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Summer School on Multidimensional Poverty Analysis

11–23 August 2014

**Oxford Department of International Development
Queen Elizabeth House, University of Oxford**

Tabita, Kenya



Rabiya, India



Stéphanie, Madagascar



Agatha, Madagascar



Dalma, Kenya



Ann-Sophie, Kenya



Valérie, Madagascar



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Population Subgroup Decomposition and Dimensional Breakdown

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18 August 2014

Session III

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Focus of This Lecture

Discuss how overall poverty can be decomposed across different population subgroups, and show maps for visual policy analysis

- Population subgroup decomposability

Discuss how poverty can be decomposed to understand the prevalence of deprivations in different dimensions among the poor

- Dimensional breakdown

Main Source of this Lecture

- Alkire S., J. E. Foster, S. Seth, S. Santos, J. M. Roche, P. Ballon, Multidimensional Poverty Measurement and Analysis, Oxford University Press, forthcoming, (Chs 5.5.2 and 5.5.3).

Population Subgroups

- Subgroups (mutually exclusive and exhaustive)
 - The population size of Matrix X is n
 - Matrix X is divided into two population subgroups
 - Group 1: X^1 with population size n^1
 - Group 2: X^2 with population size n^2
 - Note that $n = n^1 + n^2$

Inc Edu Hel				
$X =$	4	4	2	Person 1
	3	5	4	Person 2
	8	6	3	Person 3

Population Subgroups

Population Subgroup Decomposability: A poverty measure is additive decomposable if

$$P(X) = \frac{n^1}{n} P(X^1) + \frac{n^2}{n} P(X^2)$$

Then, one can calculate the contribution of each group to overall poverty:

$$C(X^1) = \frac{n^1}{n} \frac{P(X^1)}{P(X)}$$

Population Subgroups

Reconsider the following example

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$X =$	700	14	Yes	Yes	Person 1
	300	13	Yes	No	Person 2
	400	10	No	No	Person 3
	800	11	Yes	Yes	Person 4
$Z =$	500	12	Yes	Yes	

Population Subgroups

The deprivation matrix

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$g^0 =$	0	0	0	0	Person 1
	1	0	0	1	Person 2
	1	1	1	1	Person 3
	0	1	0	0	Person 4

$z =$	500	12	Yes	Yes	
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Population Subgroups

The weight vector is (1, 2, 0.5, 0.5); replace deprivation status with weight (*weighted deprivation matrix*)

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0 =$	0	0	0	0	Person 1
	1	0	0	0.5	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Population Subgroups

Who is poor when $k = 1.5$?

$\bar{g}^0 =$	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
	0	0	0	0	Person 1
	1	0	0	0.5	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Population Subgroups

Who is poor when $k = 1.5$?

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0(k) =$	0	0	0	0	Person 1
	1	0	0	0.5	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Population Subgroups

What is the M_0 of the matrix?

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0(k) =$	0	0	0	0	Person 1
	1	0	0	0.5	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Population Subgroups

What is the M_0 of the matrix? It is 15/32

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0(k) =$	0	0	0	0	Person 1
	1	0	0	0.5	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Population Subgroups

Let us divide the population into two subgroups

$$\bar{g}^0(k) =$$

Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
0	0	0	0	Person 1
1	0	0	0.5	Person 2
1	2	0.5	0.5	Person 3
0	2	0	0	Person 4

Population Subgroups

Let us divide the population into two subgroups

$\bar{g}^0(k) =$

Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
0	0	0	0	Person 1
1	0	0	0.5	Person 2
1	2	0.5	0.5	Person 3
0	2	0	0	Person 4

- M_0 for the pink group: $1.5/8 = 3/16$
- M_0 for the green group: $6/8 = 3/4$
- Overall $M_0 = ?$

Population Subgroups

Let us divide the population into two subgroups

$\bar{g}^0(k) =$

Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
0	0	0	0	Person 1
1	0	0	0.5	Person 2
1	2	0.5	0.5	Person 3
0	2	0	0	Person 4

- M_0 for the pink group: $1.5/8 = 3/16$
- M_0 for the green group: $6/8 = 3/4$
- Overall $M_0 = (1/2) \times (3/16) + (1/2) \times (3/4) = 15/32$

Contribution of Subgroup

Let us divide the population into two subgroups

$\bar{g}^0(k) =$

Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
0	0	0	0	Person 1
1	0	0	0.5	Person 2
1	2	0.5	0.5	Person 3
0	2	0	0	Person 4

- The contribution of group 1 to M_0 is $(1/2) \times (3/16) / (15/32) = 1/5$
- The contribution of group 2 to M_0 is $(1/2) \times (3/4) / (15/32) = 4/5$
- The total contribution must sum up to 1

How Does it Help to Analyze Results?

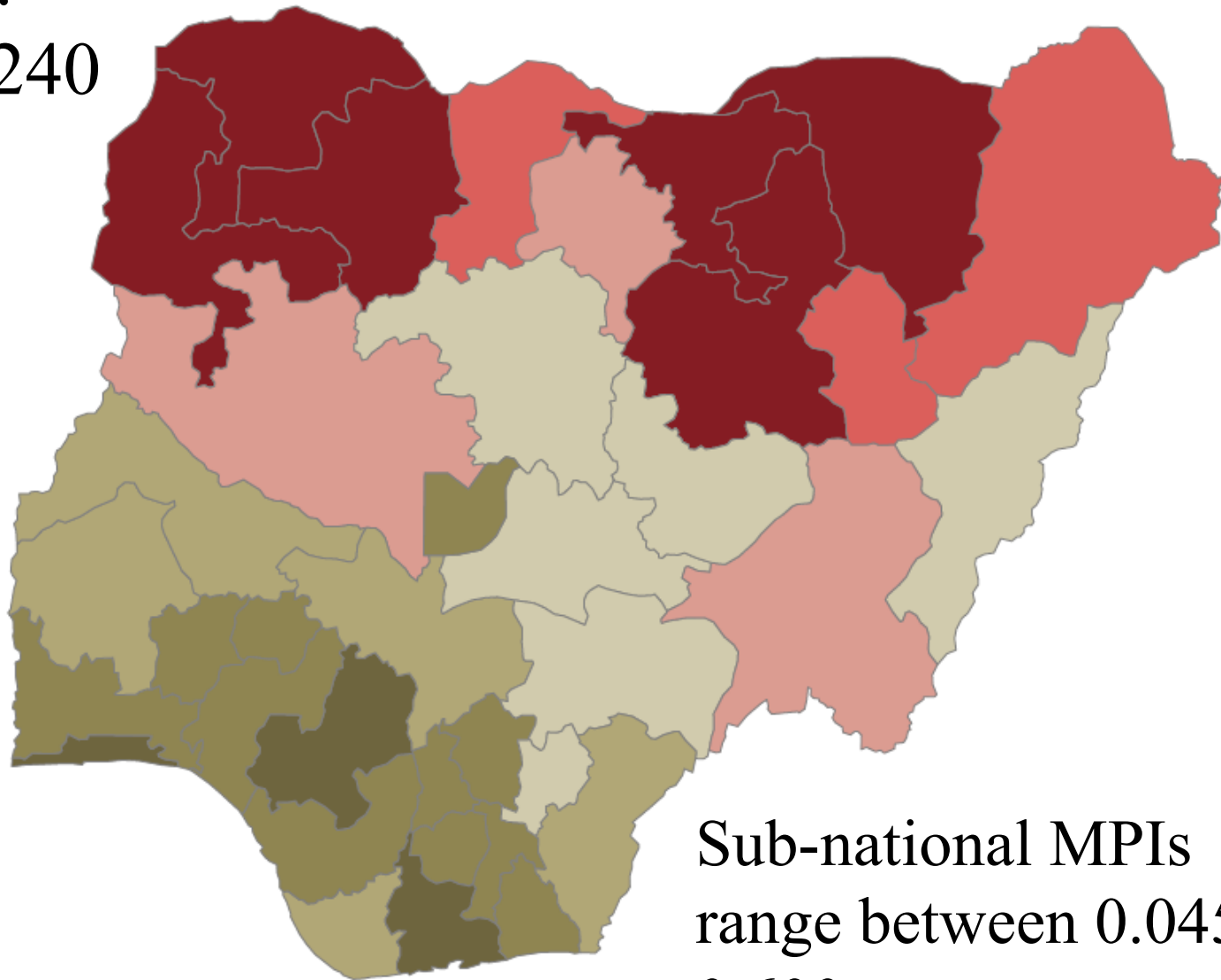
Nigeria:

MPI=0.240



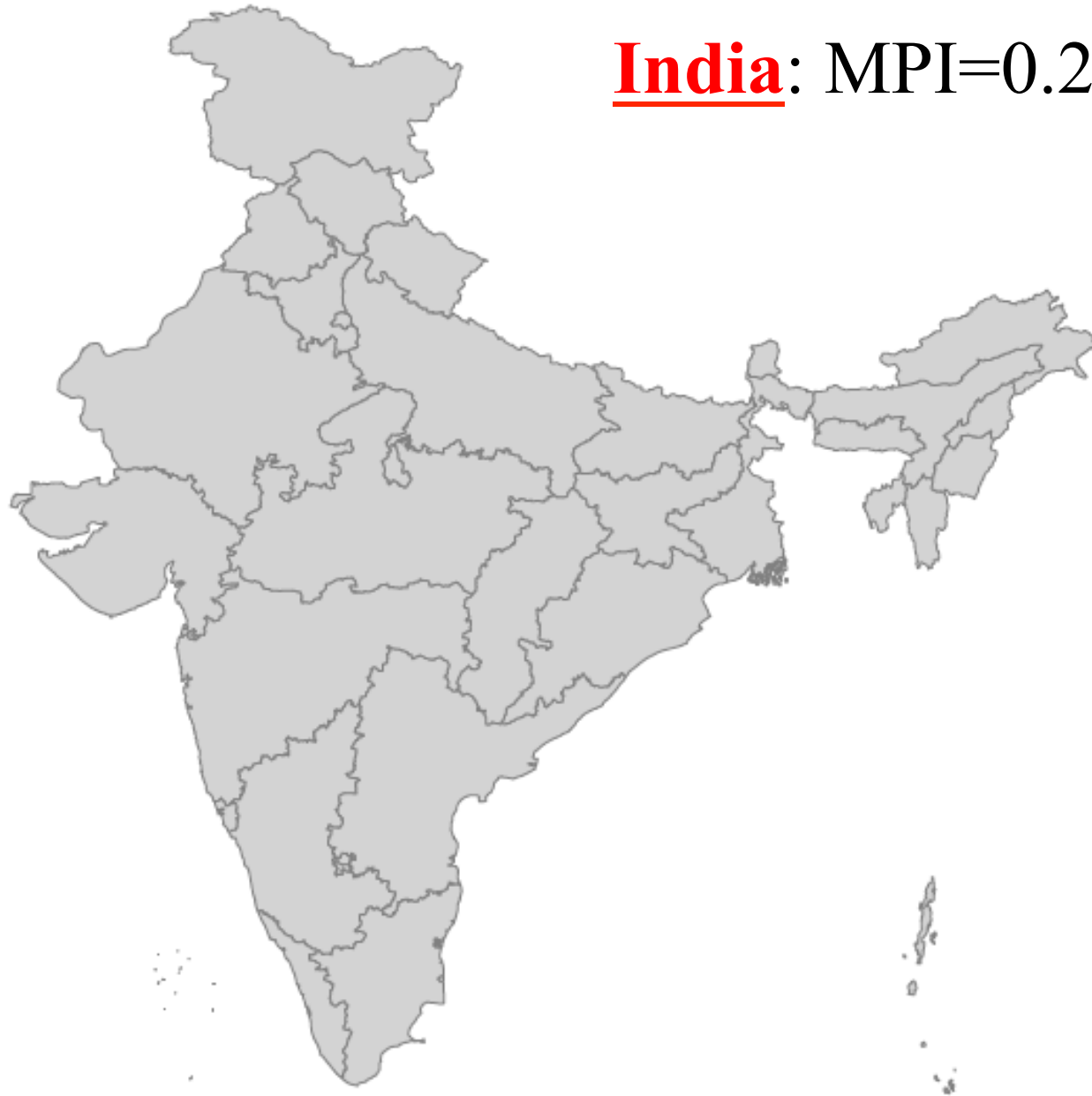
Nigeria:

