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UNIVERSITY OF OXFORD

HDCA Summer School on Capability and Multidimensional Poverty

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Tabita, Kenya Rabiya, India Stéphanie, Madagascar Agatha, Madagascar Dalma, Kenya Ann-Sophia, Kenya Valérie, Madagascar

Unidimensional Poverty Measurement

Suman Seth

Oxford Poverty & Human Development Initiative (OPHI)

Main Sources of this Lecture

- Foster and Sen (1997), Annexe of "On Economic Inequality".
- Foster (2006) "Poverty Indices"
- There are others: please see the readings list.

Preliminaries

Preliminaries

- Single dimension
  - Income (referred as 'achievement')
- Achievements of a society or a country can be represented by a vector or a distribution
- Unit of analysis may be individual or household

Preliminaries

Achievement Vector

Suppose there are four persons in a society with incomes \$9, \$4, \$15 and \$8

- Then $x = (9, 4, 15, 8)$ is a vector representing the incomes of the society

Ordered Achievement Vector

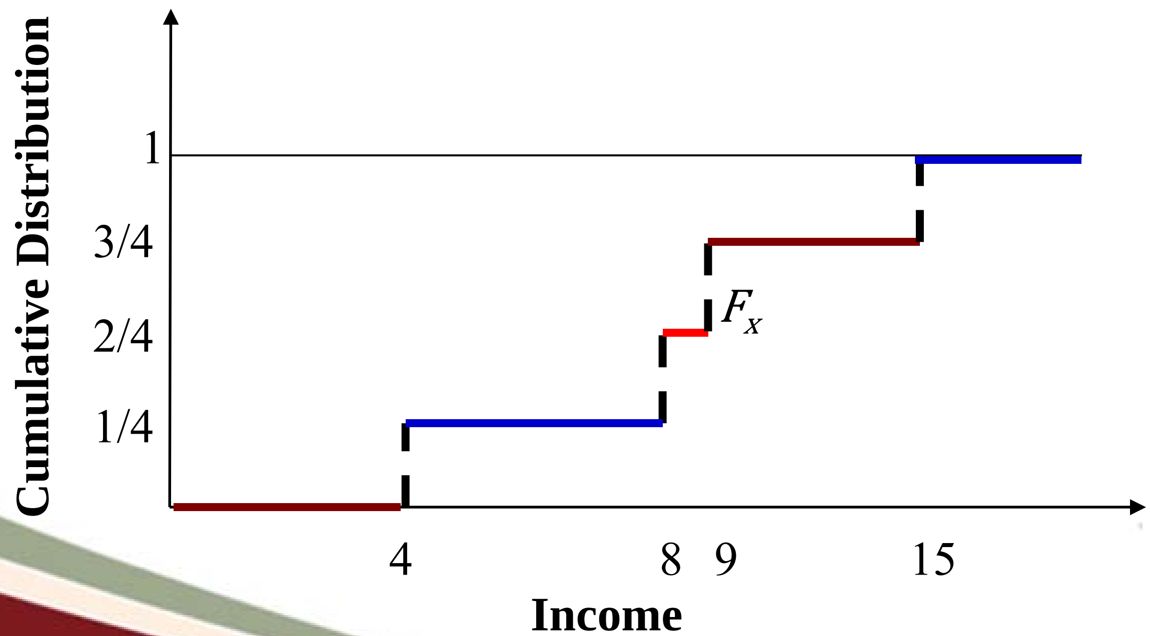
An ordered vector ranks or orders individuals by their achievements

- Ordered vector x^{ord} of x is $x^{ord} = (4, 8, 9, 15)$

Preliminaries

Cumulative Distribution Function (CDF)

The ordered vector of x = (9, 4, 15, 8) can be represented by a cdf. The cdf of distribution x is denoted by F_x

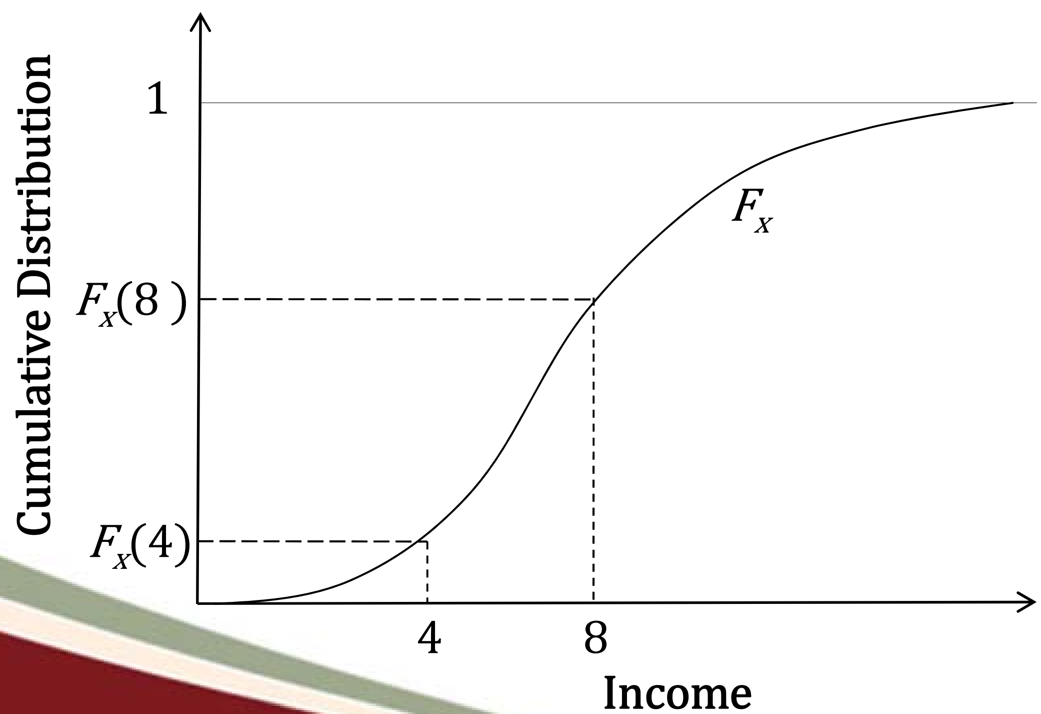


$$F_x = \begin{cases} 0 & \text{for } b<4 \\ 1/4 & \text{for } 4\leq b<8 \\ 2/4 & \text{for } 8\leq b<9 \\ 3/4 & \text{for } 9\leq b<15 \\ 1 & \text{For } b\geq 15 \end{cases}$$

Preliminaries

Cumulative Distribution Function (CDF)

For a society with large population size, a typical cdf looks like



What does a CDF tell us?

It tells us the share of the population having income less than a particular income level

e.g., $F_x(4)$ is the share of the population having income less than $4

Preliminaries

A policy maker is generally interested in the following three aspects of a distribution or a vector

- Size (Welfare), e.g. per-capita income
- Spread (Inequality), e.g. Gini coefficient
- Base (Poverty)
  - Welfare of the population below a certain level of income

In this summer school, we focus on the third aspect

Unidimensional Poverty Measurement

Poverty Measurement

Unidimensional poverty measurement involves two steps (Sen 1976): Identification and Aggregation

Identification: Who is poor?

This step dichotomises the population into *poor* and *non-poor*.

The main tool: Poverty Line (z)

Person i is poor if $x_i < z$ and is non-poor if $x_i \geq z$

x_i is the i^{th} element of vector x

Types of Poverty Lines

***Absolute Poverty Line* (z_a):** Does **not** depend on the **size** of the entire distribution. Rather usually based on the cost of a set of goods and services considered necessary for having a satisfactory life.
Example: a food poverty line: 2100 calories a day equivalent of consumption expenditure.

***Relative Poverty Line* (z_r):** Depends on the **size** of the entire distribution.
Example: half of the median income.

***Hybrid Poverty Line* (z_h):** Combinations of absolute and relative poverty lines.
Examples: $z = (z_r)^{\rho}(z_a)^{(1-\rho)}$ for $0 < \rho < 1$ (Foster, 1998);
$z = \max(z_a, \alpha + kM)$, with M being the median income, (Ravallion and Chen, 2009).

Significance of Poverty Line (z)

Poverty line enables policy makers to identify a group of people, who are subject to different social assistance programs or 'targeting'

Poverty line as 'a benchmark': the objective of a policy maker should be to bring the poor at z

- For poverty analysis, thus, additional achievement of the non-poor **above** the poverty line is ignored

Significance of Poverty Line (z)

Censored Distribution of Achievements

Having z as benchmark, allows us to create a censored distribution of x, referred as x^*, where

$$x_i^* = x_i \text{ if } x_i < z$$

and $$x_i^* = z \text{ if } x_i \geq z$$

Example: If $z = 10$ and $x = (9, 4, 15, 8)$, then $x^* = (9, 4, 10, 8)$

Second Step: Aggregation

Aggregation: *How poor is the society?*

This step construct an index of poverty summarizing the information in the censored achievement vector x^*.

