Summer School on Multidimensional Poverty Analysis

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Properties of Multidimensional Poverty Measures

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Session IV

Tabita, Kenya · Rabiya, India · Stéphanie, Madagascar · Agathe, Madagascar · Dalma, Kenya · Ann-Sophia, Kenya · Valérie, Madagascar

Main Sources of this Lecture

- Alkire S., J. E. Foster, S. Seth, S. Santos, J. M. Roche, P. Ballon, Multidimensional Poverty Measurement and Analysis, Oxford University Press, forthcoming, (Chs 2.2, 2.3, 2.5).
- Bourguignon and Chakravarty (2003): The Measurement of Multidimensional Poverty
- Alkire and Foster (2007, 2011): Counting and Multidimensional Poverty Measurement
- Please see the reading list for other

Preliminaries

- Reference population
  - We refer as '*Society*' (e.g. country, region etc.)
- Unit of measurement
  - We refer as '*Person*' (could be households)
  - Suppose there are n persons in the society (n may vary)
- Variables or dimensions for assessing poverty
  - We refer as '*Space*'
  - Suppose there are d such variables (fixed set)

Preliminaries

- Achievement: performance of a person in a dimension
  - x_{ij}: Achievement of person i (=1,…,n) in dimension j (=1,…, d)
- Achievement matrix
  - Summarizes achievements of all n persons in d dimensions
- Achievement vector of a Person
  - May contain achievements in d different dimensions
    - Standard of living, knowledge, quality of health

$$\begin{array}{ccc} & \text{Dimensions} & \\ X = & \begin{bmatrix} x_{11} & \cdots & x_{1d} \\ x_{21} & \cdots & x_{2d} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nd} \end{bmatrix} & \text{Persons} \end{array}$$

Preliminaries

- A typical achievement matrix (with 4 dimensions)

$$
\begin{array}{cc}
 & \begin{array}{cccc} \textbf{Income} & \textbf{Years of Education} & \textbf{Sanitation (Improved?)} & \textbf{Access to Electricity} \end{array} \\
X = & \left|\begin{array}{cccc} 700 & 14 & \text{Yes} & \text{Yes} \\ 300 & 13 & \text{Yes} & \text{No} \\ 400 & 10 & \text{No} & \text{No} \\ 800 & 11 & \text{Yes} & \text{Yes} \end{array}\right| \begin{array}{l} \textbf{Person 1} \\ \textbf{Person 2} \\ \textbf{Person 3} \\ \textbf{Person 4} \end{array}
\end{array}
$$

Preliminaries

- Matrix X summarizes the joint distribution of 'd' dimensions across 'n' individuals

- Row vector $x_{i\bullet} = (x_{i1},\ldots, x_{id})$ summarizes the achievements of person i in all d dimensions

- Column vector $x_{\bullet j} = (x_{1j},\ldots, x_{nj})$ summarizes the achievements in dimension j of all n persons

Measurement

Measurement of multidimensional poverty involves two major steps like unidimensional measurement

- Identification
- Aggregation

First Step: Identification

- Identification: Who is multidimensionally poor?
  - An '*identification function*', $\rho(\bullet)$, decides who should be multidimensionally poor

    $\rho(x_{i\bullet}) = 1$ if person i is multidimensionally poor
    $\rho(x_{i\bullet}) = 0$ if person i is not multidimensionally poor

  - There can be two types of identification Approaches
    - Censored Achievement Approach (Includes Counting)
    - Aggregate Achievement Approach

First Step: Identification

- Identification: Censored Achievement Approach

  - First stage: Determine whether individuals are deprived in each dimension
  - Second stage: Identify if someone is poor based on an identification function (criterion)

  - Examples:
    - Union criterion (if deprived in at least one dimension)
    - Intersection criterion (if deprived in all dimensions)
    - Intermediate criterion

First Step: Identification

Recall the achievement matrix

The deprivation cutoff vector is $z = (z_1, \ldots z_d)$

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$X =$	700	14	Yes	Yes	**Person 1**
	300	13	Yes	No	**Person 2**
	400	10	No	No	**Person 3**
	800	11	Yes	Yes	**Person 4**

First Step: Identification

Recall the achievement matrix

The deprivation cutoff vector is $z = (z_1, \ldots z_d)$

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$X =$	700	14	Yes	Yes	**Person 1**
	300	13	Yes	No	**Person 2**
	400	10	No	No	**Person 3**
	800	11	Yes	Yes	**Person 4**
$z =$	**500**	**12**	**Yes**	**Yes**	