MPI=0.240

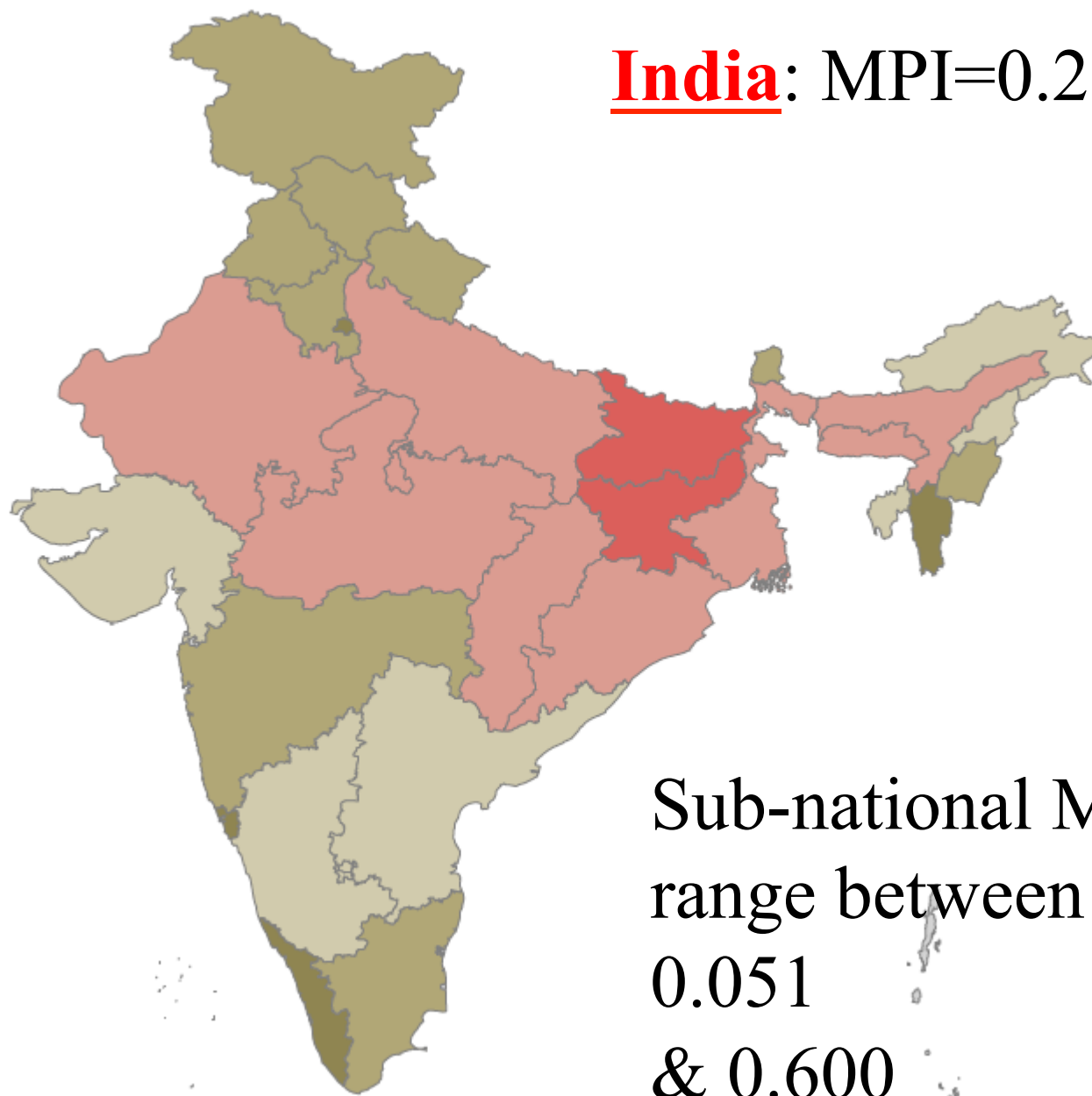


Sub-national MPIs
range between 0.045 &
0.600

India: MPI=0.283



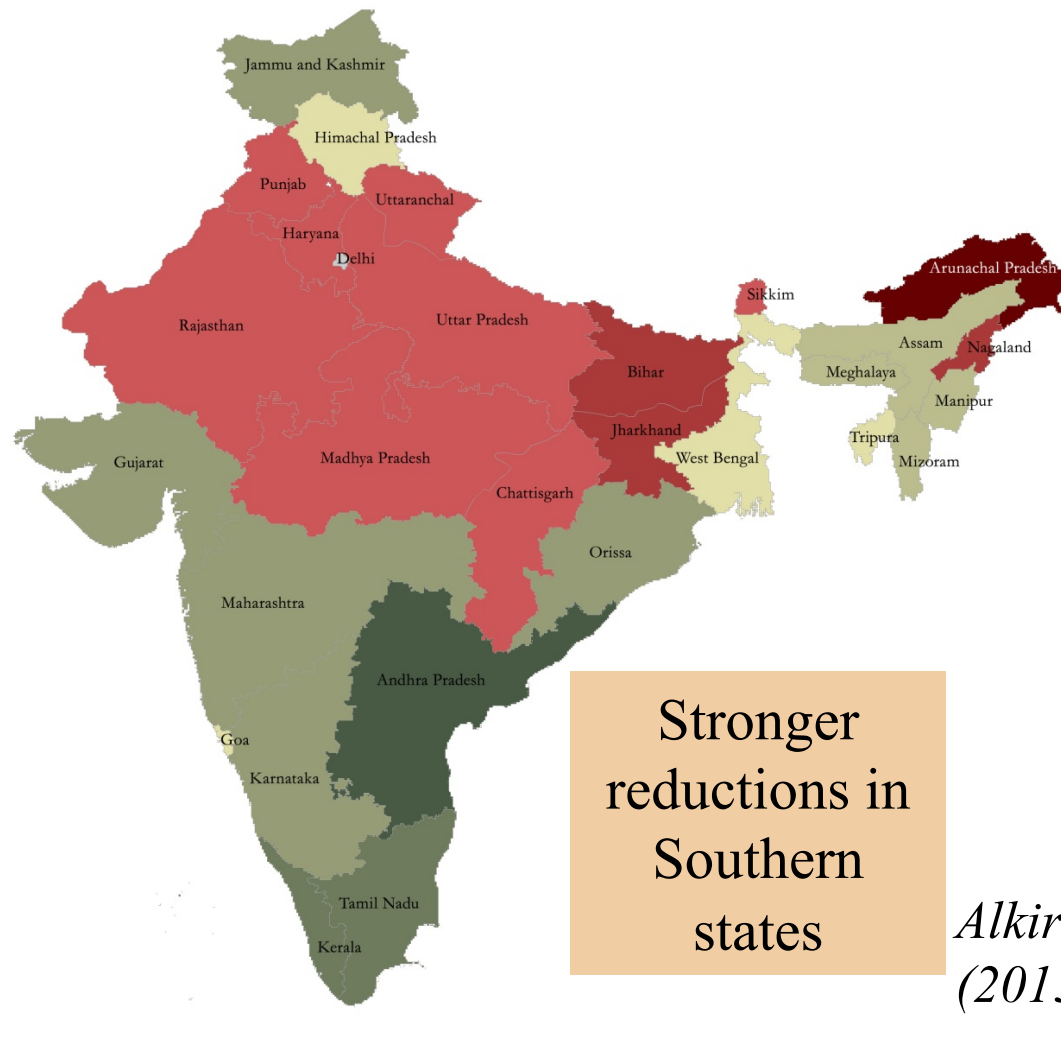
India: MPI=0.283



Sub-national MPIs
range between
0.051
& 0.600

Reduction in MPI across States 99-06

Slower
reductions in
initially
poorer states

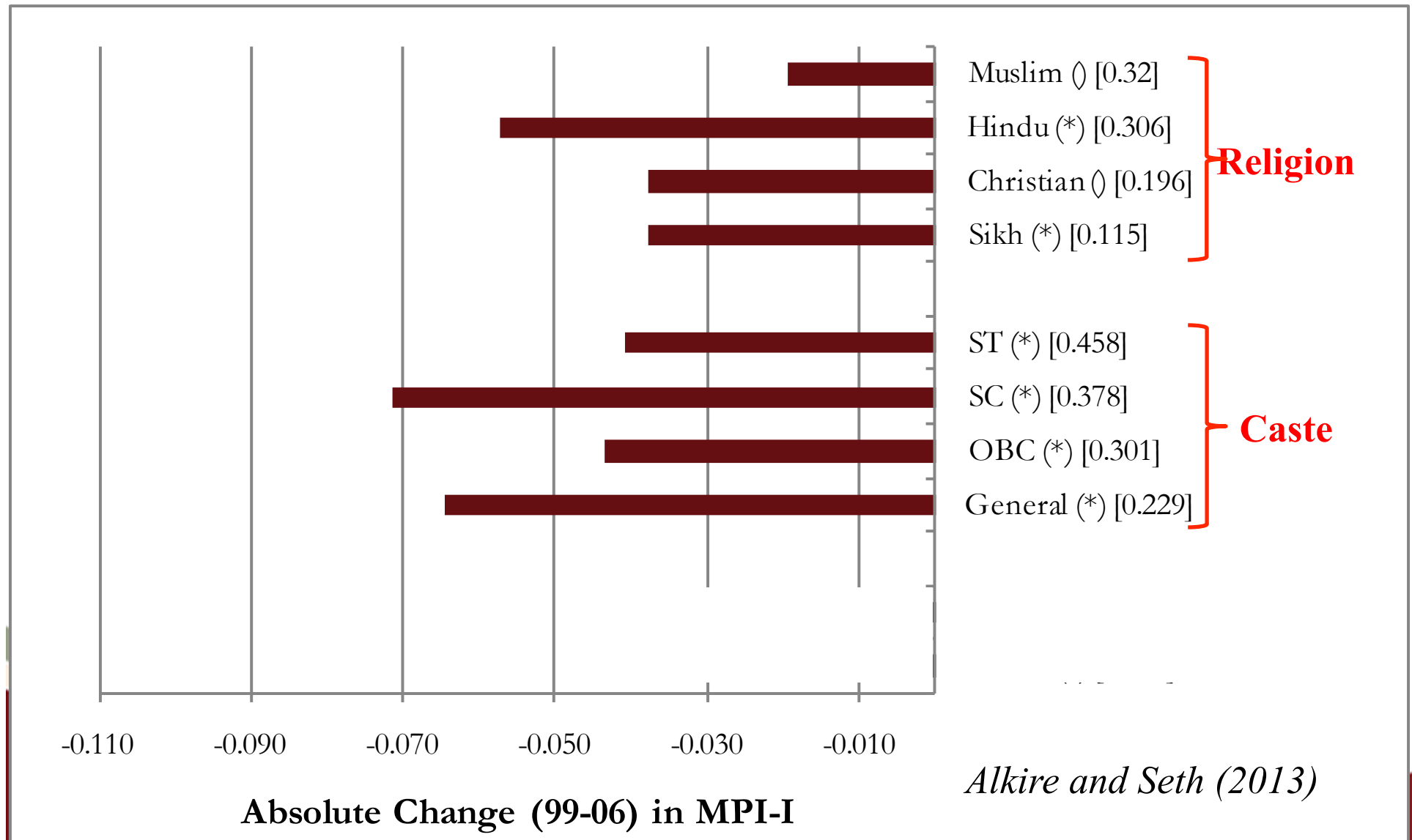


Stronger
reductions in
Southern
states

*Alkire and Seth
(2013)*

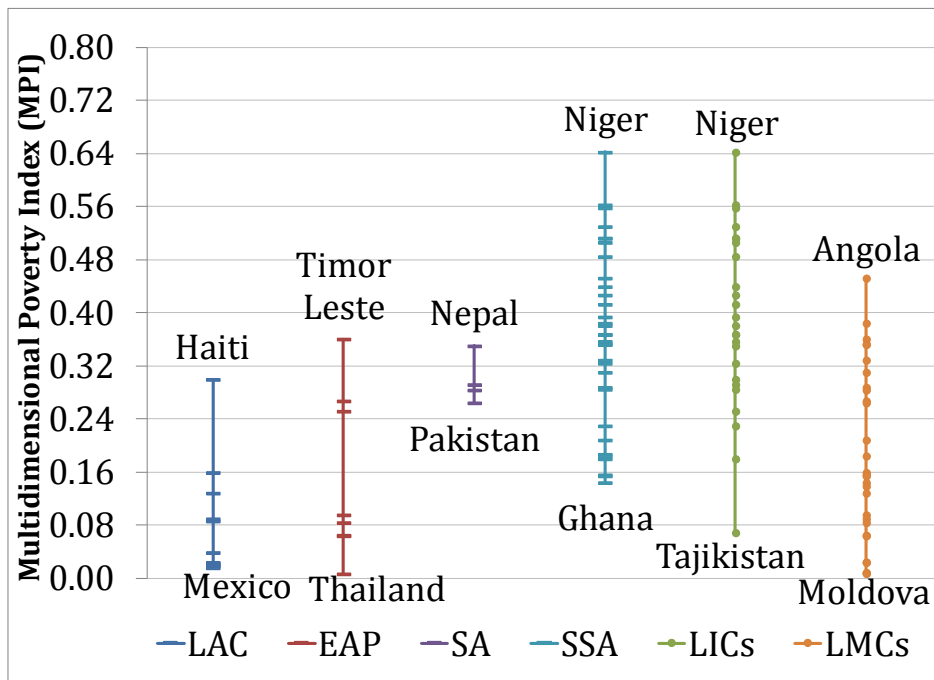
We combined Bihar and Jharkhand, Madhya Pradesh and Chhattisgarh, and Uttar Pradesh and Uttarakhand

