terms of aggregate deprivation values

Aggregation across the poor – Adjusted FGT

Reduces to FGT in single variable case

Key Measure: Adjusted headcount ratio $M_0 = HA$

H is the share of the population identified as poor, or the *incidence*

A is the average breadth or multiplicity of deprivation people suffer at the same time, or the *intensity*

Note: Relies on joint distribution

Observations

- **Satisfies many desirable axioms**
  - joint restrictions on identification and aggregation
- **Decomposability** by sub-group
  - Key for targeting
- **Breakdown** by factor after identification
  - Key for policy coordination
- **Ordinality axiom**
  - Key for applicability

Multidimensional Poverty

Suppose *many* variables or dimensions

Question

How to evaluate poverty?

Answer 1

If variables can be meaningfully combined into some overall resource or achievement variable, *traditional methods can be used*

Traditional Unidimensional Methods

Variable – income

Identification – poverty line

Aggregation – Foster-Greer-Thorbecke ’84

Example Incomes = (7,3,4,8) poverty line z = 5

Deprivation vector $g^0 = (0,1,1,0)$

Headcount ratio $P_0 = m(g^0) = 2/4$

Normalized gap vector $g^1 = (0, 2/5, 1/5, 0)$

Poverty gap $= P_1 = m(g^1) = 3/20$

Squared gap vector $g^2 = (0, 4/25, 1/25, 0)$

FGT Measure $= P_2 = m(g^2) = 5/100$

Combining Variables

- Welfare aggregation
  - Construct each person's welfare level
  - Set cutoff and apply traditional poverty index
- Problems
  - Many assumptions needed
  - Cardinal utility?
  - Comparability across people?
    - Alkire and Foster (2010) "Designing the Inequality-Adjusted Human Development Index"

Combining Variables

Price aggregation

- Construct each person's expenditure level
- Set cutoff and apply traditional poverty index

Problems

- Many assumptions needed
- Ordinal and nonmarket variables problematic
- Link to welfare tenuous (local and unidirectional)
  - Foster, Majumdar, Mitra (1990) "Inequality and Welfare in Market Economies" *JPubE*

Caveats

- Note
  - Even if an aggregate exists, it may not be the right approach
- Idea
  - Aggregate resource approach signals what *could be*
    - The budget constraint
  - Does not indicate what *is*
    - The actual bundle purchased
  - Example
    - Consumption poverty is falling rapidly in India
    - Yet 45% of kids malnourished
- Problem
  - Aggregating may hide policy relevant information can't retrieve

Multidimensional Poverty

Suppose *many* variables or dimensions

Question

How to evaluate poverty?

Answer 2

If variables cannot be meaningfully aggregated into some overall resource or achievement variable, *new methods must be used*

Multidimensional Poverty

Some people go to great lengths to avoid this fact:

Blinders approach

- Limit consideration to a subset that *can* be aggregated, and use traditional methods.
- Key dimensions ignored OPHI Missing Dimensions

Marginal methods

- Apply traditional methods separately to each variable
- Ignores joint distribution
- Where did identification go? Alkire, Foster, Santos (2011) *JEI*

Multidimensional Data

- **Income:** "What is your income per capita in dollars a day?"
  - **$13 or above (bold is non-deprived)**
  - Below $13 (non-bold is deprived)
- **Schooling:** "How many years of schooling have you completed?"
  - **12 or more**
  - 1-11 years
- **Health:** "Would you say that in general your health is - Excellent, Very good, Good, Fair, Or Poor?"
  - **Excellent, very good or good**
  - Fair or poor
- **Social Service:** "Do you have access to social service?"
  - **Yes**
  - No
- For this illustration we will assume deprivations have equal value

Multidimensional Data

Matrix of well-being scores for *n* persons in *d* domains

$$
\begin{array}{cc}
 & \text{Domains} \\
y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix} & \text{Persons}
\end{array}
$$

Multidimensional Data

Matrix of well-being scores for *n* persons in *d* domains

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 & \text{Domains} \\
y = \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & 7 & 5 & 0 \\ 12.5 & 10 & 1 & 0 \\ 20 & 11 & 3 & 1 \end{bmatrix} & \text{Persons}
\end{array}
$$

$$
z \quad (13 \quad 12 \quad 3 \quad 1) \quad \text{Cutoffs}
$$

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
y = & \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

