HDCA-OPHI Summer School, Delft, 2011
Exercise on first-order stochastic dominance

The following are the joint absolute frequencies of two wellbeing attributes in three societies:

$$A = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 150 & 50 \\ x_2 & 50 & 250 \end{array} ; \; B = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 160 & 160 \\ x_2 & 160 & 320 \end{array} ; \; C = \begin{array}{ccc} & y_1 & y_2 \\ x_1 & 60 & 45 \\ x_2 & 45 & 150 \end{array}$$

The purpose of the exercise is to perform first-order stochastic dominance tests considering both cumulative and survival functions, in order to rank the three societies. We are going to use the test for ordinal variables developed by Yalonetzky (2011). The exercise is going to be guided throughout. The exercise can be solved using Excel, STATA or a combination of both.

1. Compute the joint distributions for the three countries.
2. Compute the multidimensional headcounts assuming that the poverty lines are (x_1, y_1) and that the union approach holds.
3. Compute the multidimensional headconts assuming the same poverty lines, but now the intersection approach holds.
4. Compute the cumulative distributions, $F(x, y)$, and the survival functions, $\overline{F}(x, y)$. To do this computation pre-multiply each matrix by a lower triangular matrix of 1's and then post-multiply by an upper triangular matrix of 1's. Before testing, what is the poverty ordering that could ensue if the distributional differences between the countries are proved to be statistically significant?
5. Now we implement the testing procedure. First, for the three possible comparisons, compute $F^i(x, y) - F^j(x, y)$ and compute $\overline{F}^i(x, y) - \overline{F}^j(x, y)$. These matrices have the numerators of our z-statistics. Now the denominators...
6. For each compared pair separately (i.e. A and B, B and C, A and C), compute the pool joint distribution matrices, P^{AB}, P^{BC}, P^{AC}
7. Compute of the covariance matrix of the pooled joint distributions:

$$\Omega = \left(\frac{N^i + N^j}{N^i N^j}\right) \begin{bmatrix} p_{11}(1-p_{11}) & -p_{11}p_{12} & \cdots & \\ -p_{11}p_{12} & p_{12}(1-p_{12}) & \ddots & \vdots \\ \vdots & \ddots & p_{21}(1-p_{21}) & -p_{21}p_{22} \\ & \cdots & -p_{21}p_{22} & p_{22}(1-p_{22}) \end{bmatrix}$$

8. Compute the covariance matrix of $F^i(x,y) - F^j(x,y)$ and $\overline{F}^i(x,y) - \overline{F}^j(x,y)$ (L_2 is a 2-dimension lower triangular matrix of 1s) :

$$\begin{aligned} var\Delta F &= L_2 \otimes L_2 \Omega L_2' \otimes L_2' \\ var\Delta \overline{F} &= L_2' \otimes L_2' \Omega L_2 \otimes L_2 \end{aligned}$$

9. Construct the matrices of z-statistics dividing the elements of $F^i(x,y) - F^j(x,y)$ and $\overline{F}^i(x,y) - \overline{F}^j(x,y)$ by their respective standard errors which are the square roots of the *diagonal* elements of $var\Delta F$ and $var\Delta\overline{F}$, respectively. For instance:

$$z_{F11}^{AB} = \frac{F^A(1,1) - F^B(1,1)}{\sqrt{var\Delta F_{11}}}$$

10. Rejection criterion for a test of strict dominance (e.g. $H_0 : \Delta F \geq 0$ against $H_1 : \Delta F < 0$): All the z-statistics must have the same sign and be statistically significant.