Reduction in MPI: Castes and Religions



National & Sub-national Disparity in MPI

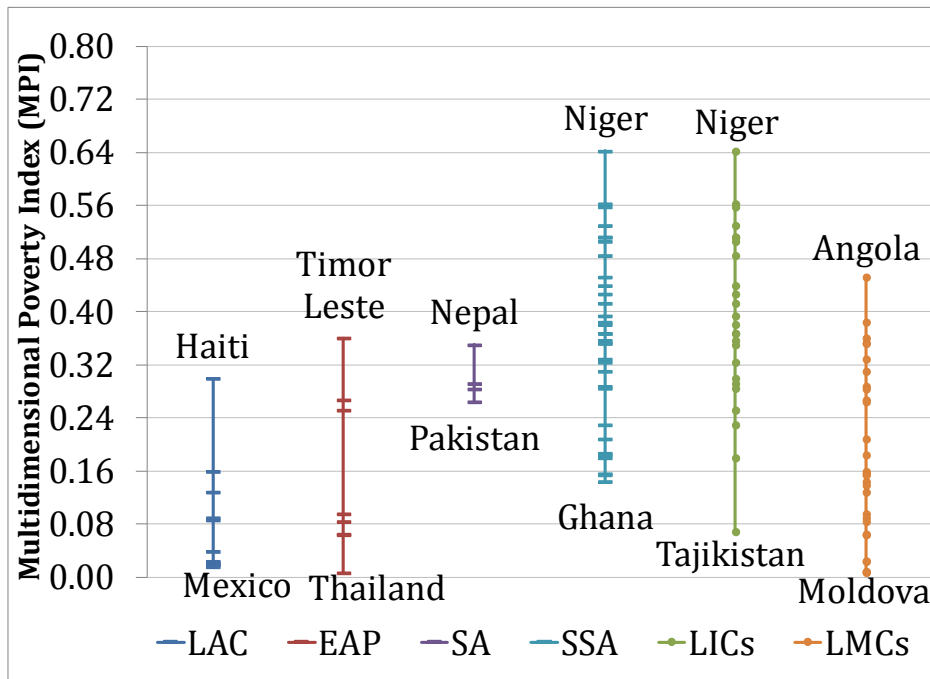
National Disparity



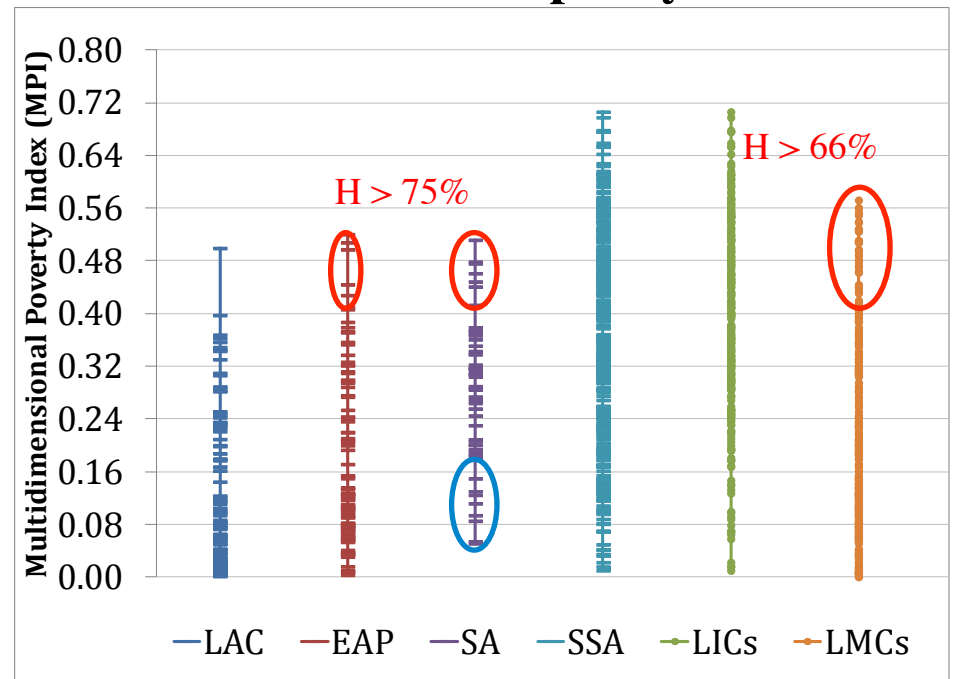
2011 MPI Data

National & Sub-national Disparity in MPI

National Disparity

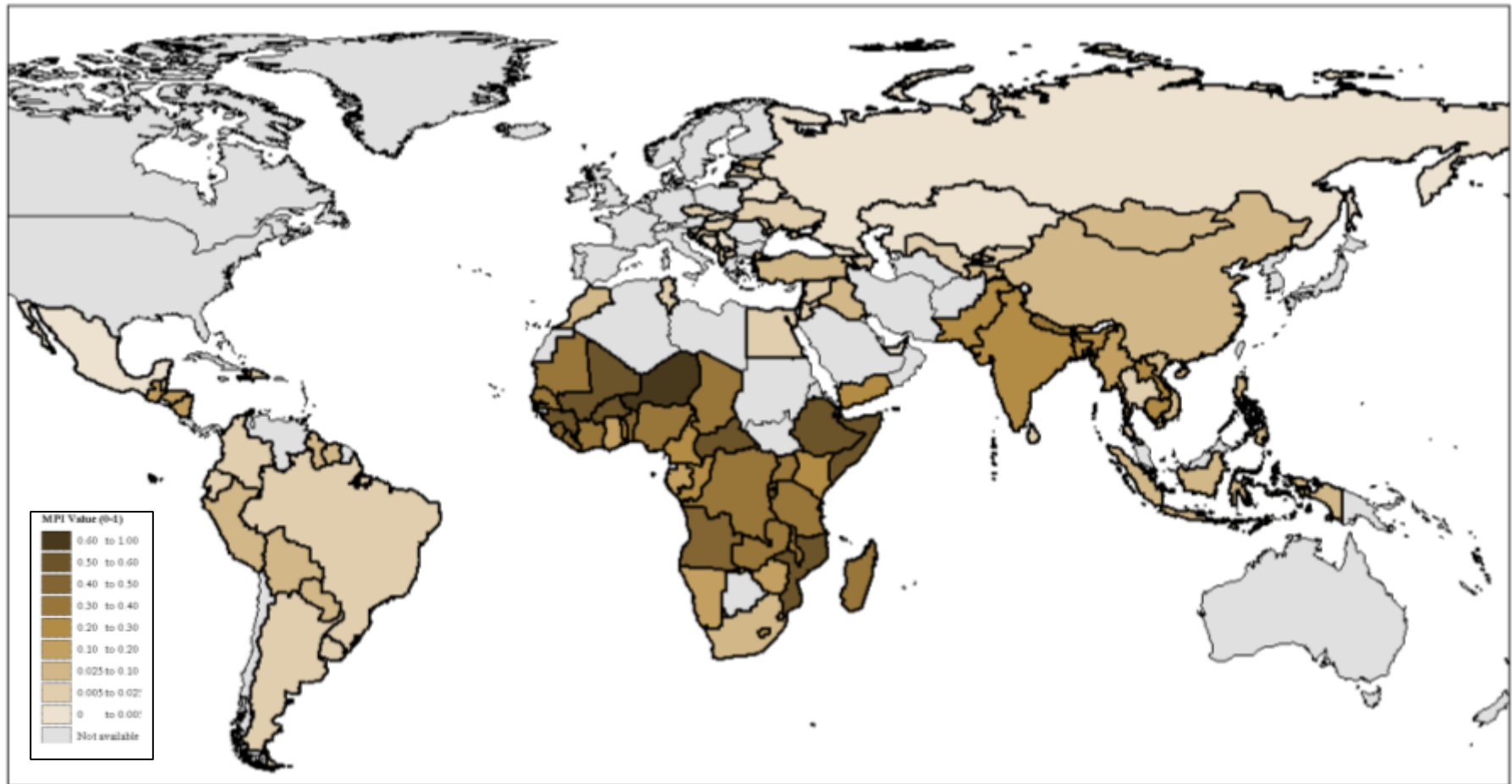