<u>These entries fall below cutoffs</u>

Deprivation Matrix

Replace entries: 1 if deprived, 0 if not deprived

$$
\begin{array}{cc}
\text{Domains} & \\
g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \text{Persons}
\end{array}
$$

Normalized Gap Matrix

Normalized gap = $(z_j - y_{ji})/z_j$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
y = & \begin{bmatrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{bmatrix} \quad \text{Persons} \\
 & \\
z & \begin{pmatrix} 13 & 12 & 3 & 1 \end{pmatrix} \quad \text{Cutoffs}
\end{array}
$$

Normalized Gap Matrix

Normalized gap = $(z_j - y_{ji})/z_j$ if deprived, 0 if not deprived

$$\begin{array}{ccc} & \text{Domains} & \\ \mathbf{g}^{1} = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0.08 & 0 & 0 \end{bmatrix} & \text{Persons} \end{array}$$

Squared Gap Matrix

Squared gap = $[(z_j - y_{ji})/z_j]^2$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
g^1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0.08 & 0 & 0 \end{bmatrix} & \text{Persons}
\end{array}
$$

Squared Gap Matrix

Squared gap = $[(z_j - y_{ji})/z_j]^2$ if deprived, 0 if not deprived

$$
\begin{array}{cc}
 & \text{Domains} \\
\mathbf{g}^2 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.176 & 0 & 1 \\ 0.002 & 0.029 & 0.449 & 1 \\ 0 & 0.006 & 0 & 0 \end{bmatrix} \text{Persons}
\end{array}
$$

Identification

$$
\begin{array}{cc}
 & \text{Domains} \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

Deprivation matrix

Identification – Counting Deprivations

$$
\begin{array}{cccc}
 & \text{Domains} & & c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & & \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix} & \text{Persons}
\end{array}
$$

Identification – Counting Deprivations

Q/ Who is poor?

$$\begin{array}{cc} & \text{Domains} \qquad\qquad c \\ g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ 2 \\ 4 \\ 1 \end{array} \quad \text{Persons} \end{array}$$

Identification – Union Approach

Q/ Who is poor?

A1/ Poor if deprived in any dimension $c_i \geq 1$

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ 2 \\ 4 \\ 1 \end{array} \quad \text{Persons}
\end{array}
$$

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Q/ Who is poor?

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\begin{array}{cccc}
 & \text{Domains} & c & \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ \underline{1} \end{array} & \text{Persons}
\end{array}
$$

Observations

Union approach often predicts very high numbers.

Charavarty et al '98, Tsui '02, Bourguignon & Chakravarty 2003 etc use the union approach

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ 2 \\ 4 \\ 1 \end{array} \quad \text{Persons}
\end{array}
$$

Identification – Intersection Approach

Q/ Who is poor?

A2/ Poor if deprived in all dimensions $c_i = d$

$$\begin{array}{cccc} & \text{Domains} & & c \\ g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & & \begin{array}{c} 0 \\ 2 \\ \underline{4} \\ 1 \end{array} & \quad \text{Persons} \end{array}$$

Observations

Demanding requirement (especially if d large)

Often identifies a very narrow slice of population

Atkinson 2003 first to apply these terms.

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$

$$\begin{array}{cc} \text{Domains} & c \\ g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ 2 \\ 4 \\ 1 \end{matrix} \end{array} \quad \text{Persons}$$

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $c_i \geq k$ (Ex: $k = 2$)

$$
\begin{array}{cc}
 & \text{Domains} \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\end{array}
\quad
\begin{array}{c} c \\ 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{array}
\quad \text{Persons}
$$

Identification – Dual Cutoff Approach

Q/ Who is poor?

A/ Fix cutoff k, identify as poor if $\mathbf{c_i \geq k}$ **(Ex: k = 2)**

$$
\begin{array}{cc}
& \text{Domains} \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\end{array}
\quad
\begin{array}{c} c \\ \mathbf{0} \\ \underline{\mathbf{2}} \\ \underline{\mathbf{4}} \\ \mathbf{1} \end{array}
\quad \text{Persons}
$$

Note

Includes both union ($k = 1$) and intersection ($k = d$)

Identification – Empirical Example

k =	H
Union 1	91.2%
2	75.5%
3	54.4%
4	33.3%
5	16.5%
6	6.3%
7	1.5%
8	0.2%
9	0.0%
Inters. 10	0.0%