For each distribution *x* and poverty line z, P(x;z) or P(x^*) indicates the level of poverty in the distribution.

We will adopt an absolute z approach and focus the discussion in terms of the indices

Axioms

Axioms
(Classification of Foster, 2006)

- Invariance Axioms
- Dominance Axioms
- Continuity
- Subgroup Axioms (Consistency and Decomposability)

Invariance Axioms

Symmetry *(Anonymity):* If vector y is obtained from vector x by a *permutation* of incomes and the poverty line remains unchanged, then $P(y;z) = P(x;z)$

y is obtained from x by a ***permutation*** of incomes if $y = Px$, where P is a permutation matrix.

Example: $z = 10$, $x = (9,4,15,8)$ and $y = (9,15,4,8)$

Why is this axiom imortant?

Invariance Axioms

Permutation Matrix

A square matrix with entries 0 or 1, with rows and columns summing up to one

Example:

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

Invariance Axioms

Replication Invariance *(Population Principle)*: If vector y is obtained from vector x by a *replication* and the poverty line remains unchanged, then P(y;z) = P(x;z)

y is obtained from x by a ***replication*** if the incomes in y are simply the incomes in x repeated a finite number of times.

Example: z = 10, x = (9,4,15,8), y = (9,9,4,4,15,15,8,8)

Why is this axiom important?

Invariance Axioms

Focus: If y is obtained from x by *an increment to a non-poor person's income* and the poverty line remains unchanged, then P(y;z) = P(x;z)

Example: z = 10, x = (9,4,15,8), y = (9,4,16,8)

Income of the non-poor person increases from \$15 to \$16, which should not alter a poverty index

Why is this axiom important?

Invariance Axioms

Scale Invariance *(Homogeneity of Zero-Degree)*: If all incomes in vector x and the poverty line z are changed by the same *proportion* $\alpha > 0$, then $P(\alpha x;\alpha z)=P(x;z)$.

Example: $z = 10$ and $x = (9,4,15,8)$; if $\alpha = 2$, then $\alpha z = 20$ and $\alpha x = (18,8,30,16)$

Why is this axiom important?

Invariance Axioms

Normalization: As long as everybody is non-poor in vector x for any poverty line z, then P(x;z) = 0. In other words, if $\min\{x\} \geq z$, then P(x;z) = 0

Example 1: z = 4 and x = (9,4,15,8)

Example 2: z = 2 and x = (9,4,15,8)

Dominance Axioms

Monotonicity: If y is obtained from x by a decrement of incomes among the poor and the poverty line remains unchanged, then P(y,z) > P(x,z)

Example 1: z = 10, x = (9,4,15,8); y = (9,4,15,7)

Example 2: z = 10, x = (4,8,9,15); y = (3,8,6,15)

Why is this axiom important?

Dominance Axioms

Transfer: If y is obtained from x by a progressive transfer among the poor, then $P(y;z) < P(x;z)$.

If income is transferred from a person to another who is not richer than the former, keeping mean income same, the transfer is called a *progressive transfer*

Example: $z = 10$, $x = (9,4,15,8)$; $y = (9,5,15,7)$

Dominance Axioms

Transfer: *Is there a limit on the amount of transfer in this axiom?*

There is a limit on the amount of transfer. Post-transfer income cannot fall below the lower pre-transfer income

What is the implication of this axiom for non-transferable dimensions?

Continuity

A technical assumption. It prevents poverty measures from changing abruptly for changes in distribution of achievements

Continuity: For any sequence x , if x' converges to x, then $P(x';z)$ converges to $P(x;z)$

Continuity

Example: $x' = (9,4,15,8)$ and $z = 10$

Suppose the income of the poorest person with income $4 is increased gradually

No change in H is observed until the income reaches $10 [$x = (9,10,15,8)$]

Then, H falls suddenly from 3/4 to 2/4 without taking any intermediate value

Subgroup Axioms

Subgroups

Suppose the population size of vector x is denoted by $n(x)$. Vector x is divided into two population subgroups: x' with population size $n(x')$ and x'' with population size $n(x'')$ such that $n(x) = n(x') = n(x'')$

Example: Let $x = (9,4,15,8)$, $x' = (9,4)$, $x'' = (15,8)$

Then, $n = 4$, $n(x') = n(x'') = 2$.

Subgroup Axioms

Subgroup Consistency: If $P(y';z) > P(x';z)$ and $P(y'';z) = P(x'';z)$, and $n(x') = n(y')$, $n(y'') = n(x'')$, then $P(y;z) > P(x;z)$

Example: Let $z=10$, $x=(9,4,15,8)$, $x'=(9,4)$, $x''=(15,8)$

Say y' is obtained from x' such that $y'=(6,4)$, while y'' is obtained from x'' such that $y'' = x''$

Then $P(y';z) > P(x';z)$ by *monotonicity*, but $P(y'';z) = P(x'';z)$, and so one would expect $P(y;z) > P(x;z)$

Subgroup Consistency

For which practical reason may this axiom be important?

– Evaluation of poverty reduction programs!

– It can be seen as an extension of monotonicity

Monotonicity requires poverty to fall when one person's poverty level is reduced. SC requires aggregate poverty to fall when one group's poverty level is reduced

However, a reduction in one group's poverty may be accompanied by both increase and fall in individual incomes

Subgroup Axioms

Additive Decomposability: A poverty measure is additive decomposable if:

$$P(x)=\frac{n(x')}{n}P(x')+\frac{n(x'')}{n}P(x'')$$

(Extendable to any number of groups)

Then, one can calculate the contribution of each group to overall poverty:

$$C(x')=\frac{n(x')P(x')}{nP(x)}$$

Additive decomposability implies subgroup consistency, but the converse does not hold

Subgroup Consistency & Additive Decomposability

P is a continuous, subgroup consistent poverty index if and only if P is a continuous, increasing transformation of a continuous, decomposable poverty index.

(Foster and Shorrocks, 1991)

Other Advanced Axioms

Transfer Sensitivity: If a transfer $t > 0$ of income takes place from a poor person with income x_i to a poor person with income x_i+d, then the magnitude of the increase in poverty must be smaller for larger x_i. (Kakwani, 1980)

In other words, a poverty measure that satisfies transfer-sensitivity places greater emphasis on progressive (regressive) transfers at the lower end of the distribution of poor than at the upper end of the distribution of poor

Poverty Measures

Classification of Measures

Basic Measures

Headcount Ratio

Income Gap Ratio

Poverty Gap Ratio

Advanced Measures

Squared Poverty Gap (Foster-Greer-Thorbecke)

Sen-Shorrocks-Thon Measure

Watts Measure

Clark-Hemming-Ulph-Chakravarty Class of Measures

Basic Poverty Measures

The Headcount Ratio (H)

The most commonly and widely used measure of poverty

It reports the proportion of the population that is poor

Thus, *ranges between 0 and 1*

If q is the number of poor in vector x with population size n, then $H = q/n$

Example: Let $z = 10$ and $x = (9,4,15,8)$, then $H = 3/4$

Basic Poverty Measures

The Headcount Ratio (H)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, scale invariance, focus, normalization, and subgroup consistency

Does not satisfy – monotonicity, transfer, transfer sensitivity, continuity

Policy Implication?

Encourages a policy maker, with limited budget, to assist the marginally poor only instead of the severely poor

Basic Poverty Measures

Income Gap Ratio (I)

It reports the average normalized income shortfall of the poor from the poverty line; *ranges between 0 and 1*

The average normalized income shortfall of the i^{th} poor is $(z - x_i)/z$. The average income shortfall of the poor is: $I = (1/q)\Sigma_q(z - x_i)/z = (z - \mu_p)/z$

Example: $x=(9,4,15,8)$; $z=10$; $\mu_p=(4+8+9)/3=7$;

Thus, $I = (10 - 7)/10 = 0.3$

Basic Poverty Measures

Income Gap Ratio (I)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, scale invariance, focus, normalization, monotonicity and subgroup consistency

Does not satisfy –transfer, transfer sensitivity, continuity

Policy Implication?

Counter intuitive: if a poor person's income increases and the person becomes non-poor, poverty increases!