Sub-national Disparity

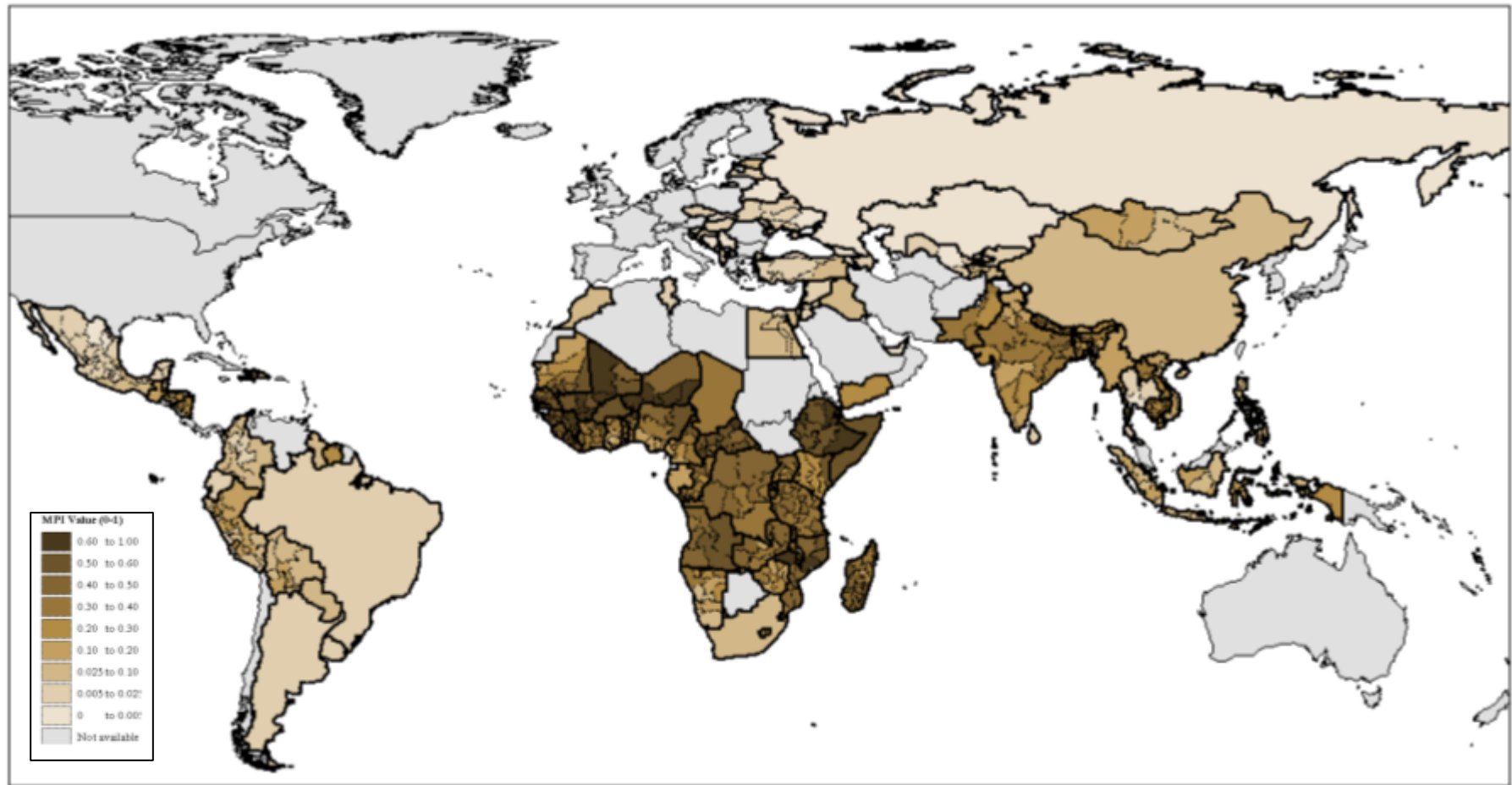


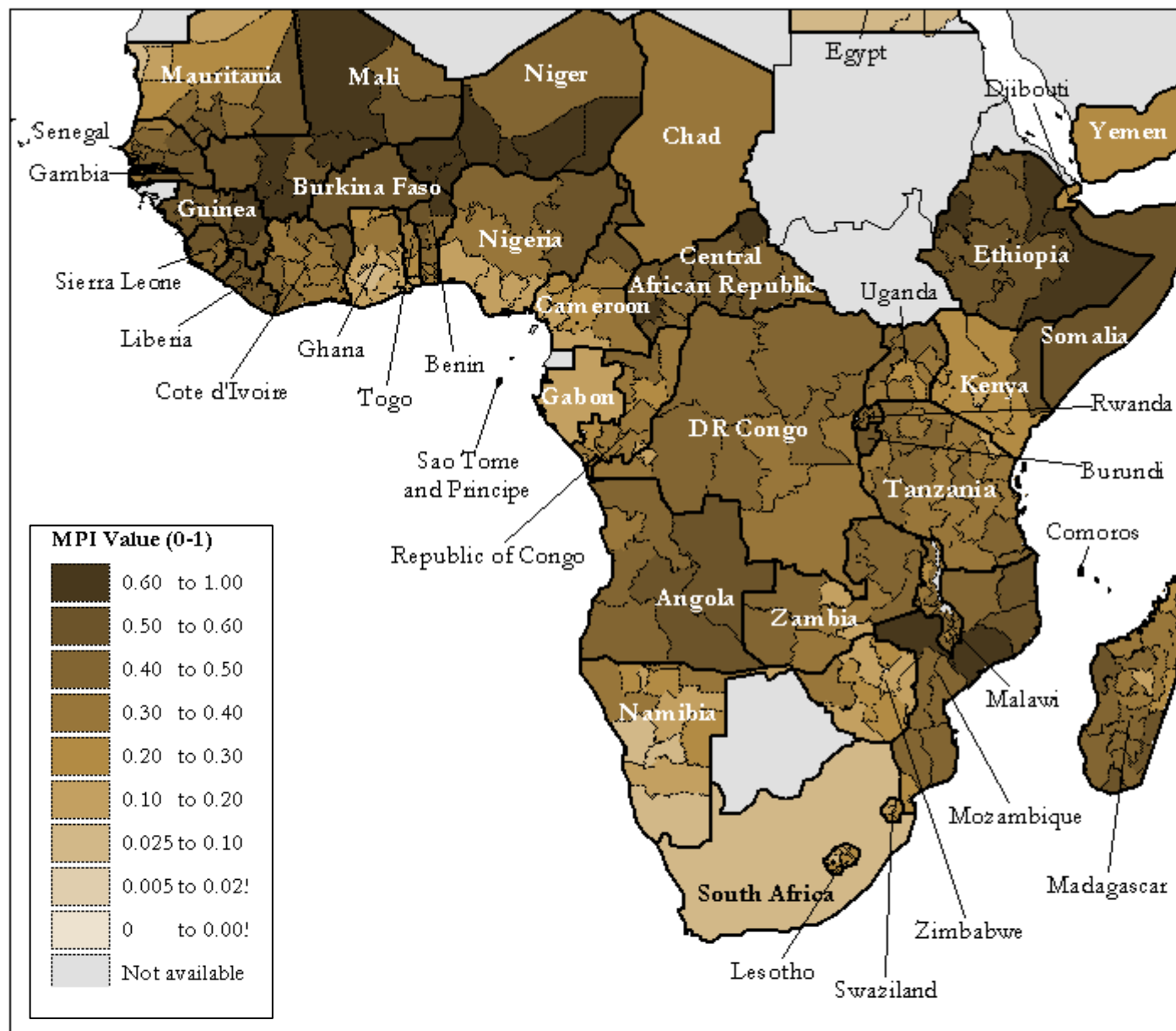
2011 MPI Data

National MPI (Use 2011 results)



Sub-national MPI (Use 2011 results)





Dimensional Breakdown

Q1: What is the difference between the raw headcount ratio and the censored headcount ratio?