Poverty in India for 10 dimensions

91% of population would be targeted using union

0% using intersection

Need something in the middle

(Alkire and Seth 2009)

Identification – Dual Cutoff Approach

Identification function is $\rho_k(y_i;z)$ where

$\rho_k(y_i;z) = 1$ if $\mathbf{c_i \geq k}$ (in which case i is poor)

and

$\rho_k(y_i;z) = 0$ if $\mathbf{c_i < k}$ (in which case i is nonpoor)

Aggregation

Censor data of nonpoor

$$\begin{array}{cccc} & \text{Domains} & c & \\ g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 1 \end{matrix} & \text{Persons} \end{array}$$

Aggregation

Censor data of nonpoor

$$\begin{array}{cccc} & \text{Domains} & c(k) & \\ g^0(k) = & \begin{bmatrix}0&0&0&0\\0&1&0&1\\1&1&1&1\\0&0&0&0\end{bmatrix} & \begin{matrix}0\\ \underline{2}\\ \underline{4}\\ 0\end{matrix} & \text{Persons}\end{array}$$

Aggregation

Censor data of nonpoor

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\begin{array}{cc}
 & \text{Domains} \quad\quad c(k) \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} \quad \text{Persons}
\end{array}
$$

Similarly for $g^1(k)$, etc

Aggregation – Headcount Ratio

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c(k) \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} \quad \text{Persons}
\end{array}
$$

Aggregation – Headcount Ratio

$$\begin{array}{lccc} & \text{Domains} & c(k) & \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons} \end{array}$$

Two poor persons out of four: **H = 1/2**

Critique

Suppose the number of deprivations rises for person 2

$$\begin{array}{ccccc} & \text{Domains} & & c(k) & \\ g^{0}(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \text{Persons} \end{array}$$

Two poor persons out of four: **H = 1/2**

Critique

Suppose the number of deprivations rises for person 2

$$
\begin{array}{ccc}
 & \text{Domains} & c(k) & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons}
\end{array}
$$

Two poor persons out of four: **H = 1/2**

Critique

Suppose the number of deprivations rises for person 2

$$
\begin{array}{c c c c}
 & \text{Domains} & c(k) & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{matrix} & \text{Persons}
\end{array}
$$

Two poor persons out of four: **H = 1/2**

No change!

Critique

Suppose the number of deprivations rises for person 2

$$
\begin{array}{cc}
 & \text{Domains} \qquad\qquad c(k) \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{c} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{array} \quad \text{Persons}
\end{array}
$$

Two poor persons out of four: **H = 1/2**

No change!

Violates 'dimensional monotonicity'

Aggregation

Return to the original matrix

$$\begin{array}{rccl} & \text{Domains} & c(k) & \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underline{1} & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{array} & \text{Persons} \end{array}$$

Aggregation

Return to the original matrix

Domains

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} c(k) \\ 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} \qquad \text{Persons}$$

Aggregation

Need to augment information

deprivation shares among poor

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \begin{array}{c} \\ 2/4 \\ 4/4 \\ \\ \end{array} & \text{Persons}
\end{array}
$$

Aggregation

Need to augment information

deprivation shares among poor

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \begin{matrix} \\ 2/4 \\ 4/4 \\ \\ \end{matrix} & \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA

$$
\begin{array}{ccc}
 & \text{Domains} & \begin{array}{cc} c(k) & c(k)/d \end{array} \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{cc} 0 & \\ \underline{2} & 2/4 \\ \underline{4} & 4/4 \\ 0 & \end{array} \quad \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\mathbf{m(g^0(k))}$

$$
\begin{array}{ccccc}
 & \text{Domains} & c(k) & c(k)/d & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \begin{array}{c} \\ 2/4 \\ 4/4 \\ \\ \end{array} & \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = **m(g⁰(k))** = 6/16 = .375

$$
\begin{array}{cccc}
 & \text{Domains} & c(k) & c(k)/d & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{matrix} & \begin{matrix} \\ 2/4 \\ 4/4 \\ \\ \end{matrix} & \text{Persons}
\end{array}
$$