Basic Poverty Measures

Poverty Gap Ratio (PG)

This measure repairs some of the problems of the headcount ratio and income gap ratio

It reports the average normalized income shortfall from the poverty line using the *censored* distribution x^*

The average normalized shortfall of the i^{th} person is $g_i^* = (z - x_i^*)/z$. The average income normalized shortfall:

$$PG = (1/n)\Sigma_n \, g_i^* = (z - \mu^*)/z = \mathbf{H \times I}$$

Basic Poverty Measures

Poverty Gap Ratio (PG)

Example: $x=(9,4,15,8)$; $z=10$. Then $x^{*}=(9,4,\mathbf{10},8)$ and $g^{*} = (0.1, 0.6, 0, 0.2)$. So, PG = 0.9/4 = 0.225

Alternatively, μ^{*} = average of elements in x^{*}. So, μ^{*} = 7.75. Thus, PG = (10 – 7.75)/10 = 0.225

PG *ranges between 0 and 1*.

1: when everybody is poor with *no income* at all

0: when there is no poor

Basic Poverty Measures

Poverty Gap Ratio (PG)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, scale invariance, focus, normalization, monotonicity, continuity and subgroup consistency

Does not satisfy –transfer, transfer sensitivity

Policy Implication?

Does not encourage a policy maker to distinguish between a marginally poor and severely poor while assisting

Advance Poverty Measures

Squared Poverty Gap (SG)

It reports the average of squared normalized income shortfalls from the poverty line using the *censored* distribution x^*. Also, known as *Foster-Greer-Thorbecke* measure

The average normalized shortfall of the i^{th} person is $g_i^* = (z - x_i^*)/z$. The average of squared normalized income shortfalls:

$$SG = (1/n)\Sigma_n (g_i^*)^2$$

Advance Poverty Measures

Squared Poverty Gap (SG)

Example: x=(9,4,15,8); z=10. Then x^*=(9,4,**10**,8) and $g^* = (0.1,0.6,0,0.2)$ and squares of poverty gap are $sg^* = (0.1^2,0.6^2,0^2,0.2^2) = (0.01,0.36,0,0.04)$.

Thus, SG = 0.41/4 = 0.102

SG *ranges between 0 and 1*.

1: when everybody is poor with *no income* at all

0: when there is no poor

Advance Poverty Measures

Squared Poverty Gap (SG)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, transfer, and subgroup consistency

Does not satisfy –transfer sensitivity

This measure can be presented as: $SG = H[I^2 + (1 - I)^2 \times C_p^{\,2}]$, where C_p is the coefficient of variation of income across the poor. *Policy implication: care for the severe poor first*

Foster-Greer-Thorbecke (FGT) Class

The FGT class of measures is defined as

$$FGT_\alpha = (1/n)\Sigma_n (g_i^*)^\alpha$$

where α is a parameter and g_i^* is the normalized income gap of the i^{th} person in x^*

For $\alpha = 0$, FGT is the Headcount Ratio

For $\alpha = 1$, FGT is the Poverty Gap Ratio

For $\alpha = 2$, FGT is the Squared Poverty Gap

Advance Poverty Measures

Sen-Shorrocks-Thon Measure (SST)

Like SG, SST is also sensitive to inequality across the poor and is written as

$$SST = HI + (1 - HI) \times \mathit{Gini}(x^*)$$

where $\mathit{Gini}(x^*)$ is the Gini Coeffient of the censored distribution

It lies between zero and one.

Advance Poverty Measures

Sen-Shorrocks-Thon Measure (SST)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, and transfer

Does not satisfy – transfer sensitivity, subgroup consistency

Not subgroup consistent means it is possible that poverty of a region *falls* and that of other remains *unchanged* but the overall poverty of the society as a whole increases

Advance Poverty Measures

Watts Measure (W)

Watts (1968) proposed the following measure of poverty

$$W = (1/n)\Sigma_n (\ln z - \ln x_i^*) = (1/n)\Sigma_n \ln(z/x_i^*)$$

where ln stands for *natural logarithm*

The lower bound of the measure is zero, it does not have any upper bound

Advance Poverty Measures

Watts Measure (W)

What axioms does this measure satisfy?

Satisfies - symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, transfer, transfer sensitivity, and subgroup consistency

Also, Additively Decomposable

A perfect measure except for few *limitations*

Advance Poverty Measures

Watts Measure (W)

Limitations?

1. It is only defined for positive income, zero incomes are not allowed
2. It does not have nice intuitive interpretation or breakdown like other poverty measures
3. It does not have any upper bound like other measures

Advance Poverty Measures

Clark-Hemming-Ulph-Chakravarty Class (CHUC)

The CHUC class of poverty measures are defined as:

$$C_\beta(x;z) = \begin{cases} 1 - \left[\dfrac{1}{n} \sum_{i=1}^{n} \left(\dfrac{x_i^*}{z} \right)^\beta \right]^{1/\beta} & \beta \leq 1, \beta \neq 0 \\ 1 - \left[\dfrac{1}{n} \prod_{i=1}^{n} \left(\dfrac{x_i^*}{z} \right) \right]^{1/n} & \beta = 0 \end{cases}$$

For $\beta = 0$, monotonic transformation of W

For $\beta = 1$, Poverty Gap Ratio

Advance Poverty Measures

Clark-Hemming-Ulph-Chakravarty Class (CHUC)

What axioms does this measure satisfy?

For $\beta<1$, satisfies - symmetry, replication invariance, focus, scale invariance, normalization, monotonicity, continuity, transfer, transfer sensitivity, and subgroup consistency.

Cons: The intuitive interpretation of the measure for $\beta<1$ is not so easy

Unidimensional Dominance

Main Sources of this Lecture

- Foster and Shorrocks (1988)
- Foster and Sen (1997), Annexe of "On Economic Inequality".
- Atkinson (1987)
- There are others: please see the readings list.

When measuring poverty…

Identification → Poverty Line

Aggregation → Poverty Measure

Entail some arbitrarieness

Which Poverty Line?

Does the choice of poverty line alters the ranking of distributions? Yes

Example: Consider two distributions:

$$x = (4,8,9,15) \text{ and } y = (3,6,12,17)$$

Which distribution has more poverty by H, if z =7?

$$H(x;z) = 1/4 \text{ and } H(y;z) = 2/4$$

Which distribution has more poverty by H, if z =10?

$$H(x;z) = 3/4 \text{ and } H(y;z) = 2/4$$

Which Measure?

Does all measures rank any two distributions in the same manner? No

Example: Consider the same two distributions:

$x = (4,8,9,15)$ and $y = (3,6,12,17)$

Which distribution has more poverty by H, if $z = 10$?

$H(x;z) = 3/4$ and $H(y;z) = 2/4$

Which distribution has more poverty by PG, if $z = 10$?

$PG(x;z) = 0.225$ and $PG(y;z) = 0.275$

Dominance Approach

To verify if one distribution dominates another distribution, do we need to calculate all measures for all possible poverty lines?

– This would be a tedious task!

Is there any useful tool for these purposes?

– Yes. A tool known as **Stochastic Dominance**
– This is closely linked to poverty ordering, where we rank different distributions

Dominance Approach

Two main types of poverty orderings:

1. Variable-line poverty orderings (focus on the identification step)

2. Variable-measure poverty orderings (address aggregation).

Variable-Line Poverty Orderings (Foster and Shorrocks, 1988)

Main procedure:

1. Choose a measure

2. Identify the condition that two distributions must satisfy so as to be able to say that one has more poverty than the other.

Definition of Poverty Ordering

x**P**y if and only if

P(x;z) ≤ P(y;z) for all z and

P(x;z) < P(y;z) for some z

x**P**y means that x has *unambiguously* **less poverty than** y with respect to poverty index P.

