Brief introduction to multidimensional stochastic dominance

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Table of contents

Introduction: Multidimensional versus unidimensional dominance

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- Multidimensional dominance is relevant for evaluation functions that map from a multivariate space. E.g. An index of well-being that depends on several aspects of wellbeing.
- In unidimensional dominance second and even third orders may be interesting/relevant. In multidimensional dominance second and higher orders are not that easy to interpret.
- By contrast, in multidimensional dominance other things matter: the joint distribution of the variables, as well as how they complement, or substitute, each other in their contributions toward the evaluation function (e.g. "increasing wellbeing").

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- We will discuss why these conditions are also relevant for poverty assessments.
- We will briefly discuss how to test these conditions.

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$$\Delta W = -\int_{x_{min}}^{x_{max}} \frac{\delta U}{\delta x}(x) \Delta F(x) dx$$

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The dominance condition is usually expressed in terms of distributions. E.g. if $\forall x : F^A(x) \leq F^B(x)$ then we say that "distribution A (first-order) dominates B".

First-order conditions for the bivariate case

With two variables we integrate by parts the following function:

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1. A condition for monotonically increasing functions with ALEP substitute arguments (e.g. $\frac{\delta^2 U}{\delta x \delta y}(x, y) \leq 0$):

$$\forall x, y : \Delta W \geq 0 \forall U \mid \frac{\delta U}{\delta i}(x, y) \geq 0 \wedge \frac{\delta^2 U}{\delta x \delta y}(x, y) \leq 0 \leftrightarrow$$

$$\Delta F^x(x), \Delta F^y(y), \Delta F(x, y) \leq 0$$

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3. A condition for monotonically increasing functions with ALEP neutral arguments (e.g. $\frac{\delta^2 U}{\delta x \delta y}(x, y) = 0$):

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Notice that, in this case, it is necessary to test $\Delta F^i(i) \leq 0$ for every variable (unlike in the previous cases).

First-order conditions for the bivariate case

4. A condition for ALL individual welfare functions that are monotonically increasing!

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Notice that, in this case, it is necessary and sufficient to test $\Delta F(x, y) \leq 0$ and $\Delta \bar{F}(x, y) \geq 0$.

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In the case of survival functions, all the signs of the cross partial derivatives of the functions for which the conditions hold are positive (e.g. $U_i > 0$, $U_{ij} > 0, U_{ijk} > 0$).

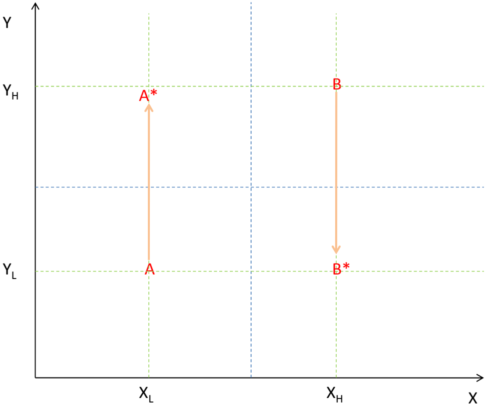
Hence when conditions on both the cumulative and the survival functions are fulfilled:

$$\forall x, y, ..., z : \Delta W \geq 0 \forall U \mid \frac{\delta U}{\delta i}, \frac{\delta^3 U}{\delta i \delta j \delta k}, ... \geq 0$$

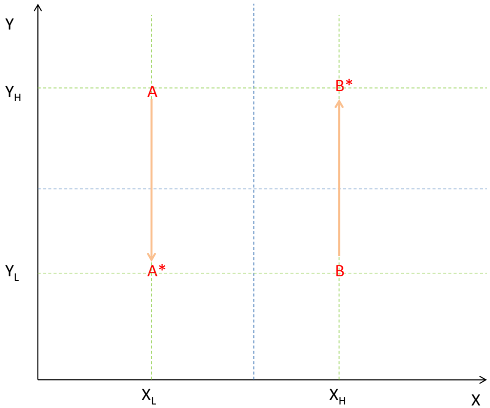
ALEP relationships and multidimensional dominance conditions

Why the conditions on cumulative distributions are associated with ALEP substitutability and the conditions on survival functions are associated with ALEP complementarity?

ALEP substitution and $\Delta F(x, y) \leq 0$



ALEP complementarity and $\Delta\bar{F}(x,y) \geq 0$



Applying dominance conditions to poverty measurement

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- Monotonicity: Higher x should not increase poverty: $\frac{\delta P_n(x;z)}{\delta x} \leq 0$

So if we define $W(X;z) = -P(X;Z)$ then we can apply the above mentioned conditions to poverty comparisons as well!

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- The logic is very intuitive: dominance conditions are based on comparing integrals of $F^A(z)$ with $F^B(z)$ for a range of z, depending on the order of dominance (e.g. for first order, just compare cumulative or survival distributions).

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There are four possible outcomes (illustrated with first-order):

1. A dominates B: $F^A(z) \leq F^B(z) \forall z \wedge \exists z | F^A < F^B(z)$

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3. No dominance because A=B: $F^A(z) = F^B(z) \forall z$
4. No dominance because A and B cross: $\exists z_1 | F^A(z_1) < F^B(z_1) \wedge \exists z_2 | F^B(z_2) < F^A(z_2)$

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Results of these tests require interpretation in order to ascertain any of the four possibilities.

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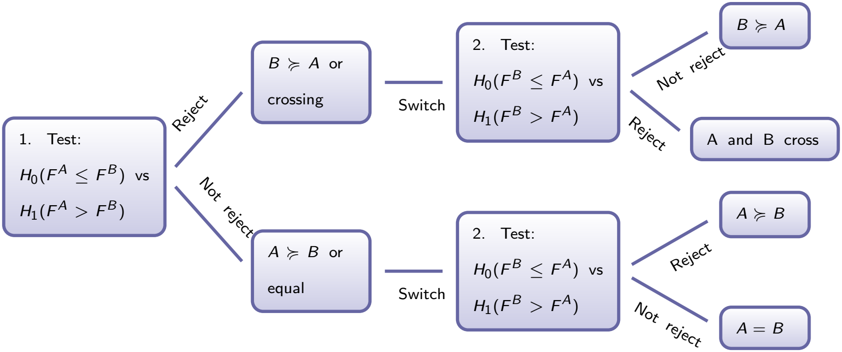

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- $\widehat{S}$ is the statistic we need. We now need to know how likely it is for this value to appear under the null hypothesis.
- There are different procedures to derive the distribution under the null hypothesis. Barrett and Donald (2003) develop two types. We are going to show one of them: a bootstrap method.

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4. If, say, $p^{A,B} < 0.01$ we reject the null hypothesis: under the hypothesis a value like $\widehat{S}$ is very unlikely.

Concluding remarks

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- When one is concerned for cardinal comparisons, e.g. between countries or across time, then dominance conditions are not that useful. Sensitivity analysis is required.
- We have assumed the variables are continuous. But these results can be easily extended to ordinal variables (e.g. see Yalonetzky, 2011). Further extensions to combinations of continuous and ordinal variables are possible.