Q2: Can the raw headcount ratio of a dimension be lower than its censored headcount ratio?

Q3: Can the censored headcount ratio of a dimension be higher than the multidimensional headcount ratio?

Q4: What is the relationship between the censored headcount ratios and M_0 ?

Q5: What kind of policy analysis can be conducted using the censored headcount ratio?

Example

An achievement matrix with 4 **dimensions**

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$X =$	700	14	1	1	Person 1
	300	13	1	0	Person 2
	400	10	0	0	Person 3
	800	11	1	1	Person 4

$z =$	500	12	1	1	
-------	------------	-----------	----------	----------	--

z is the vector of poverty lines

Example

Replace entries: 1 if deprived, 0 if not deprived

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$g^0 =$	0	0	0	0	Person 1
	1	0	0	1	Person 2
	1	1	1	1	Person 3
	0	1	0	0	Person 4

$z =$	500	12	Yes	Yes	
-------	------------	-----------	------------	------------	--

These entries fall below cutoffs

Example

What is the *uncensored Headcount Ratio* of each of the four dimensions?

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$g^0 =$	0	0	0	0	Person 1
	1	0	0	1	Person 2
	1	1	1	1	Person 3
	0	1	0	0	Person 4

Income: 2/4

Education: 2/4

Sanitation: 1/4

Electricity: 2/4

Example

Suppose the weight vector is (1, 2, 0.5, 0.5)

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$g^0 =$	0	0	0	0	Person 1
	1	0	0	1	Person 2
	1	1	1	1	Person 3
	0	1	0	0	Person 4

Example

Suppose the weight vector is (1, 2, 0.5, 0.5)

- Replace the deprivation status with the weights

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$g^0 =$	0	0	0	0	Person 1
	1	0	0	1	Person 2
	1	1	1	1	Person 3
	0	1	0	0	Person 4

Example

Suppose the weight vector is (1, 2, 0.5, 0.5)

- Replace the deprivation status with the weights

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0 =$	0	0	0	0	Person 1
	1	0	0	0.5	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Example

Suppose the weight vector is (1, 2, 0.5, 0.5). Each weight is w_j

- Replace the deprivation status with the weights

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0 =$	0	0	0	0	Person 1
	1	0	0	0.5	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Example

Suppose the weight vector is (1, 2, 0.5, 0.5)

- Construct the deprivation score vector

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0 =$	0	0	0	0	Person 1
	1	0	0	0.5	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Example

Suppose the weight vector is (1, 2, 0.5, 0.5).

- Construct the deprivation score vector

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	c
$\bar{g}^0 =$	0	0	0	0	0
	1	0	0	0.5	1.5
	1	2	0.5	0.5	4
	0	2	0	0	2

Example

If the poverty cutoff is $k = 2$, who is poor?

- Construct the deprivation score vector

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	c
$\bar{g}^0 =$	0	0	0	0	0
	1	0	0	0.5	1.5
	1	2	0.5	0.5	4
	0	2	0	0	2

Example

Let us now censor the deprivation matrix and vector

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	c
$g^0 =$	0	0	0	0	0
	1	0	0	0.5	1.5
	1	2	0.5	0.5	4
	0	2	0	0	2

Example

Let us now censor the deprivation matrix and vector

$\bar{g}^0(k)=$	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	c
	0	0	0	0	0
	0	0	0	0	0
	1	2	0.5	0.5	4
	0	2	0	0	2

The M_0 is 6/16

Dimensional Composition

There are four dimensions – denoted by $d = 4$

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0(k) =$	0	0	0	0	Person 1
	0	0	0	0	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Dimensional Composition

What is the *censored* headcount ratio of each dimension?

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0(k) =$	0	0	0	0	Person 1
	0	0	0	0	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Dimensional Composition

What is the *censored* headcount ratio of each dimension?

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0(k) =$	0	0	0	0	Person 1
	0	0	0	0	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Income: 1/4

Education: 2/4

Sanitation: 1/4

Electricity: 1/4

Uncensored vs. Censored Headcount Ratio

The uncensored headcount (UH) ratio of a dimension denotes the proportion of the population deprived in a dimension.

The censored headcount (CH) ratio of a dimension denotes the proportion of the population that is multidimensionally poor and deprived in that dimension at the same time.

M_0 and Censored Headcount Ratio

If the censored headcount ratio of indicator j is denoted by h_j , then the M_0 measure can be expressed as

$$M_0(X) = \sum_j (w_j/d) \times h_j(k)$$

where w_j is the weight attached to dimension j

Contribution of dimension j to overall poverty is

$$(w_j/d) \times [h_j/M_0(X)] \text{ for all } j$$

M_0 and UH Ratio in Union Approach

What is the relationship between the M_0 and the raw headcount ratio when a union approach is used for identifying the poor?

With a union approach, the censored headcount ratio for a dimension is its raw headcount ratio.

Thus, the M_0 with the union approach is the weighted average of the raw headcount ratios.

Dimensional Contribution

What is the contribution of the education dimension to M_0 ?

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$\bar{g}^0(k) =$	0	0	0	0	Person 1
	0	0	0	0	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

Dimensional Contribution

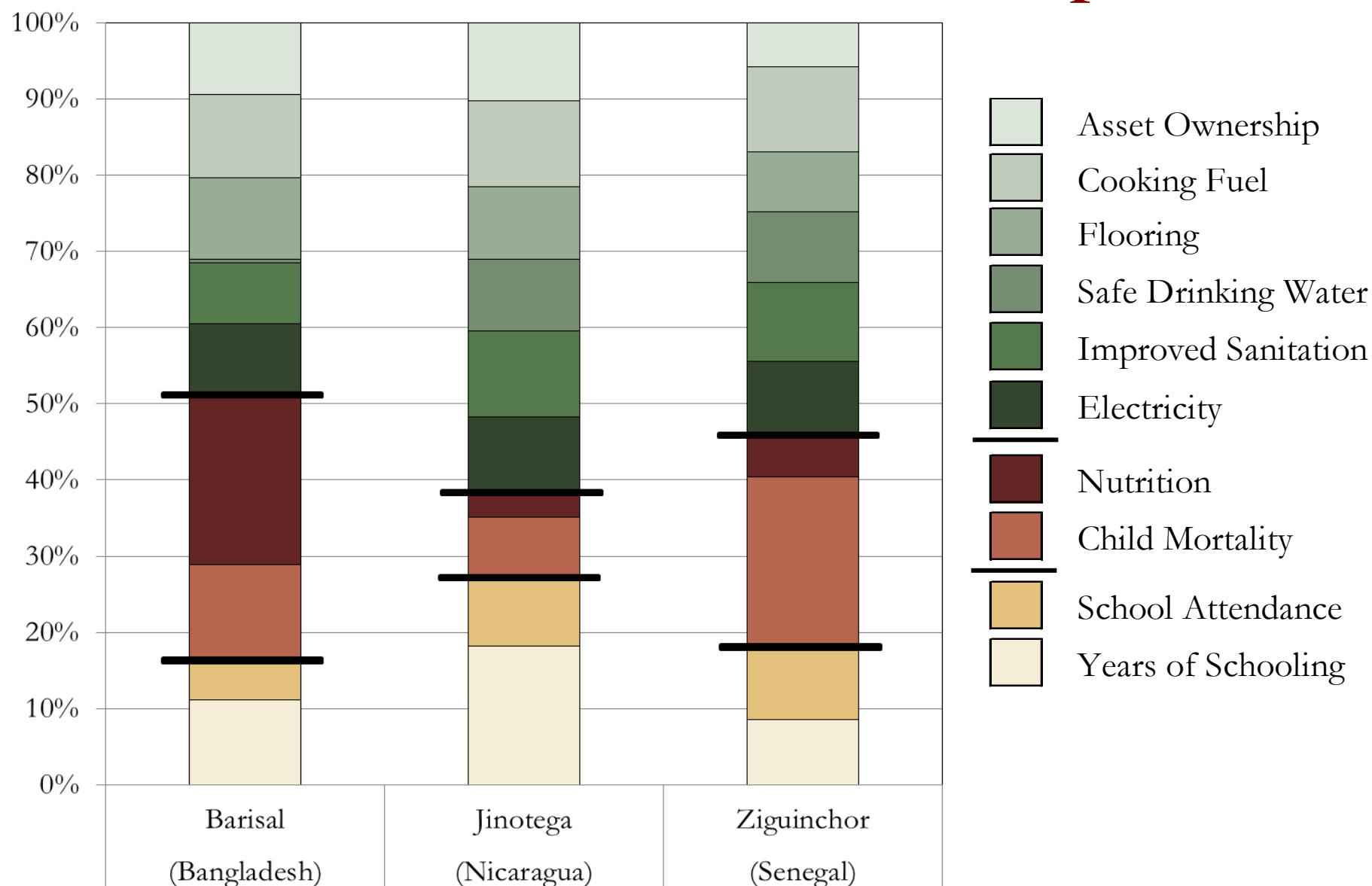
What is the contribution of the education dimension to M_0 ?