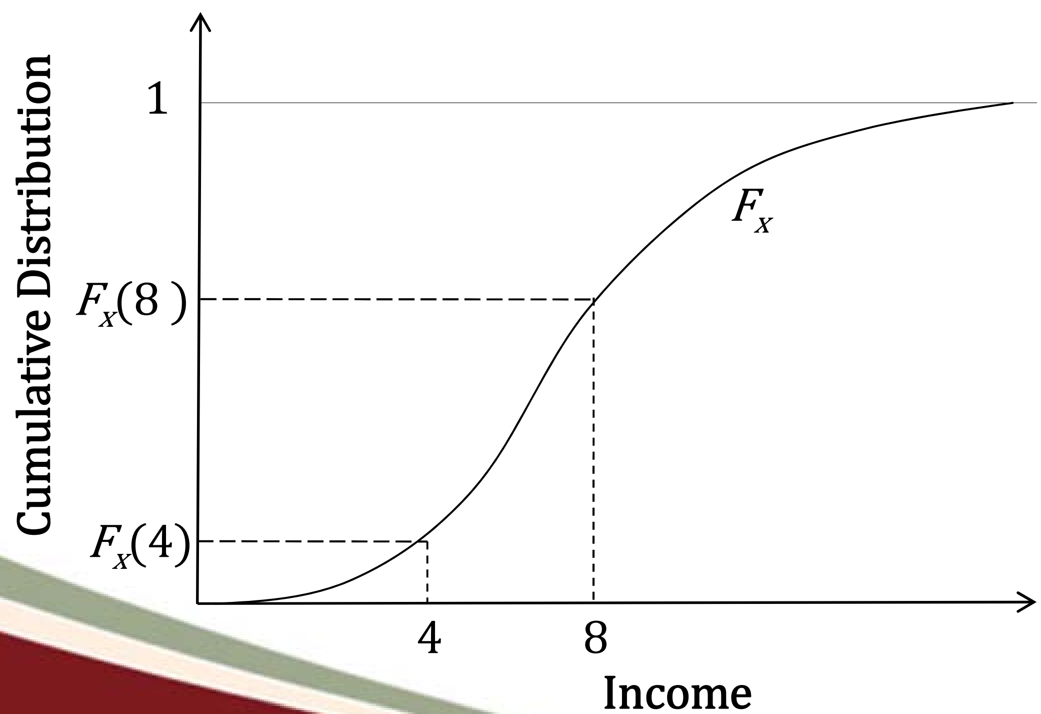
FGT Poverty Orderings

Foster and Shorrocks (1988) developed the conditions of poverty orderings for three members of the FGT family: H, PG and SG

Poverty Ordering Based on H

Recall the Concept of CDF

For a society with large population size, a typical cdf looks like



What does a CDF tell us?

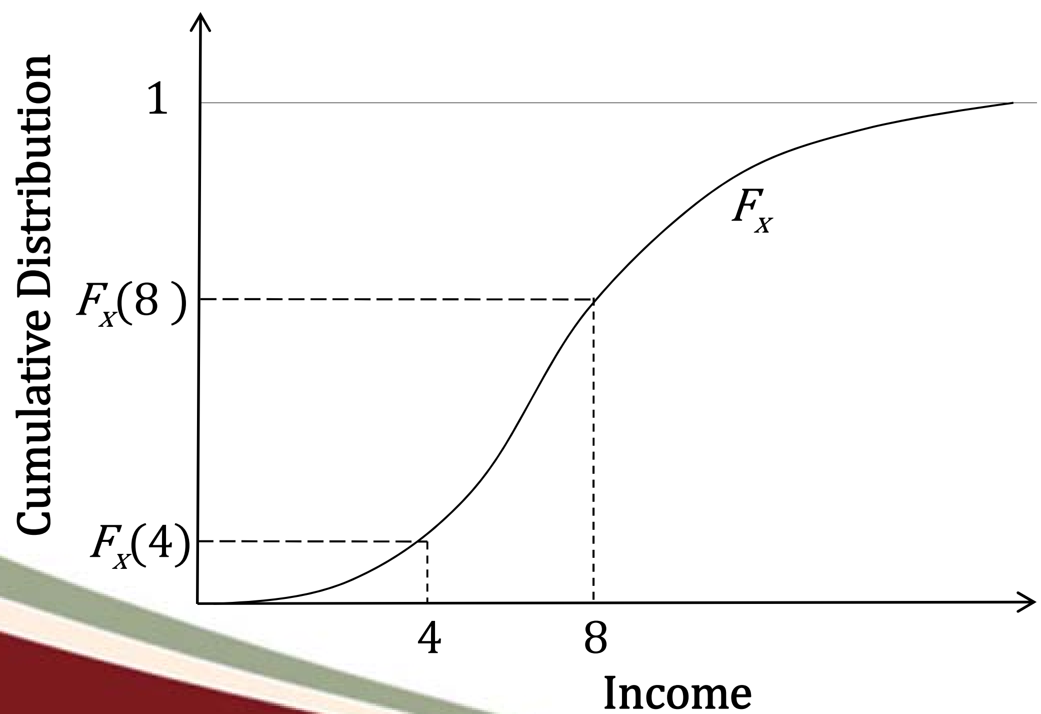
It tells us the share of the population having income less than a particular income level

e.g., $F_x(4)$ is the share of the population having income less than \$4

Poverty Ordering Based on H

Recall the Concept of CDF

For a society with large population size, a typical cdf looks like



If the poverty line is z=4, then what is H?

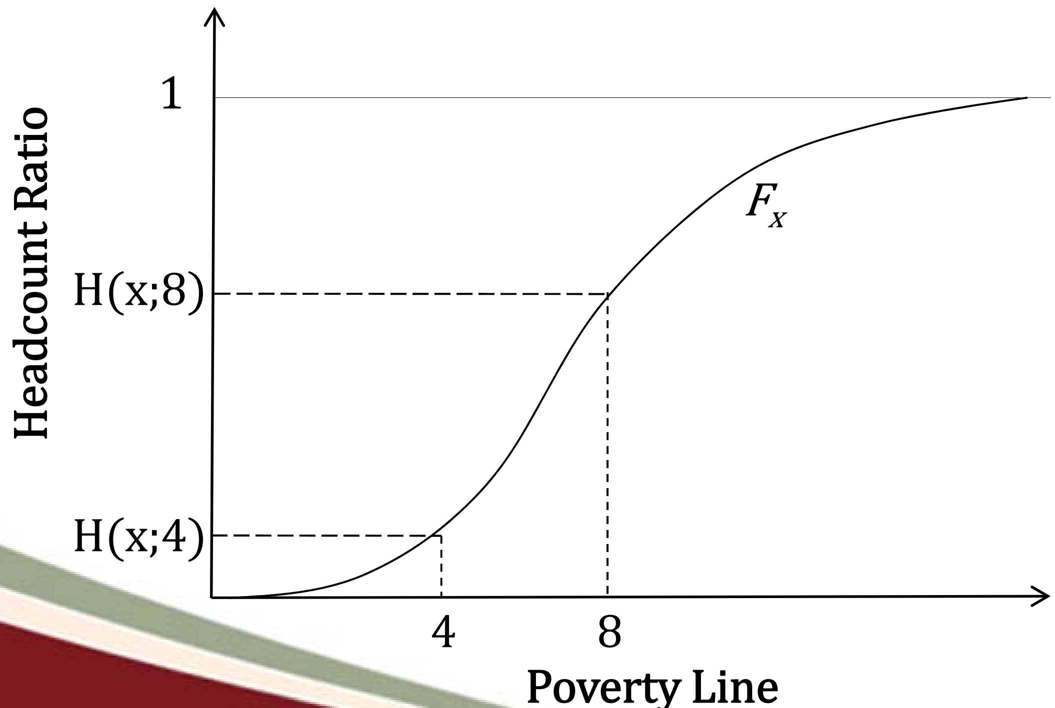
H is equal to $F_x(4)$ in this situation.

The diagram to the left may be presented as:

Poverty Ordering Based on H

Recall the Concept of CDF

For a society with large population size, a typical cdf looks like



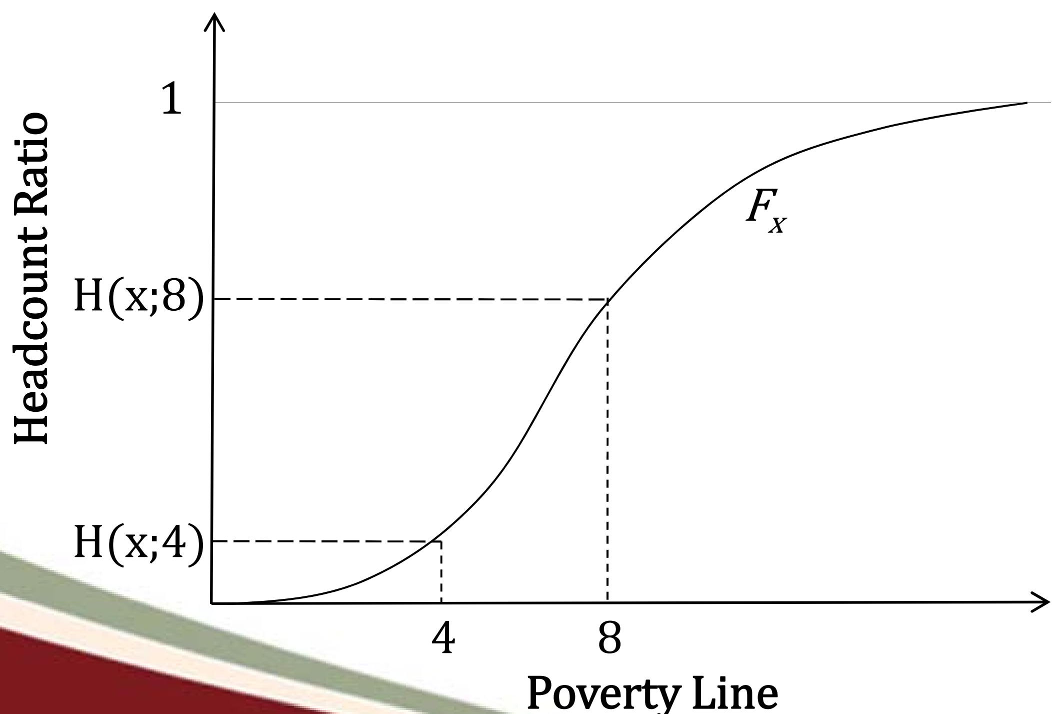
If the poverty line is z=4, then what is H?