First Step: Identification

Example: Construct the ‘Deprivation Matrix’

Replace entries: 1 if deprived, 0 if not deprived

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$X =$	700	14	Yes	Yes	**Person 1**
	300	13	Yes	No	**Person 2**
	400	10	No	No	**Person 3**
	800	11	Yes	Yes	**Person 4**
$z =$	**500**	**12**	**Yes**	**Yes**	

First Step: Identification

Example: Construct the 'Deprivation Matrix'

Replace entries: 1 if deprived, 0 if not deprived

$$g^0 = \begin{array}{cccc|l} \text{Income} & \text{Years of Education} & \text{Sanitation (Improved?)} & \text{Access to Electricity} & \\ \hline 0 & 0 & 0 & 0 & \text{Person 1} \\ \mathbf{1} & 0 & 0 & \mathbf{1} & \text{Person 2} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \text{Person 3} \\ 0 & \mathbf{1} & 0 & 0 & \text{Person 4} \end{array}$$

$$z = \begin{vmatrix} \mathbf{500} & \mathbf{12} & \mathbf{Yes} & \mathbf{Yes} \end{vmatrix}$$

These entries fall below cutoffs

First Step: Identification

Example: Equivalently 'Censored Deprivation Matrix'

$$x_{ij}^{*} = x_{ij} \text{ if } x_{ij}^{*} < z_j \text{ and } x_{ij}^{*} = z_j \text{ if } x_{ij}^{*} \geq z_j$$

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$X^{*} =$	500	12	Yes	Yes	Person 1
	300	12	Yes	No	Person 2
	400	10	No	No	Person 3
	500	11	Yes	Yes	Person 4
$z =$	500	12	Yes	Yes	

These entries fall below cutoffs

First Step: Identification

- <u>Identification</u>: Aggregate Achievement Approach

  – A person is identified as poor if her aggregate achievement falls below an aggregate poverty line

  – Let the aggregation function be denoted by ϕ

  – Then,

$$\rho(x_{i\bullet}) = 1 \qquad \text{if } \phi(x_{i\bullet}) < \underline{\phi}$$
$$\rho(x_{i\bullet}) = 0 \qquad \text{if } \phi(x_{i\bullet}) \geq \underline{\phi}$$

  – Example: Consumer Expenditure Approach

Note: No deprivation matrix was created in this situation

Second Step: Aggregation

- Aggregation: How poor is the society?
  - Based on the identification criterion, this step constructs an index of poverty $P(X;z)$ summarizing the information of the poor (*a censored matrix can be created just as in the unidimensional framework*)

Classification of Properties

- Invariance Properties
- Dominance Properties
- Subgroup Properties
- Technical Properties

- Two types
  - Natural extensions of the unidimensional properties
  - Axioms specific to the multidimensional context

Invariance Properties

Symmetry: If matrix Y is obtained from matrix X by a *permutation* of achievements and the deprivation cutoff vector z remains unchanged, then $P(Y;z) = P(X;z)$

Y is obtained from X by a ***permutation*** of incomes if $X = \Pi Y$, where Π is a permutation matrix.

Example:

$$Y = \Pi X = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 4 \\ 4 & 4 & 2 \\ 8 & 6 & 3 \end{bmatrix}$$

Invariance Properties

Replication Invariance: If matrix Y is obtained from matrix X by a *replication* and the deprivation cutoff vector z remains unchanged, then $P(Y;z) = P(X;z)$

Y is obtained from X by a replication if each person's achievement vector in X is simply repeated a finite number of times

Example:

$$X = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} \qquad Y = \begin{bmatrix} 4 & 4 & 2 \\ 4 & 4 & 2 \\ 3 & 5 & 4 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \\ 8 & 6 & 3 \end{bmatrix}$$

Invariance Properties

Scale Invariance: If all achievements in matrix X and the deprivation cutoff vector z are post multiplied by any diagonal matrix Λ, then $P(X\Lambda;z\Lambda) = P(X;z)$.

Example: $X = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix}$ $z = \begin{bmatrix} 4 & 5 & 3 \end{bmatrix}$ $\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$X\Lambda = \begin{bmatrix} 1(4) & 2(4) & 3(2) \\ 1(3) & 2(5) & 3(4) \\ 1(8) & 2(6) & 3(3) \end{bmatrix} \quad z\Lambda = \begin{bmatrix} 1(4) & 2(5) & 3(3) \end{bmatrix}$$

Useful but May be controversial!