	Income	Years of Education	Sanitation (Improved?)	Access to Electricity	
$g^0(k) =$	0	0	0	0	Person 1
	0	0	0	0	Person 2
	1	2	0.5	0.5	Person 3
	0	2	0	0	Person 4

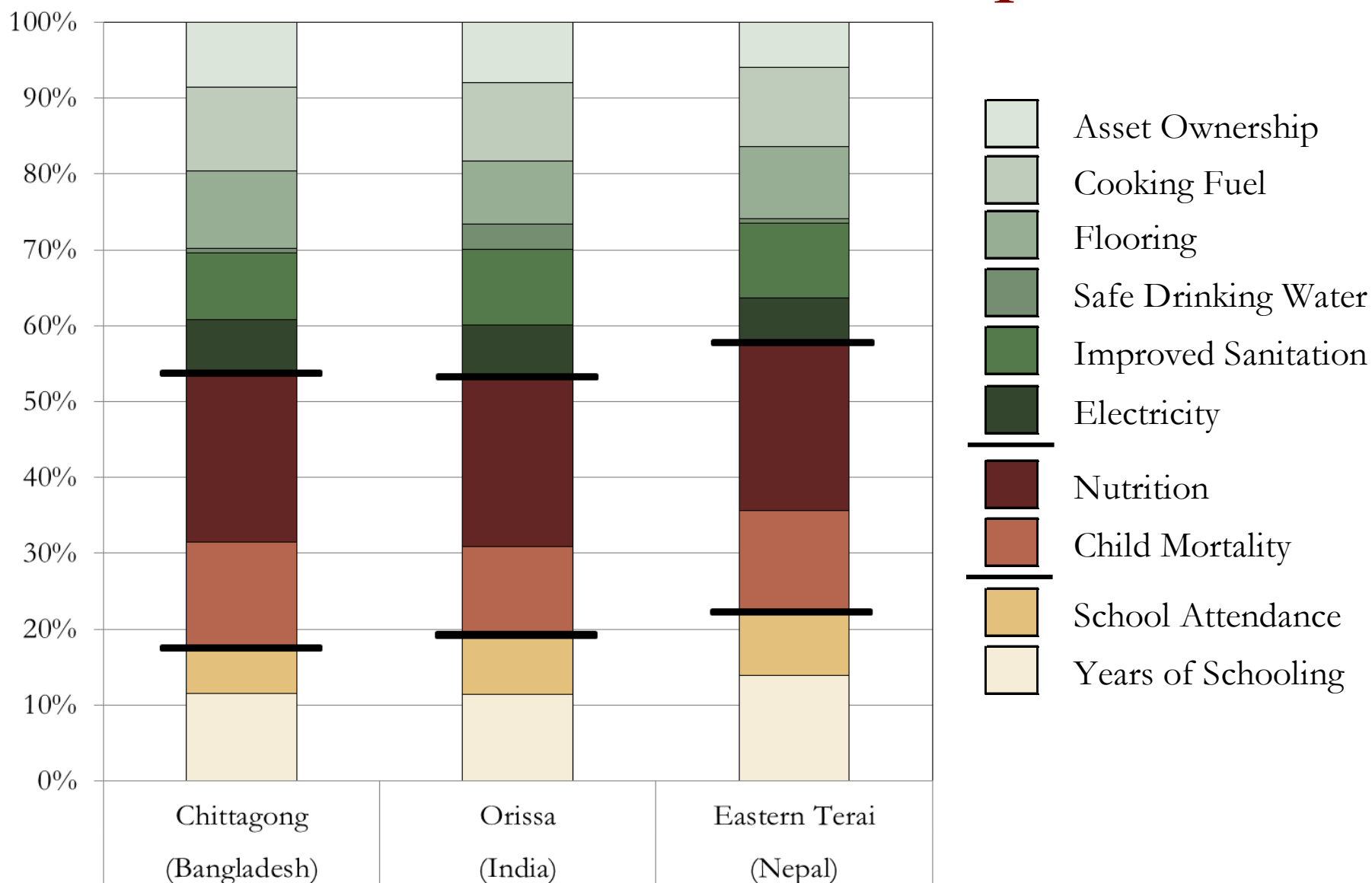
The contribution is $(2/4) \times [(2/4)/(6/16)] = 2/3$

w_E $h_E(k)$ M_0

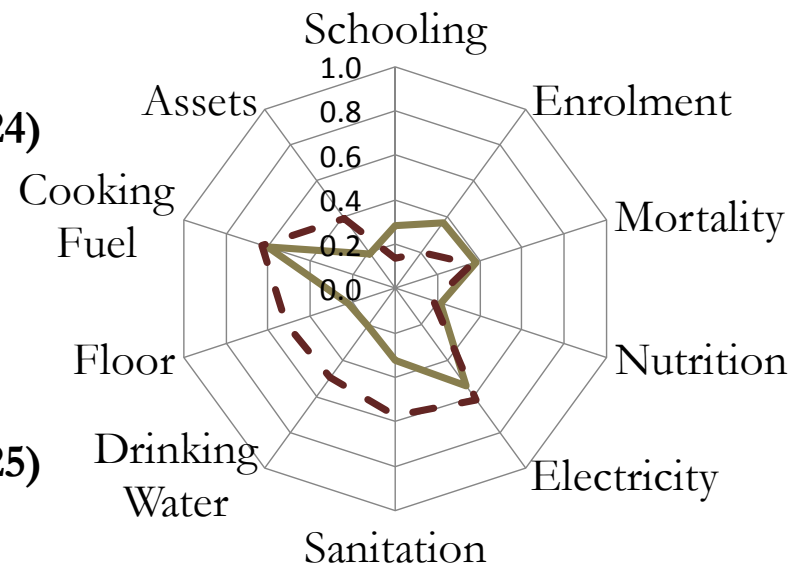
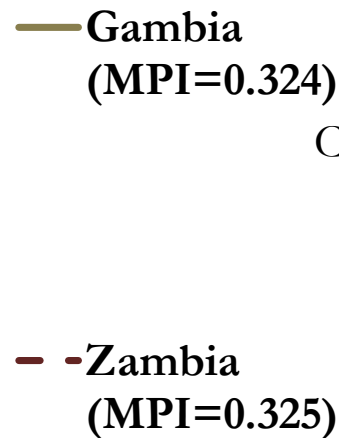
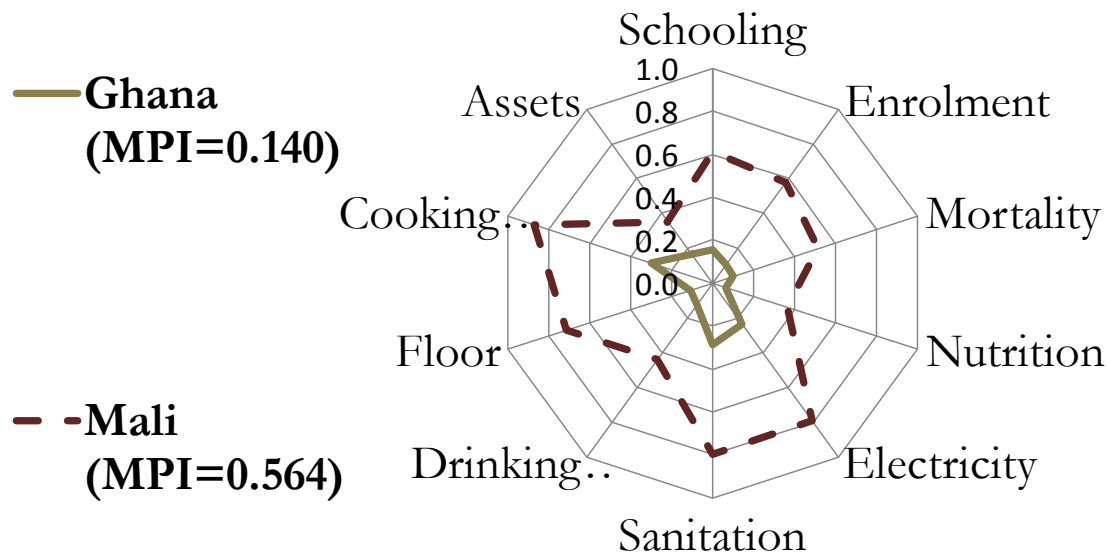
Similar MPI, but Different Composition



Different MPI, Similar Composition



Another way of Presenting Composition Graphically



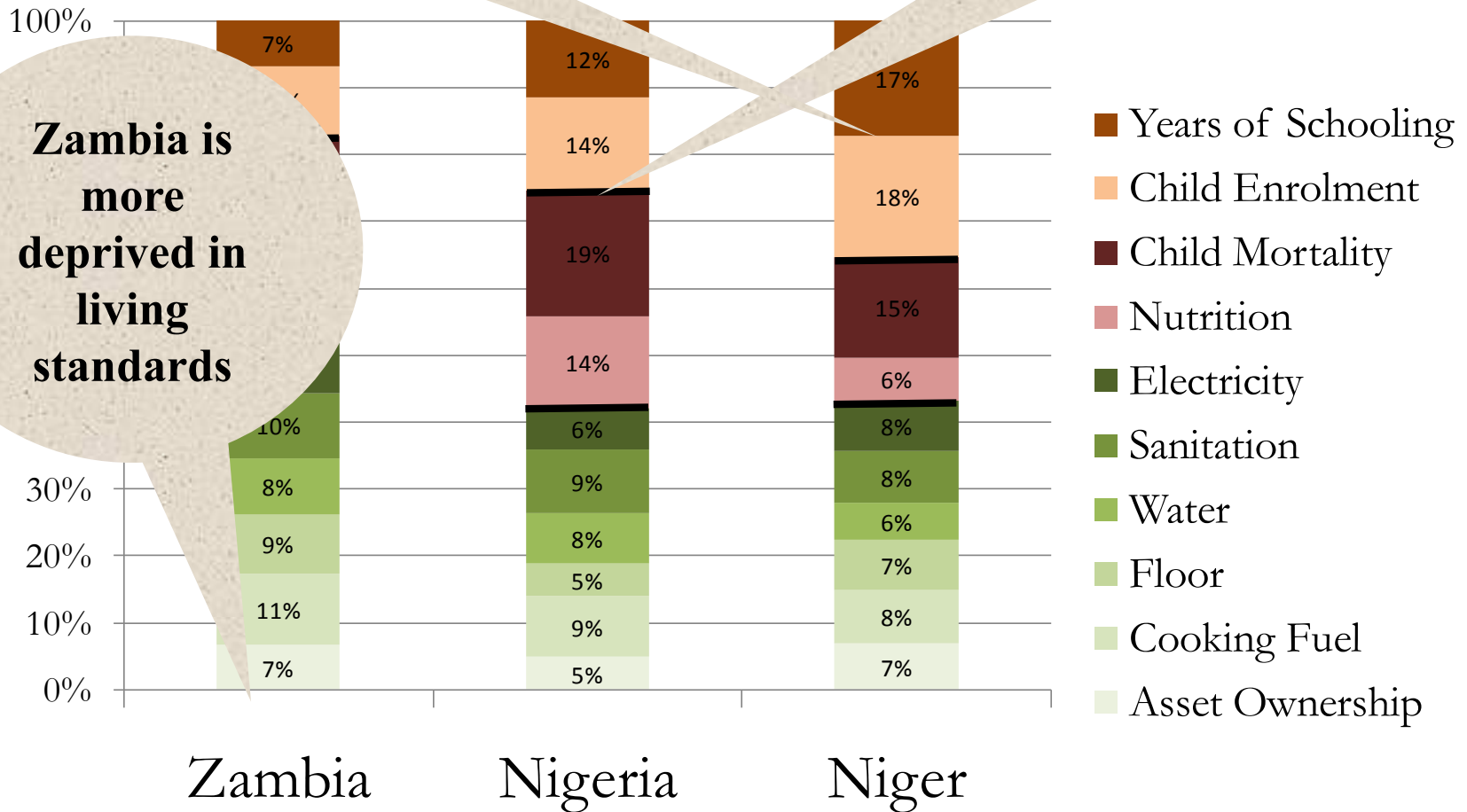
**Poverty types
(Roche 2010 for MPI Analysis)**