H is equal to $F_x(4)$ in this situation.

The diagram to the left may be presented as:

Poverty Ordering Based on H

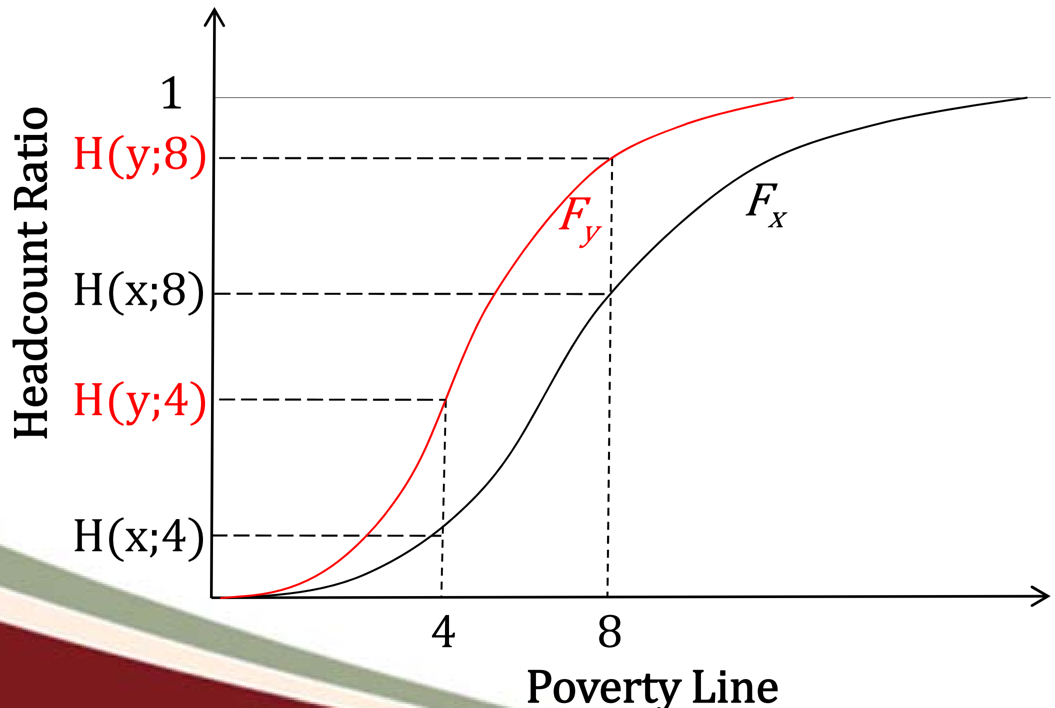
For any poverty line z, the CDF of x gives the headcount ratio



If the cdf of another distribution y, F_y, lies no where to the right of the cdf of x, then y has no lower headcount ratio than x for each and every poverty line

Poverty Ordering Based on H

For any poverty line z, the CDF of x gives the headcount ratio



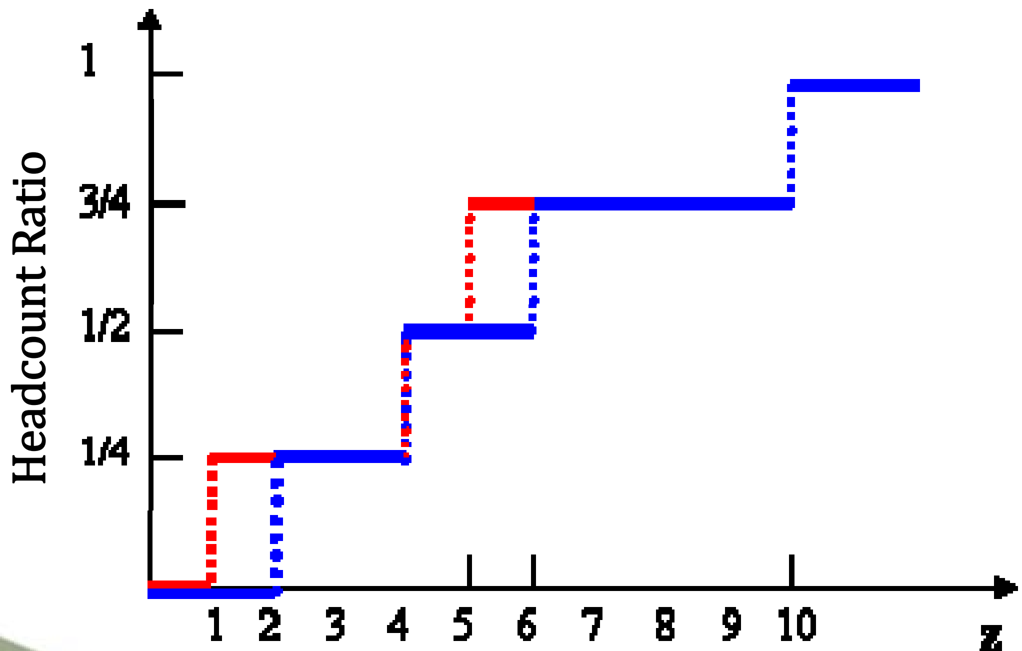
If the cdf of another distribution y, F_y, lies no where to the right of the cdf of x, then y has no lower headcount ratio than x for each and every poverty line

First order Stochastic Dominance (FSD)

x FSD y

Poverty Ordering Based on H

Example of FSD: Let x=(2,4,6,10) and y=(1,4,5,10)

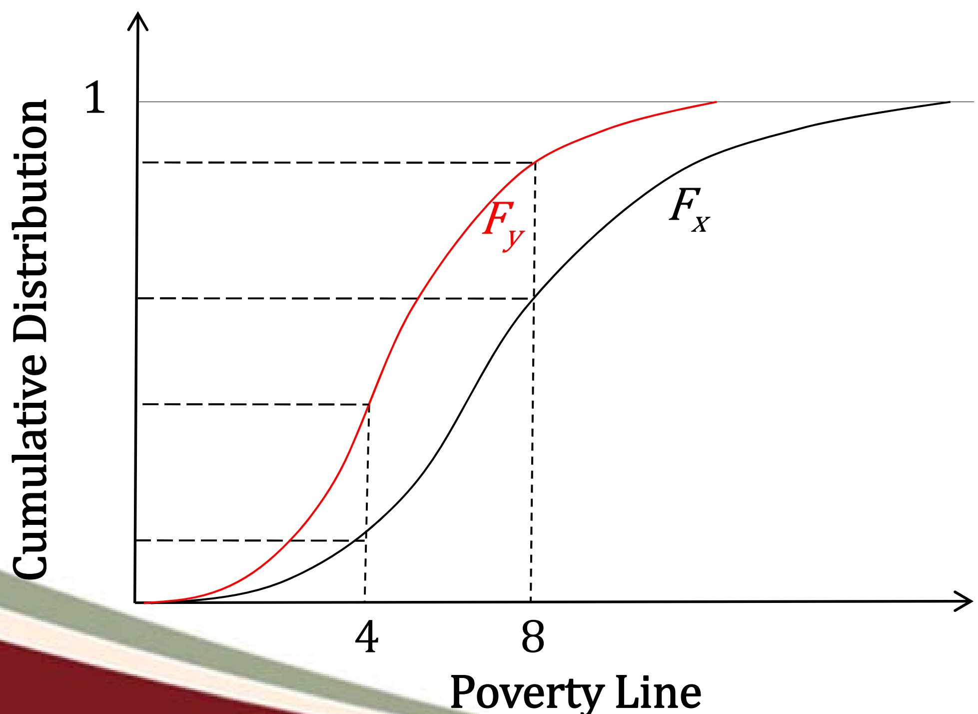


No part of y lies to the right of y

Thus, x FSD y in this case, which means x has unambiguously less poverty than y according to H

Definition of FSD

For distributions x and y, x FSD y if and only if $F_x(b) \leq F_y(b)$ for all b and $F_x(b) < F_y(b)$ for some b, where b is income



How strong is the FSD result?