A = average deprivation share among poor = 3/4

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = $\mathbf{m(g^0(k))}$ = **6/16 = .375**

$$\begin{array}{ccccc} & \text{Domains} & c(k) & c(k)/d & \\ g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 0 \\ \underline{2} \\ \underline{4} \\ 0 \end{array} & \begin{array}{c} \\ 2/4 \\ 4/4 \\ \\ \end{array} & \text{Persons} \end{array}$$

A = average deprivation share among poor = 3/4

Note: if person 2 has an additional deprivation, M_0 rises

Satisfies dimensional monotonicity

Aggregation – Adjusted Headcount Ratio

Adjusted Headcount Ratio = M_0 = HA = **m(g^0(k))** = 7/16 = .44

$$\begin{array}{ccccc}
 & \text{Domains} & c(k) & c(k)/d & \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{3} \\ \underline{4} \\ 0 \end{matrix} & \begin{matrix} \\ 3/4 \\ 4/4 \\ \\ \end{matrix} & \text{Persons}
\end{array}$$

A = average deprivation share among poor = 7/8

Note: if person 2 has an additional deprivation, M_0 rises

Satisfies dimensional monotonicity

Methodology - Adjusted Headcount Ratio

Denoted (ρ_k, M_0)

Interpretation: Similar to traditional gap

P_1 = HI and M_0 = HA

Applicability: Valid for ordinal data

Robust to monotonic transformations

Simplicity: Easy to calculate

Usefulness: Can be broken down by dimension

Robust: Dominance results

Grounded in Capability Approach: Characterization via freedom – P&X 1990

Expandable: If variables are all cardinal can go further

Pattanaik and Xu 1990 and M_0

- Freedom = the number of elements in a set.
- But does not consider the value of elements
- If dimensions are of intrinsic value and are usually valued, then *every deprivation* can be interpreted as a shortfall of intrinsic concern.
- The sum of deprivation values can be interpreted as the unfreedoms of each person
- Adjusted headcount ratio is then interpreted as a measure of unfreedoms across a population.

Aggregation: Adjusted Poverty Gap

Need to augment information of M_0 Use normalized gaps

$$
\begin{array}{cc}
\text{Domains} & \\
g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \text{Persons}
\end{array}
$$

Average **gap** across all deprived dimensions of the poor:

G= /6

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = $M_1 = M_0G = HAG$

$$\begin{array}{cc} & \text{Domains} \\ g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Persons} \end{array}$$

Average **gap** across all deprived dimensions of the poor:

G= /6

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = $M_1 = M_0G = HAG =$ **$m(g^1(k))$**

$$\begin{array}{cc} & \text{Domains} \\ g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Persons} \end{array}$$

Average **gap** across all deprived dimensions of the poor:

G= [illegible]/6

Aggregation: Adjusted Poverty Gap

Adjusted Poverty Gap = $M_1 = M_0 G = HAG =$ **$m(g^1(k))$**

$$\begin{array}{cc} & \text{Domains} \\ g^1(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Persons} \end{array}$$

Obviously, if in a deprived dimension, a poor person becomes even more deprived, then M_1 will rise.

Satisfies monotonicity

Aggregation: Adjusted FGT

Consider the matrix of squared gaps

$$\begin{array}{cc} & \text{Domains} \\ g^2(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ Persons} \end{array}$$

Aggregation: Adjusted FGT

Adjusted FGT is M_2 = **m(g^2(k))**

$$\begin{array}{cc} & \text{Domains} \\ g^2(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \text{Persons} \end{array}$$

Aggregation: Adjusted FGT

Adjusted FGT is $M_2 = \mathbf{m(g^2(k))}$

$$
\begin{array}{cc}
 & \text{Domains} \\
g^2(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

Satisfies a transfer axiom

Aggregation: Adjusted FGT Family

Adjusted FGT is $M_a = m(g^a(k))$ for $a \geq 0$

$$
\begin{array}{cc}
 & \text{Domains} \\
g^{\alpha}(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^{\alpha} & 0 & 1^{\alpha} \\ 0.04^{\alpha} & 0.17^{\alpha} & 0.67^{\alpha} & 1^{\alpha} \\ 0 & 0 & 0 & 0 \end{bmatrix} & \text{Persons}
\end{array}
$$

Theorem 1

For any deprivation values and cutoffs, the methodology $M_{k\alpha} = (\rho_k, M_{\alpha})$ satisfies: decomposability, replication invariance, symmetry, poverty and deprivation focus, weak and dimensional monotonicity, nontriviality, normalisation, and weak rearrangement for $\alpha \geq 0$; monotonicity for $\alpha > 0$; and weak transfer for $\alpha \geq 1$.