Invariance Properties

Focus: Unlike in the unidimensional framework, there are two types of focus axiom

(Type I) Focus on those identified as multidimensionally poor' (*we are not interested in those who are not multidimensionally poor*)

(Type II) Focus on dimensions where multidimensionally poor are deprived (*we are not interested in dimensions in which they are not deprived*)

Invariance Properties

<u>Poverty Focus</u> (Type I): If Y is obtained from X by *an increment to a non-poor person's achievement* and the deprivation cutoff vector remains unchanged, then $P(Y;z) = P(X;z)$

<u>Example</u>: $X = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 4 \end{bmatrix}, z = (5,6,4),$ and $g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Person 3 is not multidimensionally poor, does it matter if he/she experiences an increase in any of the dimensions?

Invariance Properties

Deprivation Focus (Type II): If Y is obtained from X by *an increment in achievements in non-deprived dimensions*, then $P(Y;z) = P(X;z)$. [Deprived vs. Poor]

Example: $X = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 4 \end{bmatrix}$, $z = (5,6,4)$, and $g^0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Suppose person 2 is considered multidimensionally poor, does it matter if he/she experiences an increment in the third dimension in which he/she is not deprived?

Invariance Properties

Focus Axioms and Types of Identification

Each of the two focus axioms is attributed to each identification technique introduced earlier

- Poverty focus is attributed to the Aggregated Achievement Approach
- Deprivation focus is attributed to the Censored Achievement Approach

Invariance Properties

Ordinality: If Y and z' are obtained from X and z as equivalent representation, then $P(Y;z') = P(X;z)$

Equivalent representation: A monotonic transformation of each dimension and its deprivation cutoff is taken

Example:

$$X = \begin{bmatrix} 4 & 9000 & 0 \\ 3 & 9000 & 1 \\ 8 & 15000 & 1 \end{bmatrix}, z = (5, 10k, 1) \rightarrow Y = \begin{bmatrix} 48 & 3.95 & 3 \\ 36 & 3.95 & 5 \\ 96 & 4.18 & 5 \end{bmatrix}, z' = (10, 4, 5)$$

$$[12x, \log(x), x+2]$$

Invariance Properties

Why is the ordinality property important?

Practical importance – real world data

– Monotonic transformations are sometimes inevitable

Scale of variables (Ch 2.3)

– Ratio scale: $y_{ij} = ax_{ij}$, $a > 0$ – Divide, Multiply (e.g. income)
– Interval scale: $y_{ij} = ax_{ij} + b$, $a > 0$ – Add, subtract (e.g. z-score)
– Ordinal: $y_{ij} = f(x_{ij})$, f is increasing – order known (e.g. access)
– Nominal or categorical: No arithmetic operator, no order (gender, ethnicity)

Dominance Properties

Monotonicity: If Y is obtained from X by a *deprived increment* among the poor and the poverty line remains unchanged, then $P(Y,z) < P(X,z)$

Y is obtained from X by a *deprived increment* if there is an increment in a deprived achievement of a multidimensionally poor

Example: $X = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix}, z = (5 \quad 6 \quad 4),\ Y = \begin{bmatrix} 4 & 4 & 3 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix}$

Person 1 is multidimensionally poor, and experiences an improvement in the third dimension.

Dominance Properties

Dimensional Monotonicity: If Y is obtained from X by a *dimensional increment among the poor*, then $P(Y;z)<P(X,z)$

Y is obtained from X by a *dimensional increment among the poor* if due to an increment in a deprived achievement of a poor, he or she becomes non-deprived in that dimension

Example: $X = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix}, z = (5 \;\; 6 \;\; 4), Y = \begin{bmatrix} 4 & 4 & 2 \\ 3 & 6 & 4 \\ 8 & 6 & 3 \end{bmatrix}$

Suppose person 2 is considered multidimensionally poor, and experiences an increment in the second dimension and is no longer deprived in it

Dominance Properties

Transfer in unidimensional context: If y is obtained from x by a progressive transfer among the poor, then $P(y;z) < P(x;z)$

Recall if income is transferred from a person to another who is not richer than the former, keeping mean income same, the transfer is called a *progressive transfer*

This is also known as *Pigou-Dalton* transfer principle

Example: $z = 10$, $x = (9,4,15,8)$; $y = (9,5,15,7)$

Dominance Properties

Bistochastic matrix (B): A matrix whose row elements and column elements sum up to one

Example: A general bistochastic matrix $\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$

Multiply a vector by a bistochastic matrix

$$\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 16 \end{bmatrix} = \begin{bmatrix} 7.6 \\ 8.8 \\ 11.6 \end{bmatrix}$$

Dominance Properties

Bistochastic matrix (B): A matrix whose row elements and column element sum up to one

Example: What bistochastic matrix is used to obtain $y = (9,5,15,7)$ from $x = (9,4,15,8)$?