If FSD holds, then there is agreement for *all* continuous poverty measures satisfying symmetry, focus, scale and replication invariance and monotonicity for all z.

What Happens When CDFs Cross?

In this case, FSD does not hold for all z and H no more provides an unambiguous ranking

Can distributions be ranked in this situation based of other measures for all z?

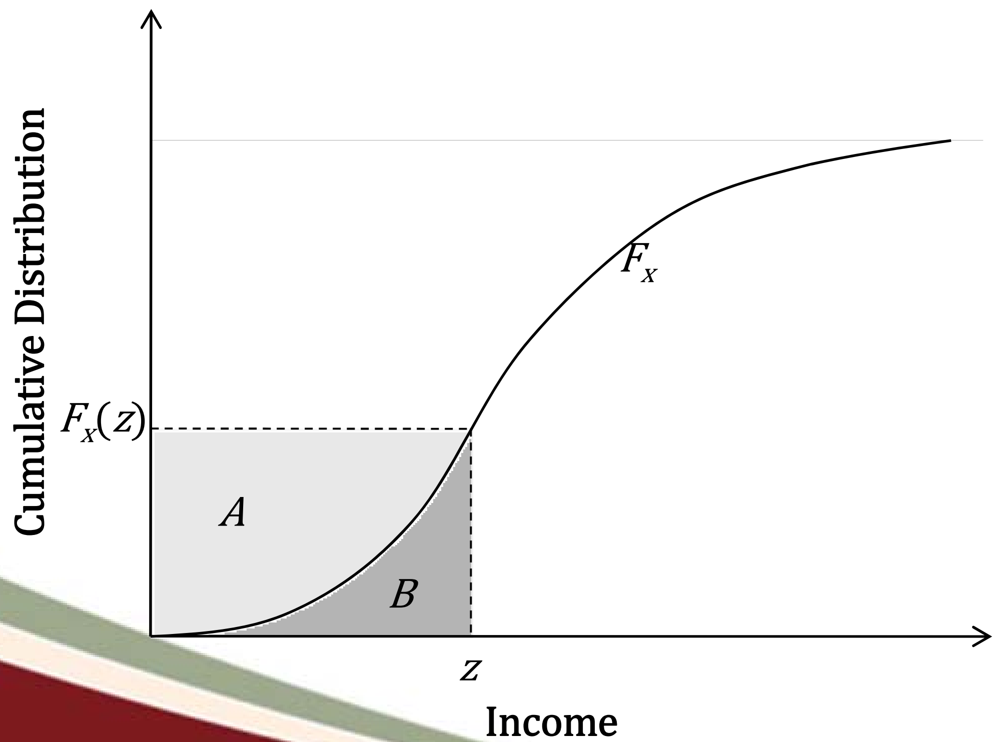
Second order stochastic dominance (SSD)

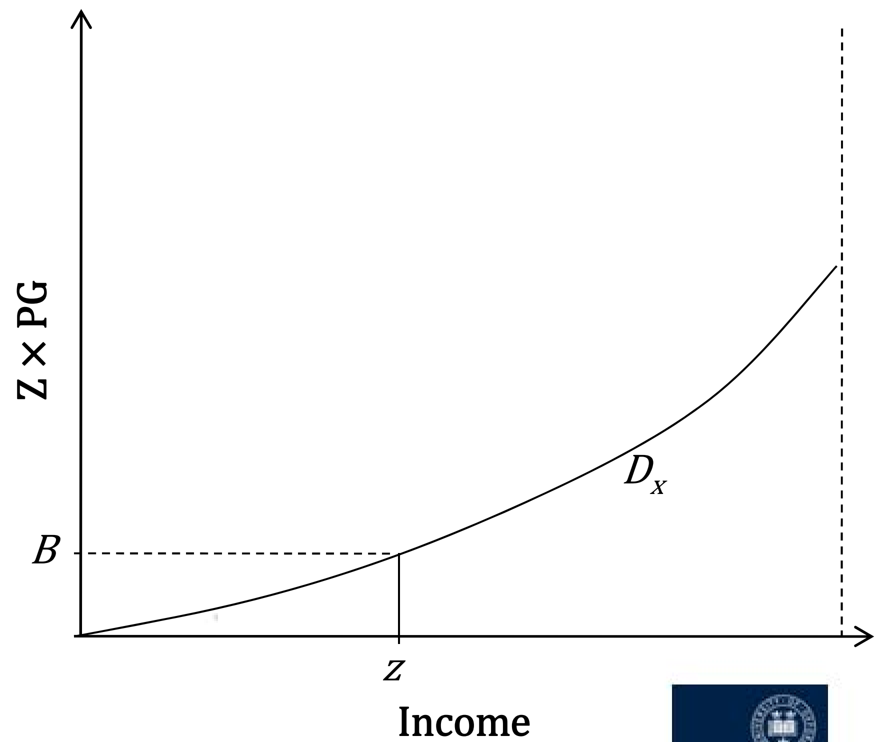
This requires comparing the area under the CDFs

The area under a CDF is closely linked to **PG** (Foster and Shorrocks, 1988)