General Case

Previously assumed value of 1 for each deprivation

With sum being d

Now allow values or weights be general: $w_j > 0$

With sum being d

Identification and aggregation steps

1) Poverty cutoff k is compared to deprivation score or sum of deprivation values
2) Aggregation matrix now has columns weighted by deprivation values, and measures are found by taking mean of matrix

General Case - Matrices

$$
\begin{array}{cc}
 & \text{Domains} \\
\mathbf{g}^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\end{array}
\quad \text{Persons}
$$

Deprivation matrix with values given by

Weighting vector $\omega = \quad (1, 1, 1, 1)$

General Case - Matrices

$$
\begin{array}{cc}
 & \text{Domains} \\
\mathbf{g}^0 = & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \text{Persons}
\end{array}
$$

Suppose instead that we have

Weighting vector $\omega =$ **(.5 , 2, 1, .5)**

General Case - Matrices

$$
\begin{array}{cc}
 & \text{Domains} \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} \quad \text{Persons}
\end{array}
$$

Deprivation matrix with

Weighting vector $\omega = \quad (.5\ ,\ 2,\ \ 1,\ \ .5)$

General Case - Identification

$$
\begin{array}{ccc}
 & \text{Domains} & & \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ 2.5 \\ 4 \\ 2 \end{matrix} & \text{Persons}
\end{array}
$$

Deprivation matrix with

Weighting vector $\omega = \quad (.5\ ,\ 2,\ \ 1,\ \ .5)$

General Case - Identification

Who is poor?

$$
\begin{array}{ccc}
 & \text{Domains} & & \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ 2.5 \\ 4 \\ 2 \end{matrix} & \text{Persons}
\end{array}
$$

Deprivation matrix with

Weighting vector $\omega = (.5\,,\ 2,\ 1,\ .5)$

General Case - Identification

Who is poor?

Let k =2

$$
\begin{array}{cccc}
 & \text{Domains} & & \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} & \begin{matrix} 0 \\ \underline{2.5} \\ \underline{4} \\ \underline{2} \end{matrix} & \text{Persons}
\end{array}
$$

Deprivation matrix with

Weighting vector $\omega = (.5\ ,\ 2,\ 1,\ .5)$

General Case - Identification

Who is poor?

Let k =2.5

$$
\begin{array}{cc}
 & \text{Domains} \\
g^0 = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 2 & 0 & 0 \end{bmatrix} \begin{array}{c} 0 \\ \underline{2.5} \\ \underline{4} \\ 2 \end{array} \quad \text{Persons}
\end{array}
$$

Deprivation matrix with

Weighting vector ω = (.5 , 2, 1, .5)

Note: Impact identification

General Case - Aggregation

How much poverty? $M_0 = HA$

Domains

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 0 \\ \underline{2.5} \\ \underline{4} \\ 0 \end{matrix}$$

Persons

Deprivation matrix with

Weighting vector $\omega = \quad (.5\,,\ 2,\ \ 1,\ \ .5)$

$H = 1/2, \;\; A = 6.5/8$

General Case - Aggregation

How much poverty? $M_0 = HA = m(g^0(k)) = 6.5/16 = .406$

$$
\begin{array}{cc}
 & \text{Domains} \\
g^0(k) = & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & .5 \\ .5 & 2 & 1 & .5 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
\quad
\begin{array}{c} 0 \\ \underline{2.5} \\ \underline{4} \\ 0 \end{array}
\qquad \text{Persons}
$$

Deprivation matrix with

Weighting vector $\omega = \quad (.5\,,\ 2,\ \ 1,\ \ .5)$

$H = 1/2,\ \ A = 6.5/8$

Properties Reviewed

- Our methodology satisfies a number of typical properties of multidimensional poverty measures:
- *Symmetry*, *Normalization*, *Poverty Focus*, *Deprivation Focus*, *Scale invariance*, *Replication invariance*, *Weak Monotonicity*, *Weak Re-arrangement*

- M_0, M_1 and M_2 satisfy *Dimensional Monotonicity, Decomposability*

- M_1 and M_2 satisfy *Monotonicity* (for [illegible] > 0) – that is, they are sensitive to changes in the depth of deprivation in all domains with cardinal data.