Composition by Ind

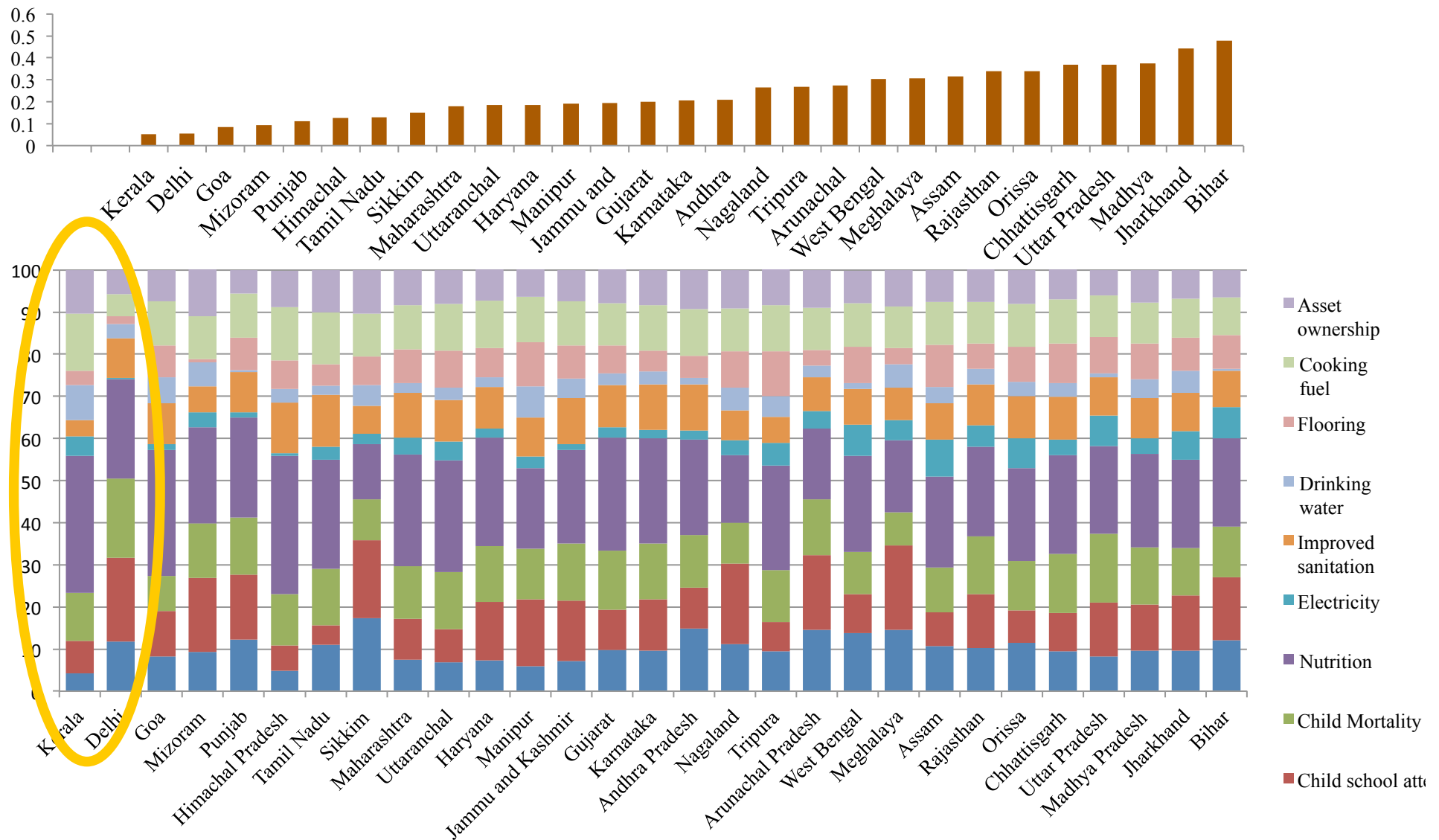
Niger is most deprived in education

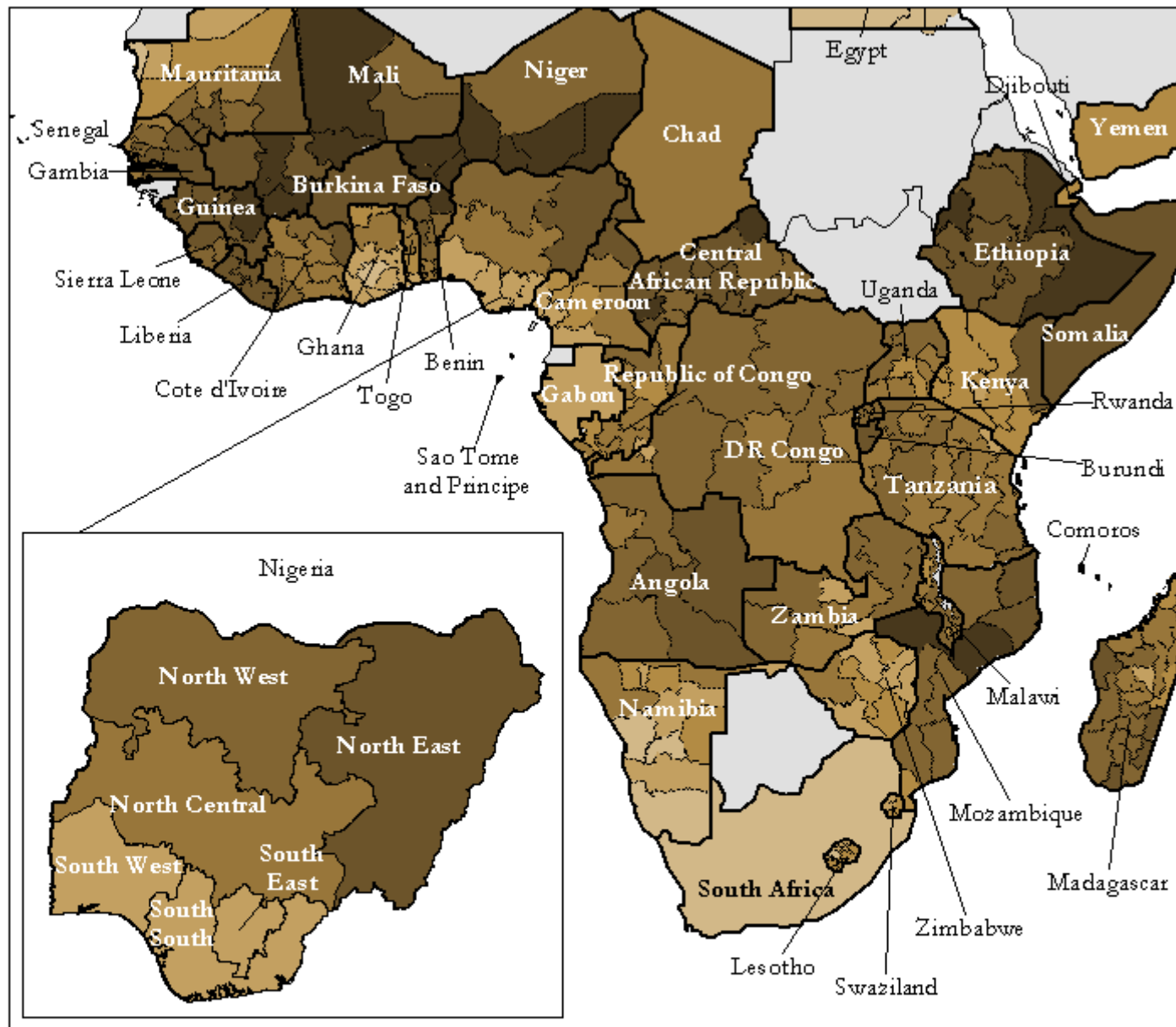
Nigeria is more deprived in health and education

Zambia is more deprived in living standards

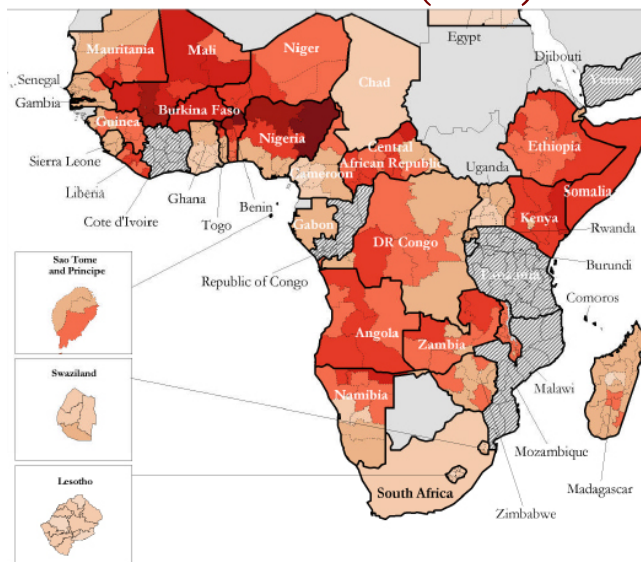


The composition of the MPI can inform policy. It can change across countries and states.