Poverty Ordering Based on PG

Area B under CDF of x is equal to z times PG

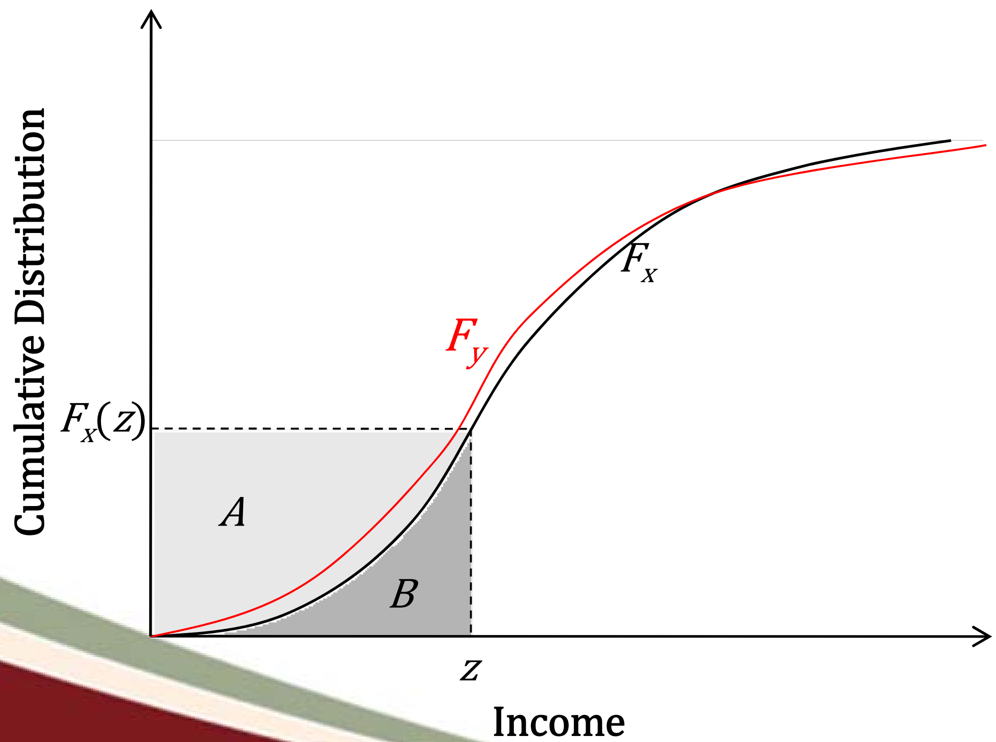


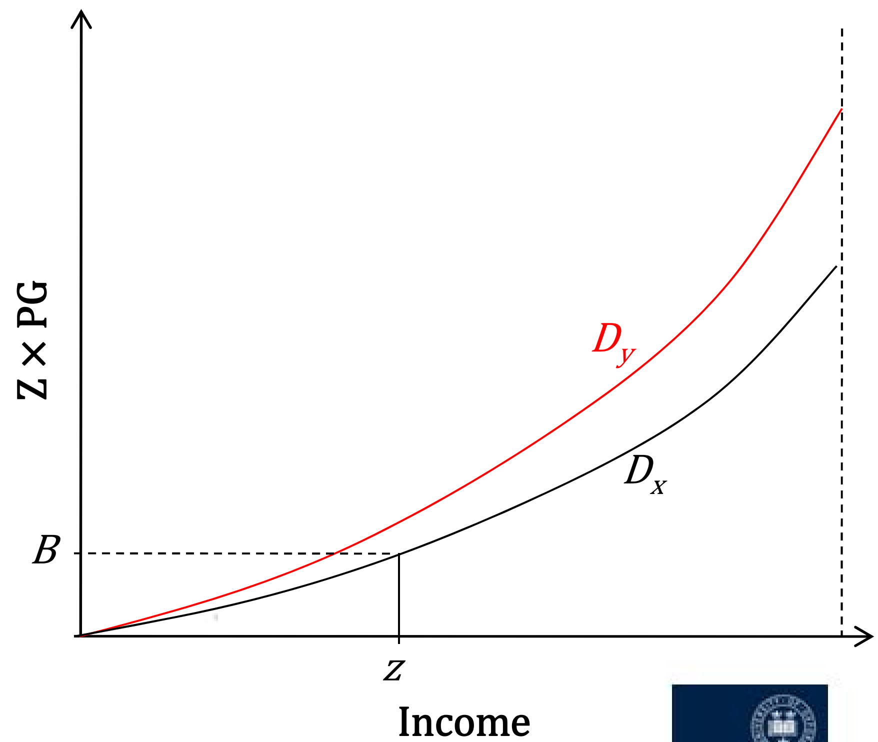


Poverty Ordering Based on PG

Area B under CDF of x is equal to z times PG

x SSD y





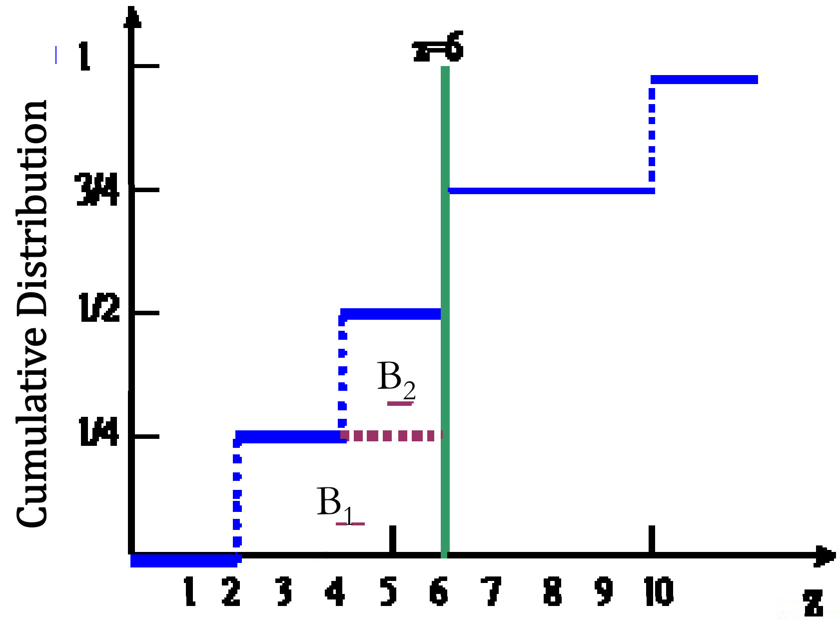
Poverty Ordering Based on PG

How to calculate the area under the CDF?

Example: Let x=(2,4,6,10) and z = 6

Area=(1/4)[(6-2)+(6-4)]=

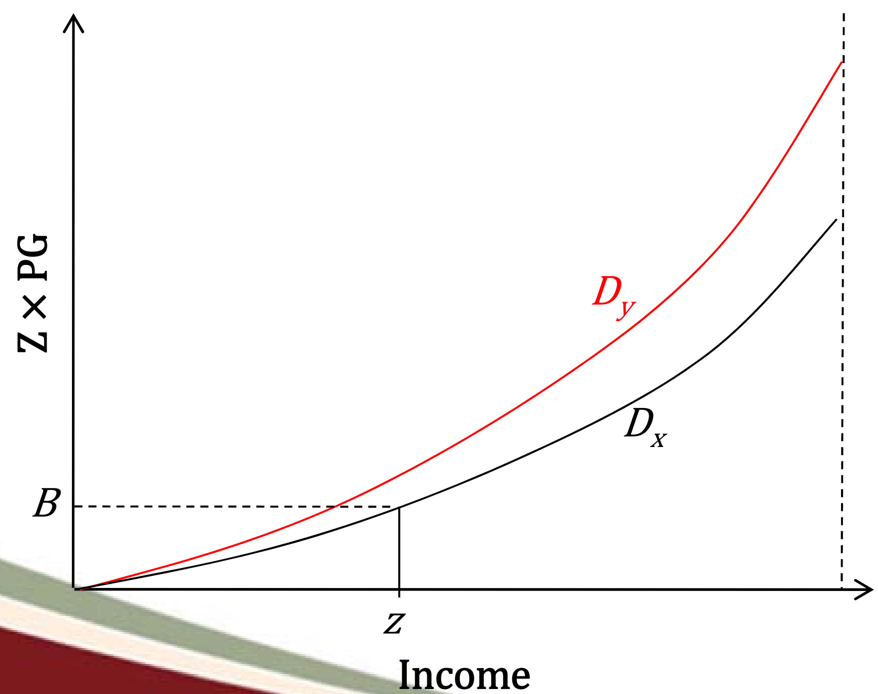
=(B_1+B_2)=1.5



D_x is called deficit curve of x

Definition of SSD

For distributions x and y, x SSD y if and only if $D_x(b) \leq D_y(b)$ for all b and $D_x(b) < D_y(b)$ for some b, where b is income



How strong is the SSD result?

If SSD holds, then there is agreement for *all* continuous poverty measures satisfying symmetry, focus, scale and replication invariance, monotonicity and transfer for all z.

What Happens When D_x and D_y Cross?

In this case, SSD does not hold for all z and PG no more provides unambiguous ranking

Can distributions be ranked in this situation based of other measures for all z?

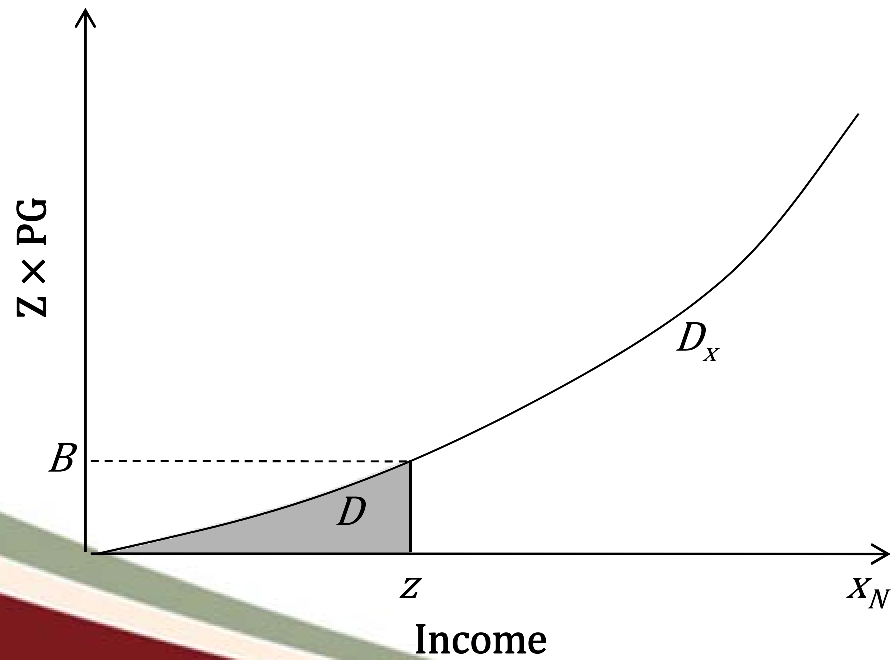
Third order Stochastic Dominance (TSD)