It is $$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0.25 \\ 0 & 0 & 1 & 0 \\ 0 & 0.25 & 0 & 0.75 \end{bmatrix}$$

Dominance Properties

<u>Uniform Majorization</u> (UM): Y is obtained from X by a *Uniform Majorization among the poor* (an averaging of achievements among the poor) if $Y = BX$, where B is an $n \times n$ bistochastic matrix but not a permutation matrix, and $b_{ii}=1$ for every non-poor person i in Y.

$$X = BY = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 3.5 & 4.5 & 3 \\ 3.5 & 4.5 & 3 \\ 8 & 6 & 3 \end{bmatrix}, \text{ and } z = [5 \;\; 6 \;\; 5]$$

Achievements of the first two persons (poor) were smoothed

Dominance Properties

Transfer: If Y is obtained from X by a uniform majorization among the poor (an averaging of achievements among the poor), then $P(Y;z) < P(X;z)$.

Weak Transfer: If Y is obtained from X by a uniform majorization among the poor (an averaging of achievements among the poor), then $P(Y;z) \leq P(X;z)$.

Note: The stronger version is not compatible with the focus axioms

Dominance Properties

Rearrangements

$$X = \begin{array}{c} \begin{array}{ccc} \text{Income} & \text{Education} & \text{Health} \end{array} \\ \begin{bmatrix} 7 & 7 & 2 \\ 3 & 3 & 8 \\ 10 & 10 & 12 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Person 1} \\ \text{Person 2} \\ \text{Person 3} \end{array} \qquad Y = \begin{array}{c} \begin{array}{ccc} \text{Income} & \text{Education} & \text{Health} \end{array} \\ \begin{bmatrix} 7 & 7 & 8 \\ 3 & 3 & 2 \\ 10 & 10 & 12 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Person 1} \\ \text{Person 2} \\ \text{Person 3} \end{array}$$

$$z = \begin{bmatrix} 4 & 5 & 3 \end{bmatrix}$$

Is the pattern of poverty same in both societies?

If not, what is the difference?

Dominance Properties

Both matrices have the same marginal distribution for each dimension, different joint distribution

Require a property sensitive to joint distribution (Atkinson & Bourguignon, 1982; Boland & Proschan, 1988).

The property is intrinsic to the multidimensional case

Dominance Properties

$$X = \begin{bmatrix} 7 & 7 & 2 \\ 3 & 3 & 8 \\ 10 & 10 & 12 \end{bmatrix} \qquad Y = \begin{bmatrix} 7 & 7 & 8 \\ 3 & 3 & 2 \\ 10 & 10 & 12 \end{bmatrix}$$

Ways to call the data transformation:

From X to Y:

Association increasing rearrangement

Correlation-increasing transfer

Correlation increasing switch

From Y to X:

Association decreasing rearrangement

Dominance Properties

Question…

How do you think poverty should change under an *association decreasing rearrangement*?

Dominance Properties

$$X = \begin{bmatrix} 7 & 7 & 2 \\ 3 & 3 & 8 \\ 10 & 10 & 12 \end{bmatrix} \qquad Y = \begin{bmatrix} 7 & 7 & 8 \\ 3 & 3 & 2 \\ 10 & 10 & 12 \end{bmatrix}$$

- If dimensions are ***substitutes***, poverty should ***decrease***
- If dimensions are ***complements***, poverty should ***increase***
- If dimensions are neither substitute nor complements, poverty should ***not change***.