- M_2 satisfies *Weak Transfer* (for [illegible] > 1).

Implementations: Choosing k

- Depends on: purpose of exercise, data, and weights
  - "In the final analysis, how reasonable the identification rule is depends, *inter alia,* on the attributes included and how imperative these attributes are to leading a meaningful life." (Tsui 2002 p. 74).
- E.g. a measure of Human Rights; data good = union
- Targeting: according to category (poorest 5%). Or budget (we can cover 18% - who are they?)
- Poor data, or people do not value all dimensions: $k<d$
- Some particular combination (e.g. the intersection of income deprived *and* deprived in any other dimension)

Implementation: Robustness for *k*

- *Theorem 2* Where a and a' are the respective attainment vectors for y and y' in Y $(a_i = d - c_i)$, we have:
- (i) $y\ H\ y'$ $a\ FD\ a'$
- (ii) $a\ FD\ a'$ $y\ M_0\ y'$ $a\ SD\ a'$, and the converse does not hold.

(i) akin to Foster Shorrocks: first order dominance over attainment vectors ensures that multidimensional headcount is lower (or no higher) for all possible values of k – and the converse is also true.

(ii) shows that M_0 is implied by first order dominance, and implies second order, in turn

Example - Indonesia

Deprivation	Percentage of Population
Expenditure	30.1%
Health (*BMI*)	17.5%
Schooling	36.4%
Drinking Water	43.9%
Sanitation	33.8%

Example - Indonesia

Number of Deprivations	Percentage of Population
One	26%
Two	23%
Three	17%
Four	8%
Five	2%

Identification as *k* varies

Cutoff *k*	**Percentage of Population**
1	74.9%
2	49.2%
3	26.4%
4	9.7%
5	1.7%

And interpretation?

Equal Weights			
Measure	k=1 (Union)	k=2	k=3 (Intersection)
H	0.577	0.225	0.039
M_0	0.280	0.163	0.039
M_1	0.123	0.071	0.016
M_2	0.088	0.051	0.011

General Weights

Measure	$k = 0.75$ (Union)	$k = 1.5$	$k = 2.25$	$k = 3$ (Intersection)
H	0.577	0.346	0.180	0.039
M_0	0.285	0.228	0.145	0.039
M_1	0.114	0.084	0.058	0.015
M_2	0.075	0.051	0.036	0.010

And interpretation?

Equal Weights			
Measure	k=1 (Union)	$\boldsymbol{k}$=2	k=3 (Intersect
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M_0 = H for intersection

General Weights

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And interpretation?

If all persons have *maximal* deprivation, then G=1, so M_0 = M_1. **Low gap** if M_0 is **higher than** M_1.

M_0 = H for intersection

eights

	k=1 (Union)	k=2	k=3 (Intersect
	0.577	0.225	0.039
M_0	0.280	0.163	0.039
M_1	0.123	0.071	0.016
M_2	0.088	0.051	0.011

General Weights

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And interpretation?

If all persons have *maximal* deprivation, then G=1, so M_0 = M_1. **Good** if M_0 is **different** from M_1.

M_0 = H for intersection

…hts			
	k=1 (Union)	$\boldsymbol{k}$=2	k=3 (Intersect…)
	0.577	0.225	0.039
	0.280	0.163	0.039
M_1	0.123	0.071	0.016
M_2	0.088	0.051	0.011

General Weights

Measure	k = 0.75 (Union)	k = 1.5	k = 2.25	k = 3 (Intersection)
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M_2	0.075	0.051	0.036	0.010

Weights affect relevant k **values.**

AF Method: Decompositions

By Population Subgroup

M_α Poverty

H Headcount

A Intensity

Post-identification: By Dimension

Censored Headcount

Percentage Contribution

All draw on <u>censored</u> matrix

misunderstood

Informal Glossary of Terms

Deprivation: if $y_{id} < z$ person i is **deprived** in y_d

Poverty: if $c_i \leq k$ person i is poor.

Deprivation cutoffs: the z cutoffs for each dimension

Poverty cutoff: the overall cutoff k

Dimension: for AF – a column in the matrix having its own deprivation cutoff (sometimes called an 'indicator')

Joint distribution: showing the simultaneous or coupled deprivations a person/hh has