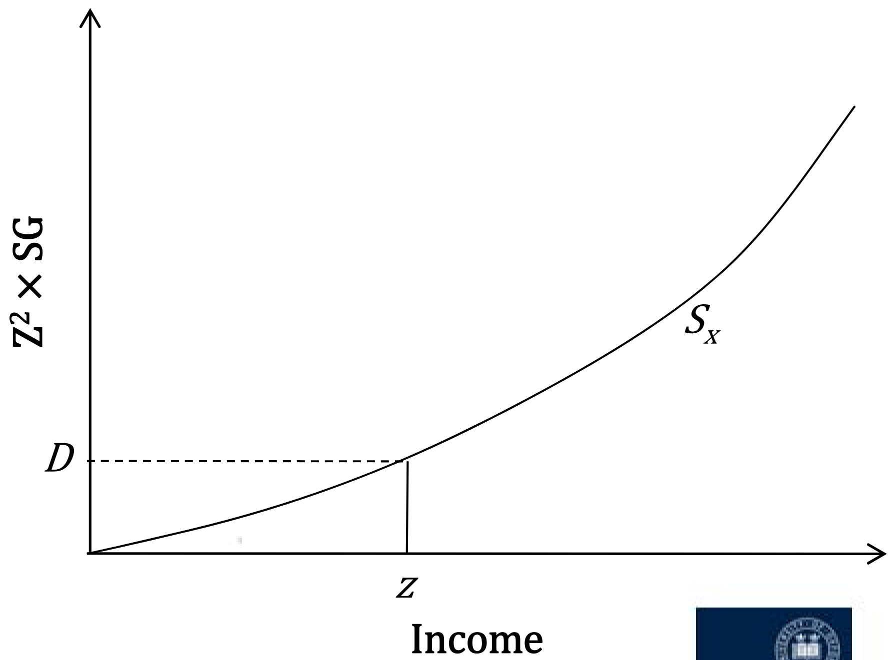
This requires comparing the area under deficit curves

The area under a deficit curve is closely linked to **SG** (Foster and Shorrocks, 1988)

Poverty Ordering Based on SG

Area B under the deficit curve is equal to $z^2 \times SG$

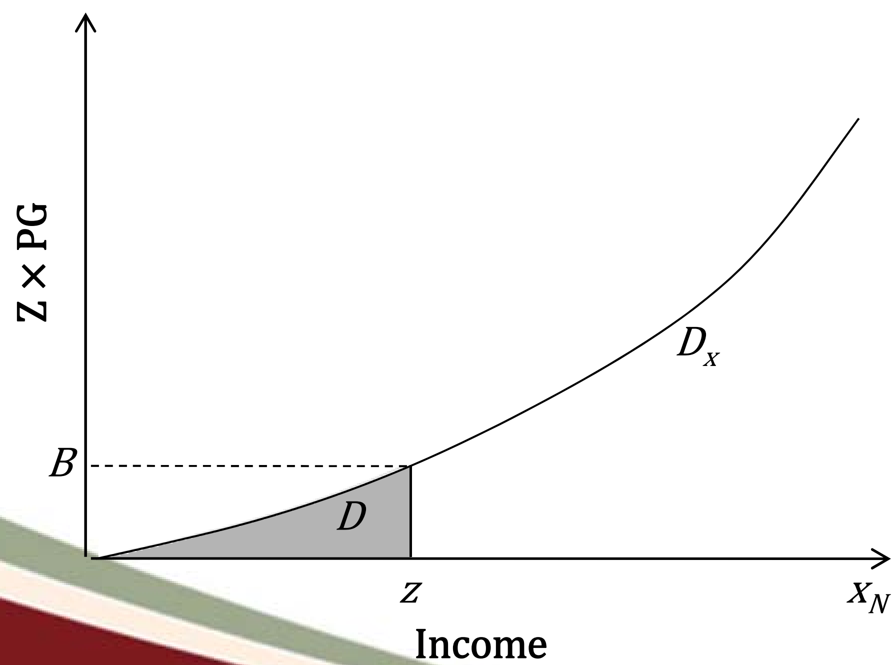


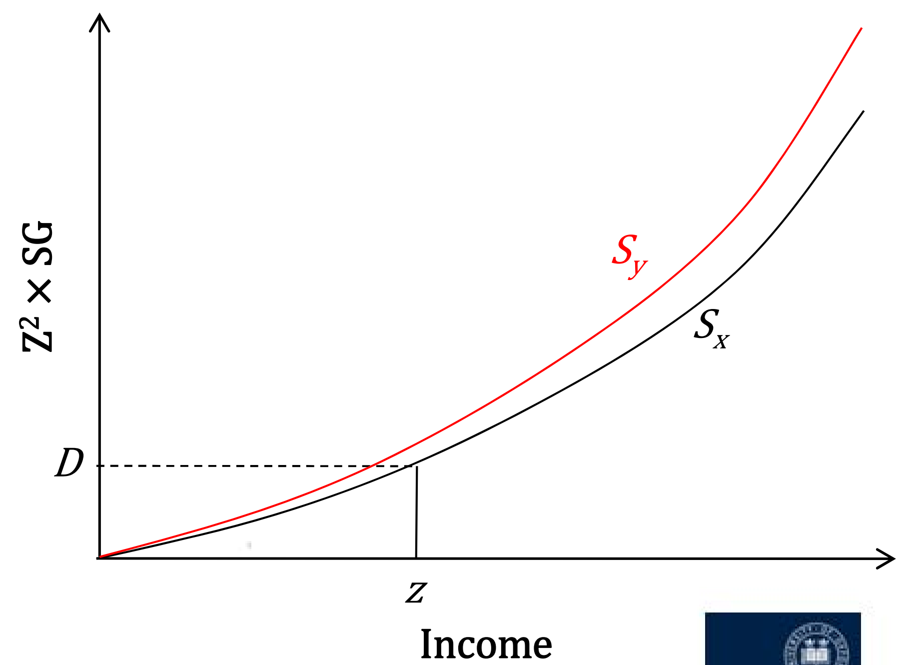


Poverty Ordering Based on SG

Area B under the deficit curve is equal to $z^2 \times SG$

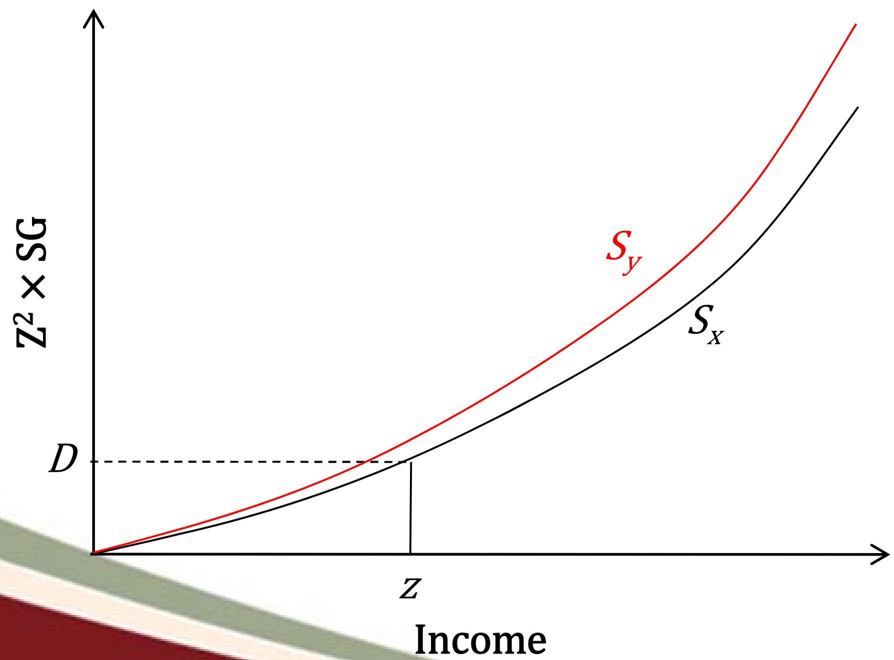
x TSD y





Definition of TSD

For distributions x and y, x TSD y if and only if $S_x(b) \leq S_y(b)$ for all b and $S_x(b) < S_y(b)$ for some b, where b is income



How strong is the TSD result?

If TSD holds, then there is agreement for *all* continuous poverty measures satisfying symmetry, focus, scale and replication invariance, monotonicity, transfer and transfer sensitivity for all z.

Limited Range Poverty Orderings

While deciding the precise value of the poverty line may be difficult, agreement is likely to occur on an interval Z.

So now the poverty ordering would be defined as x**P(Z)**y when

$$P(y;z) \geq P(x;z) \text{ for all } z \text{ in } Z$$

$$\text{and} > \text{for some } z \text{ in } Z$$

By restricting the values of z, the obtained ranking P(Z) will be "more complete" than the P ranking (but less general)

Limited Range Poverty Orderings

Indeed, looking at extremely high poverty lines, does not make sense. So now we can set an upper bound z^*, so that the relevant range is $Z^*=(0,z^*)$, and **P*** being the poverty ordering.

One can, then, work with the censored distribution, ‘ignoring’ incomes above z^*, i.e., replacing them by z^*:

$$x_i(z^*)=\min(x_i,z^*) \text{ for } i=1,\ldots,n(x)$$