Bourguignon and Chakravarty (2003)

Dominance Properties

Association decreasing deprivation rearrangement among the poor

In this case, the rearrangement takes place among the poor and only among their deprived dimensions

Dominance Properties

Example: Not an association decreasing deprivation rearrangement among the poor

$$Y = \begin{bmatrix} 7 & 7 & 8 \\ 3 & 3 & 2 \\ 10 & 10 & 7 \end{bmatrix} \quad X = \begin{bmatrix} 7 & 7 & 7 \\ 3 & 3 & 2 \\ 10 & 10 & 8 \end{bmatrix} \quad z = \begin{bmatrix} 4 & 5 & 3 \end{bmatrix}$$

Example: An association decreasing deprivation rearrangement among the poor

$$Y = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 3 & 2 \\ 10 & 10 & 12 \end{bmatrix} \quad X = \begin{bmatrix} 3 & 4 & 8 \\ 2 & 3 & 2 \\ 10 & 10 & 8 \end{bmatrix} \quad z = \begin{bmatrix} 4 & 5 & 3 \end{bmatrix}$$

Dominance Properties

Deprivation Rearrangement (Substitutes): If Y is obtained from X by an association-decreasing deprivation rearrangement among the poor, then $P(Y;z) < P(X;z)$.

Converse Deprivation Rearrangement (Complements): If Y is obtained from X by an association decreasing rearrangement among the poor, then $P(Y;z) > P(X;z)$.

Weaker versions with $\geq$ and $\leq$, respectively

Dominance Properties

Dimensional Transfer: If Y is obtained from X by a dimensional rearrangement among the poor, then $P(Y;z) < P(X;z)$

Dimensional rearrangement among the poor is a association decreasing rearrangement that switches a deprivation with a non-deprivation between two poor persons

Example: Deprivation rearrangement among the poor but not dimensional rearrangement

$$Y = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 3 & 2 \\ 10 & 10 & 12 \end{bmatrix} \qquad X = \begin{bmatrix} 3 & 4 & 8 \\ 2 & 3 & 2 \\ 10 & 10 & 8 \end{bmatrix} \qquad z = \begin{bmatrix} 4 & 5 & 3 \end{bmatrix}$$

Subgroup Properties

- Subgroups (mutually exclusive and exhaustive)
  - The population size of Matrix X is n
  - Matrix X is divided into two population subgroups
    - Group 1: X^1 with population size n^1
    - Group 2: X^2 with population size n^2
    - Note that $n = n^1 + n^2$

$$X = \begin{array}{c} \begin{array}{ccc} \textbf{Inc} & \textbf{Edu} & \textbf{Hel} \end{array} \\ \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} \end{array} \begin{array}{l} \\ \textbf{Person 1} \\ \textbf{Person 2} \\ \textbf{Person 3} \end{array}$$

Subgroup Properties

Subgroup Consistency: If Y is obtained from X, such that (i) $P(Y^1;z) > P(X^1;z)$, (ii) $P(Y^2;z) = P(X^2;z)$, and (iii) the population size of each group remains unchanged, then $P(Y;z) > P(X;z)$

Population Subgroup Decomposability: A poverty measure is additive decomposable if

$$P(X) = \frac{n^1}{n} P(X^1) + \frac{n^2}{n} P(X^2)$$

Recall: *decomposability implies subgroup consistency, but the converse does not hold*

Subgroup Properties

$$X = \begin{array}{cl} \begin{array}{ccc} \textbf{Inc} & \textbf{Edu} & \textbf{Hel} \end{array} & \\ \begin{bmatrix} 4 & 4 & 2 \\ 3 & 5 & 4 \\ 8 & 6 & 3 \end{bmatrix} & \begin{array}{l} \textbf{Person 1} \\ \textbf{Person 2} \\ \textbf{Person 3} \end{array} \end{array}$$

<u>Dimensional Breakdown</u>: It is a *purely multidimensional* concept, where the overall poverty can be expressed as an weighted average of dimensional deprivations of the poor

Subgroup Properties

Dimensional Breakdown: If $P_j(x_{\cdot j};z)$ summarizes the post-identification deprivation profile of the society in dimension j

Then, $P(X;z) = w_1 P_1(x_{\cdot 1};z) + \cdots + w_d P_d(x_{\cdot d};z)$

where w_j is the weight (normalized) assigned to dimension j

For *union criterion*, it is the factor decomposability by Chakravarty, Mukherjee and Ranade (1998)

$$P_j(x_{\cdot j};z) = P_j(x_{\cdot j};z_j)$$

Technical Properties

- <u>Normalization</u>
  - A poverty measure should be bounded between 0 and 1
- <u>Continuity</u>
  - A poverty measure should be continuous on the achievements
- <u>Non-triviality</u>
  - A poverty measure should take at least two distinct values

Thank you. Questions.