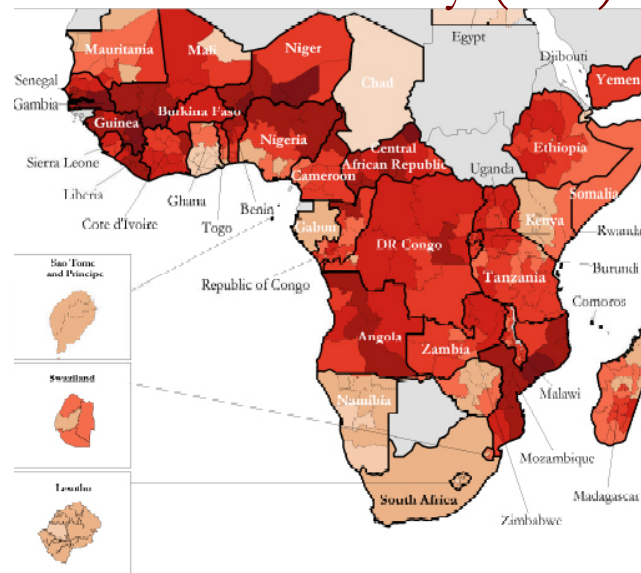




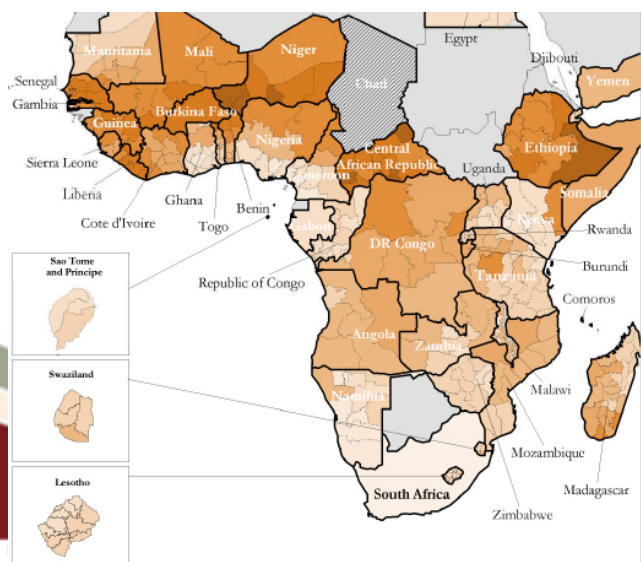
Nutrition (CH)



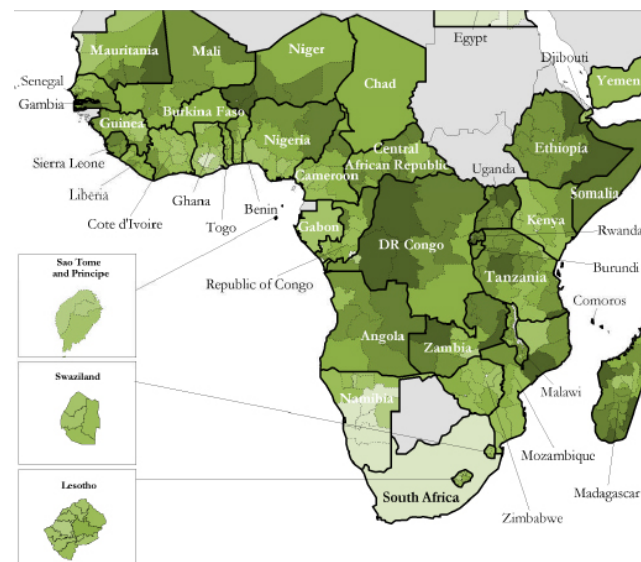
Child Mortality (CH)

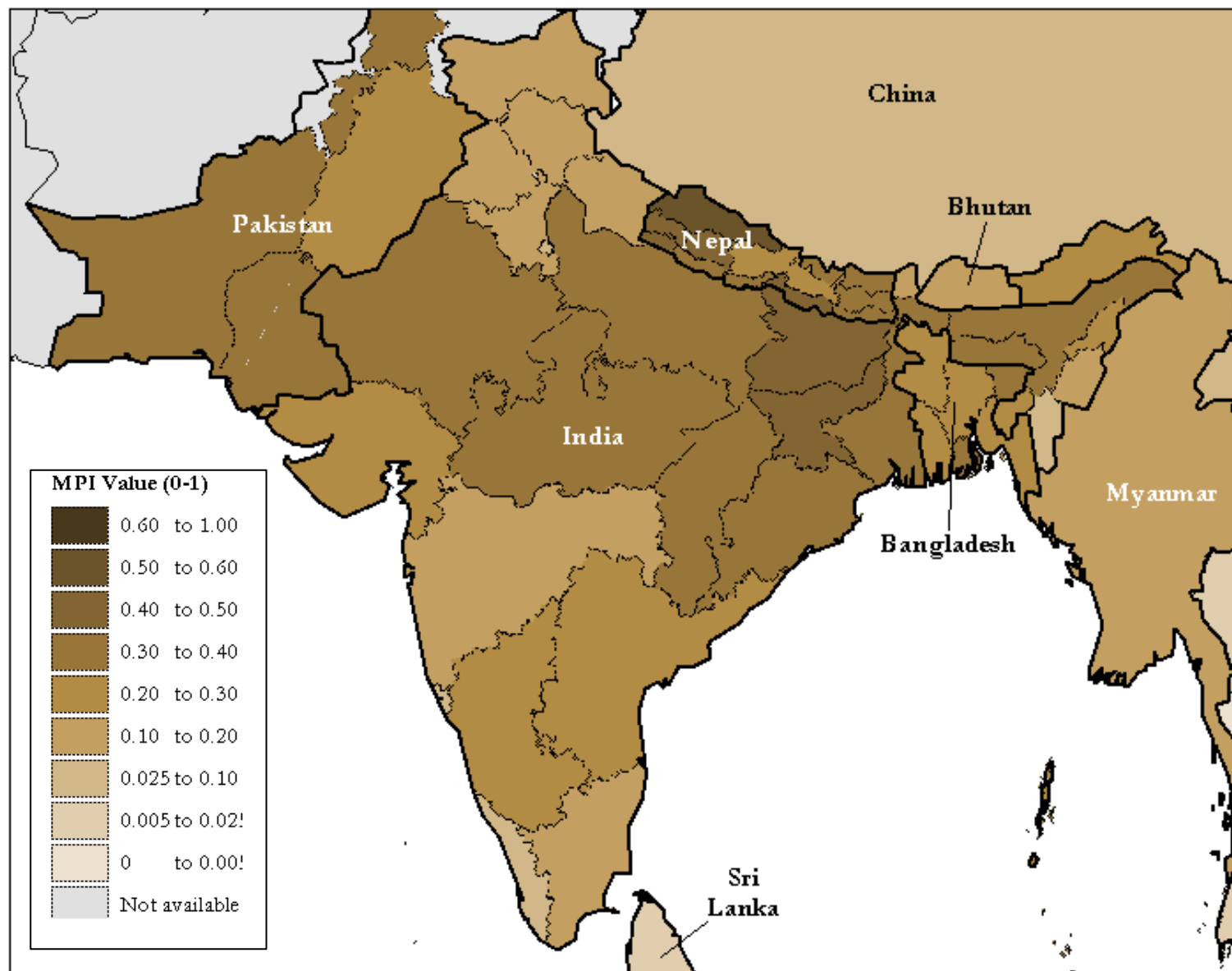


School Attendance (CH)

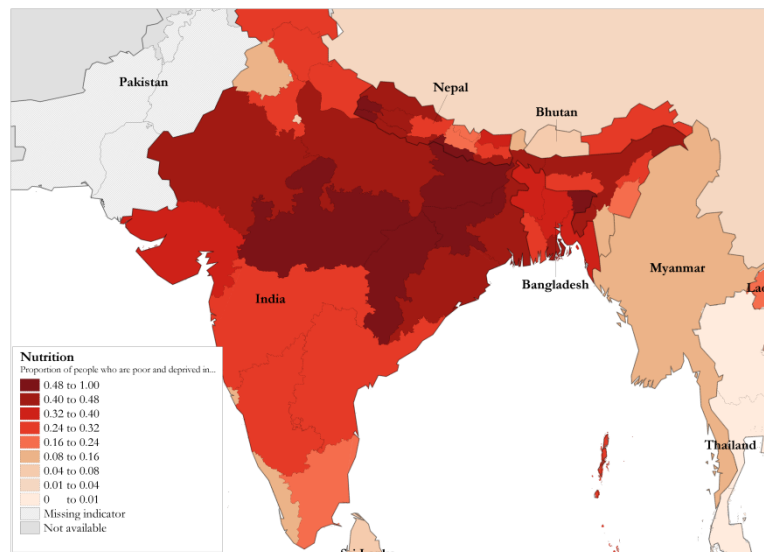


Safe Drinking Water (CH)

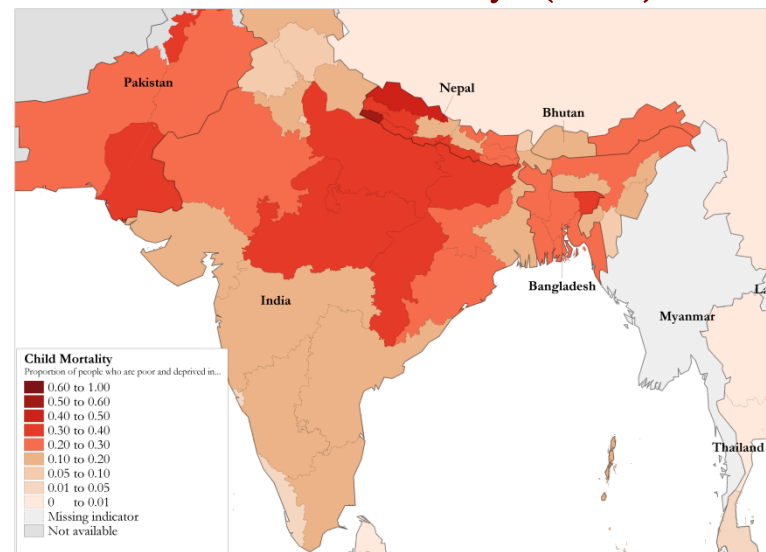




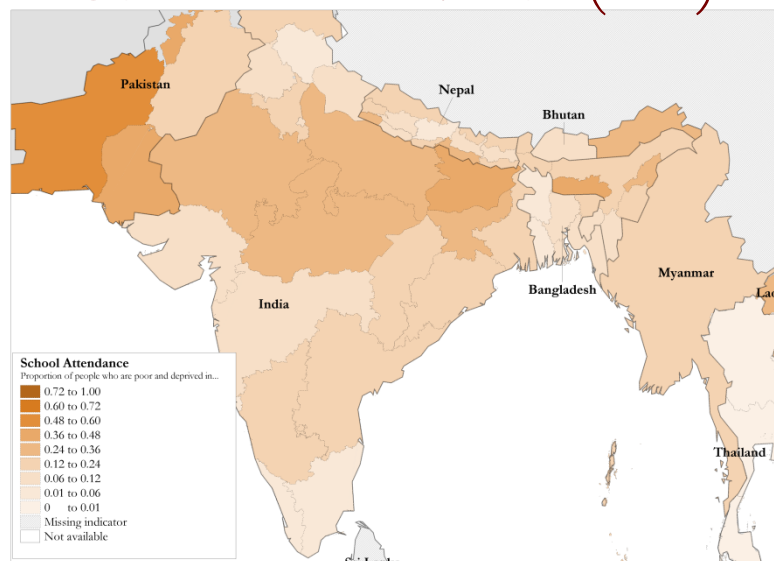
Nutrition (CH)



Child Mortality (CH)



School Attendance (CH)



Safe Drinking Water (CH)

